Frontispiece]

THE GREAT NEBULA IN ANDROMEDA

This photograph was taken at the Royal Observatory, Greenwich, with a thirty-inch reflector, the exposure being one hour forty minutes, and is reproduced by the courtesy of the Astronomer Royal.

[*See* Chapter VIII

ADMIRALTY NAVIGATION MANUAL

VOLUME III

1938

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The Lords Commissioners of the Admiralty have decided that a new Admiralty Navigation Manual is required for the information and guidance of the Officers of His Majesty's Fleet; for this purpose the present Manual has been compiled in three volumes under the direction of the Captain, H.M. Navigation School, Portsmouth, by the Staff of H.M. Navigation School assisted by the Dean and Staff of the Royal Naval College, Greenwich.

By the publication of this Manual, the Admiralty Manual of Navigation, 1928, is superseded and may be destroyed.

BY COMMAND OF THEIR LORDSHIPS.

*Admiralty, S.W.*1.
October, 1937.

PREFACE

The Admiralty Navigation Manual, 1938, consists of three Volumes :

Volume I is a practical guide for executive officers covering the syllabus laid down for examination in Navigation and Pilotage for the rank of Lieutenant, but omitting the study of nautical astronomy.

Volume II is the text book of nautical astronomy completing the above syllabus.

Volume III is based on the syllabus for officers qualifying in Navigation and deals solely with advanced subjects and mathematical proofs not included in Volumes I and II. It will be unnecessary for executive officers in general to study this Volume.

Thanks are due for the information given by the Hydrographic Department, the Naval and Marine Divisions of the Meteorological Office, various instrument manufacturers, and to Messrs. Rich and Cowan for the use of extracts from *On the Bridge*, by Captain J. A. G. Troup, R.N.

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CHAPTER I

THE SHAPE OF THE EARTH

The navigational problems dealt with in Volume II of this Manual were solved on the assumption that the Earth's shape is spherical, and it was stated that this assumption was justified because the actual departure from the spherical shape is, in general, too small to matter. For the purposes of an accurate survey, however, and also when an accurate estimation of a ship's position is required after a north–south course has been followed for a long

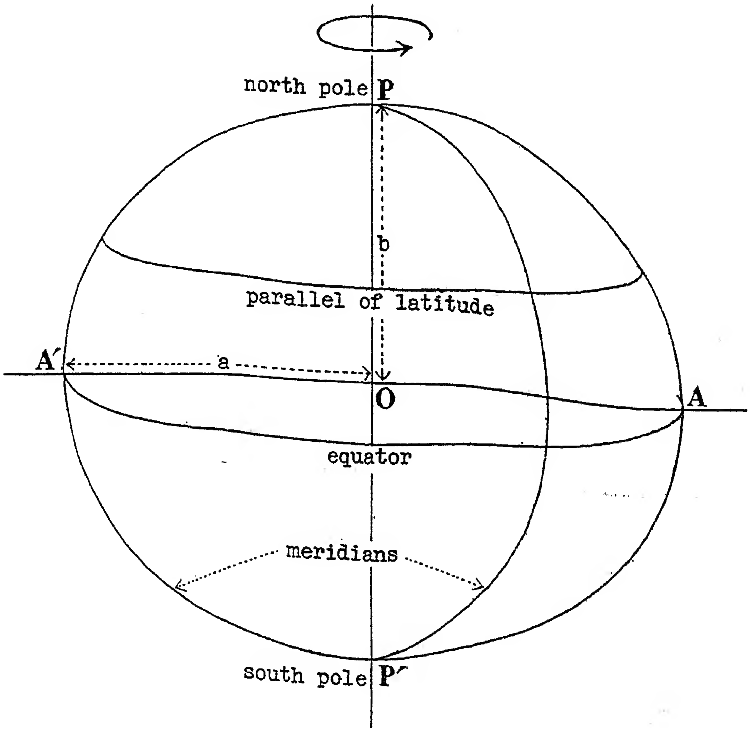


FIGURE 1.

distance in low latitudes, it is not sufficient to assume that the Earth is a sphere: the flattening of the polar regions must be taken into account. This flattening gives the Earth a shape that is approximately ellipsoidal.

Figure 1 shows the ellipsoid that is formed when the ellipse $PAP'A'$ is rotated about POP'. The surface of this ellipsoid corresponds to the sea-level surface of the Earth.

The Ellipticity or Compression. The extremity A of the major axis OA describes the circumference of a circle, the plane of which is at right-angles to the axis POP', and this circle is the equator. P and P' are the north and south poles, and any plane section through them cuts the surface in an ellipse such as $PAP'A'$ itself. PAP' is thus a typical meridian, and it is seen that its radius varies from OP at the pole to OA at the equator.

This polar radius, b, is equal to 3,950·01 statute miles, and the equatorial radius, a, is equal to 3,963·35 statute miles. Hence :

$$(a-b)=13{\cdot}34 \text{ miles}$$

The *ellipticity* or *compression* is defined by the quantity c where :

$$b=a(1-c)$$

i.e.

$$c=\frac{a-b}{\ }$$

The compression is therefore :

$$\frac{13{\cdot}34}{3{,}963{\cdot}35} \text{ or } \frac{\ }{297}$$

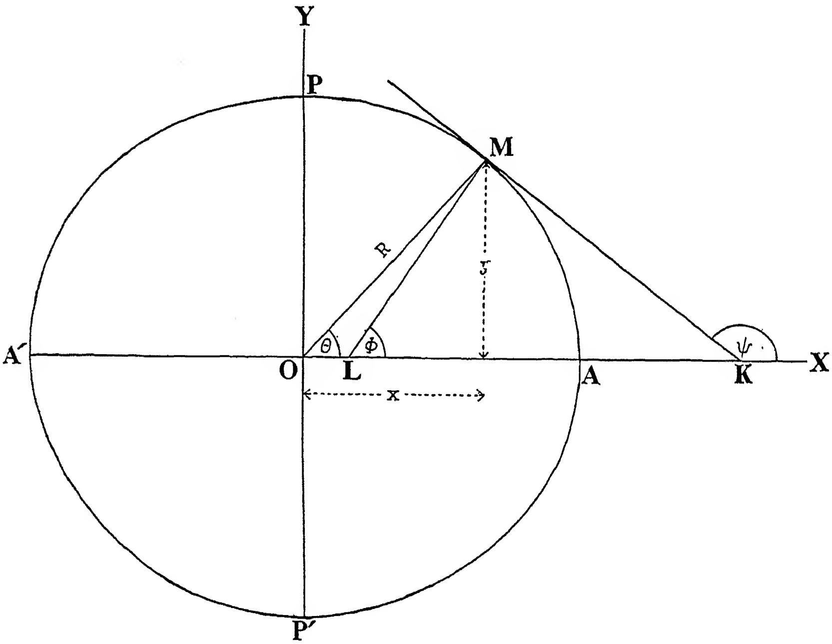


FIGURE 2.

Geographical and Geocentric Latitudes. In figure 2, M is a point on the meridian PAP', and MK is the tangent to the meridian at M. If the normal to this tangent, LM, cuts OA in L, the angle MLA is called the *geographical latitude* of M, and denoted by ϕ. The direction LM is the direction of the zenith of an observer at M.

The angle MOA is called the *geocentric latitude* of M, and is denoted by θ.

Reduction of the Latitude. The geometry of the triangle OML shows that the angle OML is equal to $(\phi-\theta)$. This angle is called the *reduction of the latitude.*

The amount of this reduction is given by the formula $r=c \sin 2\phi$.

To Establish the Formula $r=c$ sin 2ϕ. If the distance of the point M from the polar axis OP is x, and its distance from the major axis OA is y, these distances or co-ordinates are connected by the equation of the ellipse on which M lies ; that is :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

By differentiation :

$$\frac{x}{a^2}+\frac{y}{b^2}\cdot\frac{dy}{dx}=0$$

$$\therefore \qquad \frac{dy}{dx}=-\frac{b^2}{a^2}\cdot\frac{x}{y}$$

If ψ is the angle which the tangent MK makes with the x-axis, then, since the slope of the tangent is measured by the differential coefficient :

$$\tan\psi=\frac{dy}{dx}=-\frac{b^2}{a^2}\cdot\frac{x}{y}$$

But ψ is equal to $(\phi+90°)$ since ML is perpendicular to MK. Hence :

$$\tan\psi=-\cot\phi$$

$$\therefore \qquad \cot\phi=\frac{b^2}{a^2}\cdot\frac{x}{y}$$

$$=\frac{b^2}{a^2}\cot\theta$$

$$=(1-c)^2\cot(\phi-r)$$

i.e.
$$\tan(\phi-r)=(1-c)^2\tan\phi$$

But r is an angle sufficiently small for tan r to be taken as r when expressed in circular measure. Also c is a small quantity. Therefore, if terms in c^2, cr and c^2r are neglected, the formula can be simplified. Thus :

$$\tan\phi-\tan r=(1+\tan\phi\tan r)\,(1-c)^2\tan\phi$$

$$\tan\phi-r=(1+r\tan\phi)\,(1-2c)\tan\phi$$

$$\tan\phi-r=(1+r\tan\phi-2c)\tan\phi$$

$$\therefore \qquad r(1+\tan^2\phi)=2c\tan\phi$$

i.e.
$$r=c\sin 2\phi$$

If r is now expressed in seconds of arc instead of circular measure, the formula becomes:

$$r \sin 1'' = c \sin 2\phi$$

i.e.

$$r = 206{,}265'' \times c \sin 2\phi$$

When c is given its correct value, this expression becomes:

$$r = 694''\cdot 5 \times \sin 2\phi$$

Hence, if ϕ, the geographical latitude, is 0° or 90°, r vanishes, and the geocentric latitude is equal to the geographical.

The reduction has its greatest value when $\sin 2\phi$ is equal to unity; that is when ϕ is equal to 45°. The greatest reduction is thus about 11′·6.

To Find the Radius for a Given Latitude. In figure 2, the required radius is OM, and if this length is denoted by R, it follows that x is $R \cos \theta$ and y is $R \sin \theta$. Hence, by substituting for x and y in the equation of the ellipse:

$$\frac{R^2 \cos^2 \theta}{a^2} + \frac{R^2 \sin^2 \theta}{b^2} = 1$$

—or, since b^2 is equal to $a^2\ (1-c)^2$:

$$R^2[(1-c)^2 \cos^2 \theta + \sin^2 \theta] = a^2(1-c)^2$$

When terms in c^2 are neglected, this equation becomes:

$$R^2(1-2c \cos^2 \theta) = a^2(1-2c)$$

i.e.

$$R = a\left[\frac{1-2c}{1-2c \cos^2 \theta}\right]^{\frac{1}{2}}$$

When the right-hand side is expanded by the Binomial Theorem, the terms in c^2 and higher powers again being omitted, the equation becomes:

$$R = a(1-c)\ (1+c \cos^2 \theta)$$

i.e.

$$R = a(1-c \sin^2 \theta)$$

Also R can be expressed in terms of the geographical latitude without appreciable error by substituting ϕ for θ, since ϕ differs from θ by the quantity r which is small. Thus:

$$R = a(1-c \sin^2 \phi)$$

This equation shows that R is greatest at the equator, where ϕ is 0°, and least at the poles, where ϕ is 90°.

The Length of One Minute of Latitude. In figure 3, M and M' are two points close together on the meridian AP, the geographical latitude of M being ϕ and that of M', $\phi + \triangle\phi$, where $\triangle\phi$ signifies a small increment. The co-ordinates of M are (x, y) and, since

the length of MM' is small, those of M' are $[(x+\triangle x), (y+\triangle y)]$. The length of MM' is denoted by $\triangle s$. Then:

$$\frac{QM'}{MM'}=\sin QMM'=\cos\phi$$

i.e.
$$\frac{\triangle y}{\triangle s}=\cos\phi$$

In the ordinary notation of the calculus, this may be written:

$$\frac{ds}{dy}=\sec\phi$$

—and it gives the limiting ratio of an element of the meridian AP to an element of the y-ordinate. The problem is to find the ratio

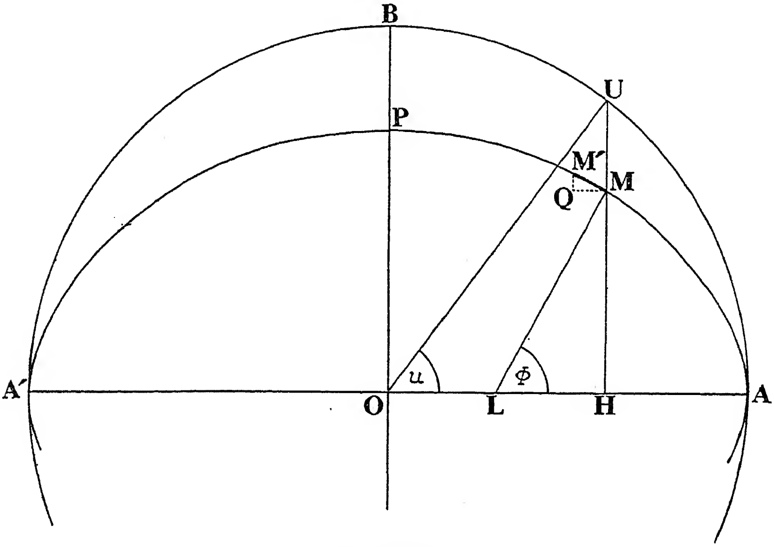


FIGURE 3.

of an element of the meridian to an element of the geographical latitude; that is $ds/d\phi$. But:

$$\frac{ds}{dy}=\frac{ds}{d\phi}\cdot\frac{d\phi}{dy}$$

i.e.
$$\frac{ds}{d\phi}=\sec\phi\,\frac{dy}{d\phi}$$

It is therefore necessary to find $dy/d\phi$.

This can be done by considering the geometry of the circle ABA', of which AA' is a diameter. HM produced cuts this circle in U, and the radius OU makes an angle u with the x-axis. If this constant radius is denoted by a, then:

$$OH=OU\cos u$$

i.e.
$$x=a\cos u$$

From the equation of the ellipse, it follows that :

$$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$$

$$=1-\cos^2 u$$

i.e. $$y=b \sin u$$

On page 3 it was proved that :

$$\tan (\phi-r)=(1-c^2) \tan \phi$$

i.e. $$\frac{y}{x}=(1-c)^2 \tan \phi$$

Hence, by substitution, it follows that :

$$(1-c)^2 \tan \phi=\frac{b \sin u}{a \cos u}$$

$$=(1-c) \tan u$$

i.e. $$\tan u=(1-c) \tan \phi$$

By differentiation :

$$\sec^2 u \frac{du}{d\phi}=(1-c) \sec^2 \phi$$

i.e. $$\frac{du}{d\phi}=(1-c) \sec^2 \phi \cos^2 u$$

But, since y is equal to $b \sin u$:

$$\frac{dy}{du}=b \cos u$$

i.e. $$\frac{dy}{d\phi}=b \cos u \frac{du}{d\phi}$$

Hence, by substitution :

$$\frac{dy}{d\phi}=b(1-c) \sec^2 \phi \cos^3 u$$

$$=\frac{a(1-c)^2 \sec^2 \phi}{(1+\tan^2 u)^{\frac{3}{2}}}$$

But $\tan u$ is equal to $(1-c) \tan \phi$. Therefore :

$$\frac{dy}{d\phi}=\frac{a(1-c)^2 \sec {}^2\phi}{[1+(1-c)^2 \tan {}^2\phi]^{\frac{3}{2}}}$$

$$=\frac{a(1-c)^2 \cos \phi}{[\cos^2 \phi+(1-c)^2 \sin^2 \phi]^{\frac{3}{2}}}$$

$$=a(1-c)^2 \cos \phi[1-2c \sin {}^2\phi+c^2 \sin {}^2\phi]^{-\frac{3}{2}}$$

When this expression is expanded by the Binomial Theorem and powers of c higher than the first are neglected :

$$\frac{dy}{d\phi}=a(1-2c)\cos\phi[1+3c\sin^2\phi]$$
$$=a\cos\phi[1-2c+3c\sin^2\phi]$$
$$=a\cos\phi[1-\frac{c}{2}(1+3\cos 2\phi)]$$

It is now possible to find $\frac{ds}{d\phi}$. Thus, from page 5:

$$\frac{ds}{d\phi}=\sec\phi\frac{dy}{d\phi}$$
$$=a[1-\frac{c}{2}(1+3\cos 2\phi)]$$

$$\therefore \qquad \triangle s=a[1-\frac{c}{2}(1+3\cos 2\phi)]\triangle\phi$$

Now $\triangle\phi$ is a small angle expressed in circular measure. But the problem is to find the length of MM' when $\triangle\phi$ is one minute of arc. Therefore 1 sin 1′ must be substituted for $\triangle\phi$. When this is done, and a and c are given their true values, the length of arc measured along the meridian between the latitudes ϕ and $(\phi+1')$ is found to be :

$$(6{,}077{\cdot}1-30{\cdot}7\cos 2\phi)\text{ feet}$$

This is the theoretical expression for the *nautical mile.*

The Nautical Mile. It is seen from this expression that the nautical mile is, strictly, a quantity which varies with the latitude, and it should be defined thus :

> *A nautical mile at any place is the length of an arc of the meridian subtending an angle of* 1′ *at the centre of curvature of the place.*

It is shortest at the equator where ϕ is 0°, and its length is (6,077·1—30·7) or 6,046·4 feet, and largest at the poles where ϕ is 90°, and its length is (6,077·1+30·7) or 6,107·8 feet.

Since the radius of the equator is a, the length of an arc of the equator subtending an angle of one minute at the centre will be a sin 1′. This works out to be 6,087·2 feet, and provides a unit known as a *geographical mile.*

For the purposes of navigation, the convenience of having a fixed or standard unit is obvious, and the unit decided upon is 6,080 feet. In normal practice, the errors arising from this assumption are negligible because only small differences of latitude occur. The following example, however, shows that the error can become large enough to be measured.

A ship in latitude 6°N., steams for 36 hours on a course 180° at 20 knots. What is her latitude at the end of the run?

The distance recorded by an accurate patent log would be $720 \times 6{,}080$ feet, and if no allowance is made for the ellipsoidal shape of the Earth, the d'lat would be taken as 720′S., so that the latitude of her position at the end of the run would be 6°S. But in the neighbourhood of the equator the length of arc of the meridian corresponding to a difference of one minute in geographical latitude is 6,046·4 feet, not 6,080. The number of minutes in the d'lat is therefore:

$$\frac{720 \times 6{,}080}{6{,}046{\cdot}4}$$

The true d'lat is thus 724′S., and the latitude of the ship's position at the end of the run is 6°04′S.

If it is possible to obtain frequent observations, it is clear that no account need be taken of the alteration in the nautical mile, but where long distances are steamed without observations, in the equatorial regions especially, a correction should be applied to allow for the actual variation of the nautical mile from its assumed value of 6,080 feet. In all other navigational problems this variation need not be taken into account.

In survey work, the greatest possible accuracy is necessary, and it is not sufficient to regard the Earth as a sphere. The formulæ employed are therefore derived from the principles outlined in this chapter.

CHAPTER II

MIDDLE LATITUDE

When a ship follows a steady course, her track cuts each meridian at the same angle and is known as a *rhumb line* or *loxodrome.*

In figure 4, for example, the course between F and T is given by the angle between any meridian and the rhumb line joining

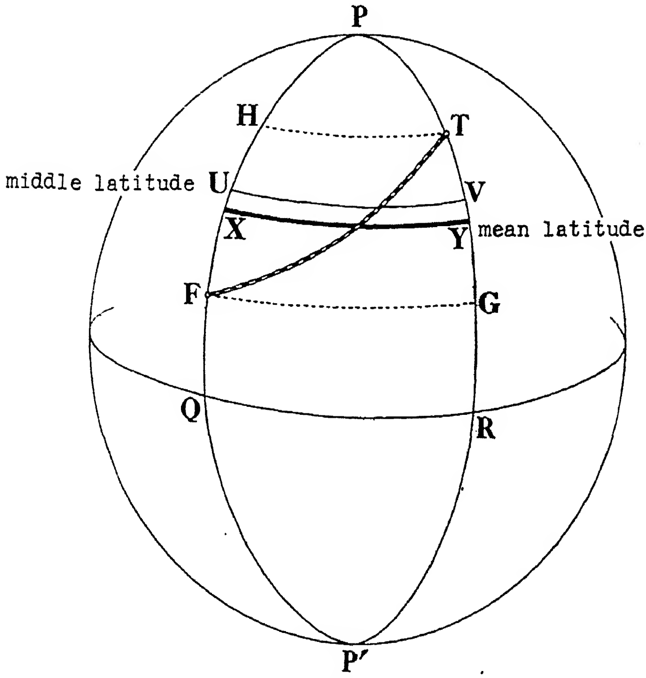


FIGURE 4.

F to T, and the length of this rhumb line, measured in nautical miles, is always referred to as *the distance* between F and T.

It was shown in Volume II that the difference of latitude between F and T, and the departure, were connected with the distance and course by the accurate formulæ :

$$\text{d'lat} = \text{distance} \times \cos(\text{course})$$
$$\text{departure} = \text{distance} \times \sin(\text{course})$$

Mean Latitude. Departure is defined as distance moved in an east–west direction, and when F and T lie as shown in figure 4, the departure is clearly greater than HT but less than FG. Also,

if the difference of latitude between F and T is not considerable, the departure is approximately equal to the arc XY of a parallel, the latitude of which is the average or arithmetical mean of the latitudes of F and T. This latitude is called the *mean latitude.*

Since QR is a measure of the d'long between F and T, and the length of the arc XY is equal to QR cos QX, it follows that (page 21, Volume II) :

$$\text{d'long} = \text{departure} \times \sec \text{(mean latitude)}$$

or
$$\text{departure} = \text{d'long} \times \cos \text{(mean latitude)}$$

These formulæ, however, are accurate only when F and T lie on the same parallel of latitude, and it must be remembered that although the error resulting from their use is negligible when the d'lat is small, that error is considerable when long distances and large differences of latitude are involved. In these circumstances, a *middle latitude* must be found or another method adopted.

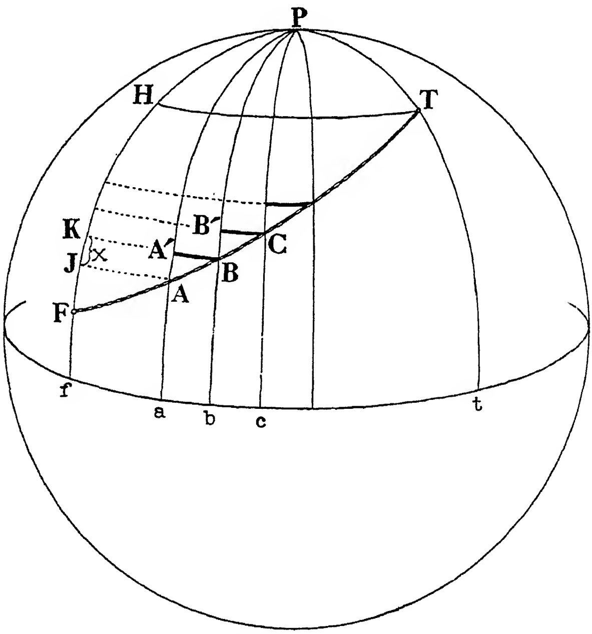


FIGURE 5.

Middle Latitude. Since the departure is greater than HT and less than FG, it must be exactly equal to the arc of some parallel UV. The latitude of this parallel is called the *middle latitude*, and if it is denoted by l, then :

$$QR = UV \sec l$$

i.e.
$$\text{d'long} = \text{departure} \times \sec l$$

This is an accurate formula, but l must be known before it can be used. The problem is therefore to find l.

In figure 5, the latitudes of F and T are denoted by l_F and l_T, and the difference of latitude between them, FH, is divided into n equal parts of length x. JK is one of these parts. Then :

$$\text{d'lat}=nx=l_T-l_F$$

If parallels of latitude are now drawn through the points J, K . . ., intersecting the rhumb line FT in A, B . . . and the meridians through these points of intersection in A', B' . . ., n small triangles are formed. Moreover these triangles are equal because in each the side of which AA' is typical is x ; the angle at A' is 90° ; and the angle at A is the course, which is constant between F and T. The length of the arc of which $A'B$ is typical is thus the same for each triangle, and if the triangles are made sufficiently small (that is, if n is made sufficiently large) for the conditions for evaluating an accurate departure to be realized, the departure between F and T is the sum of the elements $A'B$. Thus :

$$\text{departure}=ny$$

—where y is the length of $A'B$. Also, the d'long corresponding to the element $A'B$ is ab, and :

$$ab=A'B \sec (\text{latitude } B)$$

i.e. $$ab=y \sec (\text{latitude } K)$$

By adding all these elements ab, bc . . ., the d'long is obtained, the formula being :

$$\text{d'long}=y[\sec (l_F+x)+\sec (l_F+2x)+\ .\ .\ .\ \sec l_T]$$

Or, since the departure is equal to ny :

$$\text{d'long}=\text{departure}\ \frac{[\sec (l_F+x)+\sec (l_F+2x)+\ .\ .\ .\ \sec l_T]}{n}$$

But the middle latitude, l, is given by :

$$\text{d'long}=\text{departure}\times\sec l$$

Hence, by equating these two values of the d'long :

$$\sec l=\frac{1}{n}[\sec (l_F+x)+\sec (l_F+2x)+\ .\ .\ .\ +\ \sec l_T]$$

The quantity sec l is thus the mean of the secants of the latitudes of the successive parallels.

Written in the integral form in order that the value of sec l may be found, the equation is :

$$\sec l=\frac{1}{nx}[\sec (l_F+x)+\sec (l_F+2x)+\ .\ .\ .\ +\sec l_T]x$$

$$=\frac{1}{(l_T-l_F)}[\sec (l_F+x)+\sec (l_F+2x)+\ .\ .\ .\ +\sec l_T]x$$

Then, as n becomes larger, x grows progressively smaller, and, in the limit :

$$\mathrm{Lt}_{x\to 0}[\sec(l_F+x)+\sec(l_F+2x)+\ .\ .\ .\ +\sec l_T]x$$

$$=\int_{l_F}^{l_T}\sec l\, dl$$

$$=\log_e \tan\left(\frac{\pi}{4}+\frac{l_T}{2}\right)-\log_e \tan\left(\frac{\pi}{4}+\frac{l_F}{2}\right)$$

The middle latitude, l, is thus given by :

$$\sec l=\frac{1}{(l_T-l_F)}\left[\log_e \tan\left(\frac{\pi}{4}+\frac{l_T}{2}\right)-\log_e \tan\left(\frac{\pi}{4}+\frac{l_F}{2}\right)\right]$$

In order to avoid the labour of evaluating this formula, *Inman's Tables* give the difference between l and the mean latitude, $\frac{1}{2}(l_F+l_T)$, for the arguments mean latitude and d'lat. It should be noted, however, that the argument in the left-hand column is incorrectly headed ' Mid.Lat.'.

The following example shows the advantage of using these tables in preference to the actual formula.

A ship steams from F to T, a distance of 1,200′, *on a course* 050°. *The position of F is* 30°*N.*, 40°*W. What is the position of T and the error that is introduced by using the mean-latitude formula ?*

By traverse table :

d'lat=1,200′ cos 50°=771′N.
departure=1,200′ sin 50°=919′E.

The approximate formula giving the d'long is :

d'long=departure × sec (mean latitude)

Hence :

lat. F	30°00′	departure 919′	log	2·963 32
d'lat	12°51′	mean latitude 36°25′·5	log sec	0·094 40
lat. T	42°51′		log d'long	3·057 72
mean latitude 36°25′·5			∴ d'long=1,142′ E.	

Thus, by the approximate formula, the d'long is 19°02′E.

For the purpose of evaluation, the accurate formula is conveniently written :

$$\sec l=\frac{1}{(l_T-l_F)}\left[\log_{10} \tan\left(\frac{\pi}{4}+\frac{l_T}{2}\right)-\log_{10} \tan\left(\frac{\pi}{4}+\frac{l_F}{2}\right)\right]\log_e 10$$

The value of $(l_T - l_F)$ is 12°51′, expressed in arc, but in the formula it must be expressed in circular measure. It is then 0·22427 radians. Hence :

$$\sec l = \frac{1}{0{\cdot}22427}\left[\log_{10}\tan 66°25'{\cdot}5 - \log_{10}\tan 60°\right]\log_e 10$$
$$= \frac{2{\cdot}3026}{0{\cdot}22427} \times 0{\cdot}12156$$
$$= 1{\cdot}24829$$

The middle latitude is therefore 36°46′.

The correction to be added to the mean latitude to give the middle latitude (from *Inman's Tables*) is 20′·5. This, added to 36°25′·5, gives the 36°46′ found by evaluating the formula. The correct d'long is therefore :

$$919' \sec 36°46'$$
$$= 1147'\text{E.}$$
$$= 19°07'\text{E.}$$

The position of T is thus 42°51′N., 20°53′W., and the error in longitude introduced by using the mean-latitude formula is 5′.

CHAPTER III

MAP PROJECTIONS

Since a chart is necessarily a plane surface, and a plane surface cannot be fitted to a sphere, a perfect representation of the Earth's features in relative size, shape and position, is impossible on a chart. There will be distortion in at least one essential feature. Charts are therefore designed for special purposes, and the graticules on which the charts are built are chosen accordingly.

Definition of a Projection. A graticule is a network of lines representing the meridians and parallels of latitude, and any arrangement of this network is known as a projection. A map projection is thus a representation of the Earth's meridians and parallels upon a plane surface. It is not necessarily a projection in the geometrical sense. In the Mercator chart, for example, the graticule is constructed on a mathematical basis in no way connected with perspective.

The Principal Classes of Projections. Projections may be grouped in two classes. The first and larger class is derived from the cone, and includes all conical, zenithal and cylindrical projections and their modifications. The second includes the 'conventionals' such as Mollweide's equal-area and Cassini's projection by rectangular co-ordinates.

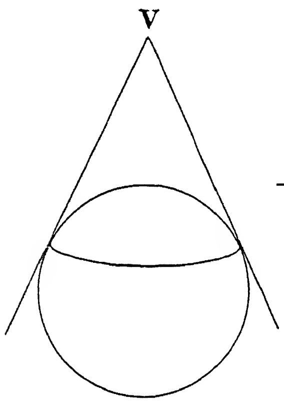
As the name suggests, a *conical projection* is one in which the connexion with the Earth can be established by rolling the plane surface that forms the chart into a cone. Figure 6a shows a simple example of such a connexion.

If the angle at the vertex of the cone is increased, it is clear that the vertex of the cone approaches the surface of the sphere until, when the angle reaches 180°, the cone itself has spread into a plane tangential to the sphere, and the vertex has become the tangent point. The projection is then said to be *zenithal*. (Figure 6b.) Since the true bearings, and therefore the azimuths, of places are correctly drawn from the vertex or central point of such a projection, the name *azimuthal* is sometimes used instead of *zenithal*. The *gnomonic projection* belongs to this class, and is one example of a perspective projection as distinct from a mathematical.

When the angle of the cone decreases to zero, the vertex recedes to infinity and the cone degenerates into a cylinder. (Figure 6c.) The projection is then said to be *cylindrical*. For civil purposes

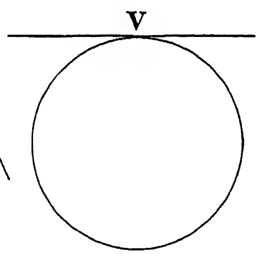
this class of projection is not important, but it includes the Mercator graticule, and that, for navigational purposes, is most important. It is described in detail in Chapter IV.

The Sub-Divisions of the Principal Classes. Since distortion is inevitable in any representation of the Earth's surface on a plane chart, it is clear that the cartographer must content himself with compromise, and that he must frame his graticule to give him accuracy in one direction at the expense of accuracy in another. If, for example, he wishes to know the great-circle routes for aircraft leaving London, he can choose a gnomonic projection with London as the centre. This will give him little idea of the relative sizes and shapes of the countries traversed by these routes, but it will tell him exactly where these routes lie, and the routes themselves will be shown as straight lines.



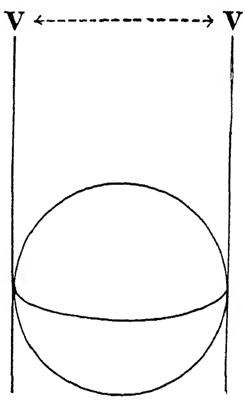
CONICAL

Figure 6a.



ZENITHAL

Figure 6b.



CYLINDRICAL

Figure 6c.

One arrangement of the graticule, in spacing and inclination of the component lines, will thus produce a chart on which the true bearings of objects from a certain point are correctly shown, and another arrangement, such as Mollweide's, will give equal areas between parallels and therefore be suitable for any distribution or supply map where the purpose is to afford a comparison between the productive resources of particular areas.

As already stated, these arrangements are achieved either by a geometrical projection from some definite point so that the surface of the Earth is shown in actual perspective, or by some mathematical rule which gives the required result. Conventional projections are necessarily built according to a mathematical rule, but the projections derived from the cone may be either mathematical or true perspective projections.

A survey of the projections in actual use will show not only how the graticule can be constructed for some definite purpose, but also how most graticules are derived from the conical projection.

Simple Conical Projection with one Standard Parallel. Although this projection is drawn upon the tangent cone, it is *not* a geometrical projection because the parallels are equally spaced.

Figure 7 shows the cone resting on the sphere with V directly above P, and touching the sphere along the parallel of latitude LL'. This parallel is known as the *standard parallel.* All points on LL are the same distance, LV, from V. If the cone is now cut along a generator and unrolled, the parallel appears as the arc of a circle of

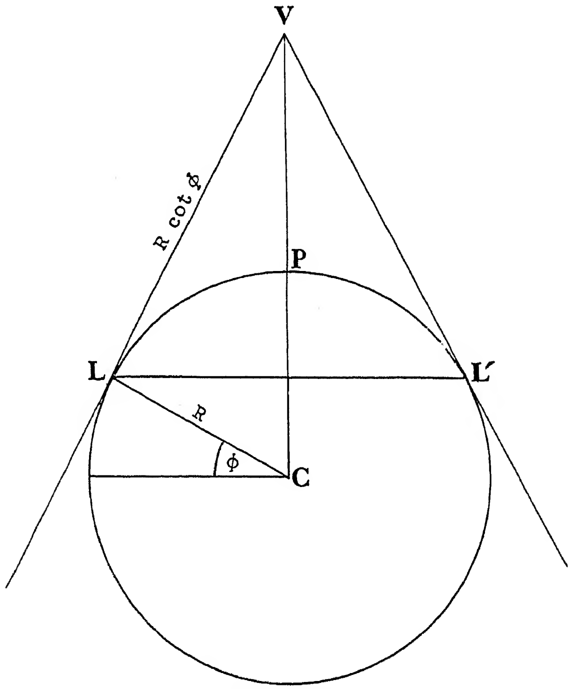


FIGURE 7.

radius LV, and the meridians are equally-spaced straight lines radiating from the centre of this circle. Clearly the angle between any two of these meridians is only a fraction of the angle between the actual meridians on the sphere, and this fraction, n, is called the *constant of the cone.*

If R is the radius of the Earth and ϕ the latitude of the standard parallel, the length of the parallel is $2\pi R \cos \phi$, and if an arc proportional to this length is drawn with radius $R \cot \phi$, it represents the standard parallel on the projection. Since this arc subtends at the centre of the circle an angle equal to $2\pi \sin \phi$, the constant of the cone is $\sin \phi$.

The parallels of latitude are drawn as arcs of circles about *V* as centre, the distances of these parallels from the standard parallel being equal to their actual distances on the Earth reduced to the scale of the map. The parallels are thus equally spaced; the pole is represented by an arc of finite length; and the graticule of the simple conical projection with one standard parallel appears as shown in figure 8.

Although this projection is useful for showing countries that are not too large, it is not an equal-area projection. Nor is it *orthomorphic*; of 'right shape', that is. Both the conical pro-

SIMPLE CONIC PROJECTION

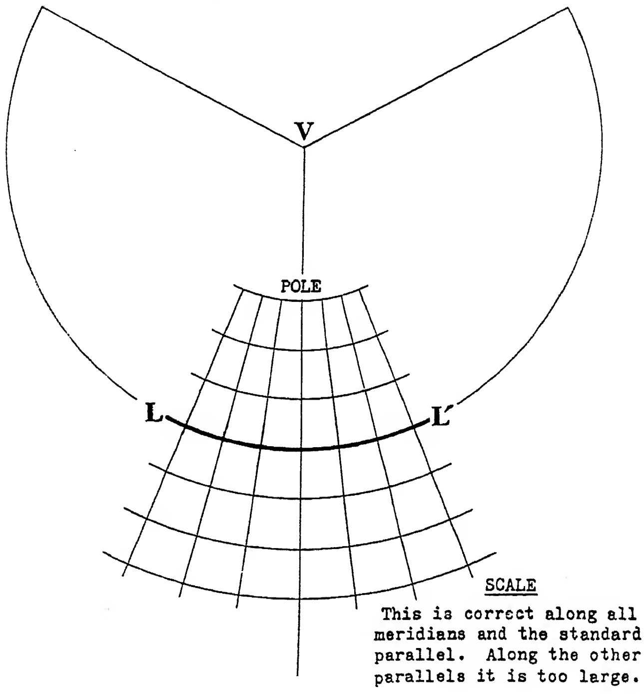


FIGURE 8.

jection with two standard parallels and Bonne's projection are more successful, especially when the country covers a wide belt of latitude. For this reason the conical projection with one standard parallel is not often used.

The Orthomorphic Property of Maps. A projection is said to be orthomorphic when the scale along a radial or meridian is equal to the scale along the parallel in the immediate neighbourhood of any point, and the radial and the meridian are at right angles to each other. Correctness of shape thus applies only to small areas. The scale of a graticule in one latitude may not be the same as the

scale in another, so that the shape which a land mass assumes on the graticule differs considerably from its shape on the Earth ; but, so long as the scale along the meridian is equal to the scale along the parallel at any point, the immediate neighbourhood of that point is just as correctly shown as the immediate neighbourhood of a point some distance removed in latitude. On a Mercator chart of the World, for example, the area round Cape Farewell in Greenland is just as correctly shown as the estuary of the Orinoco in South America, but Greenland, as a whole, appears slightly larger

BONNE'S PROJECTION

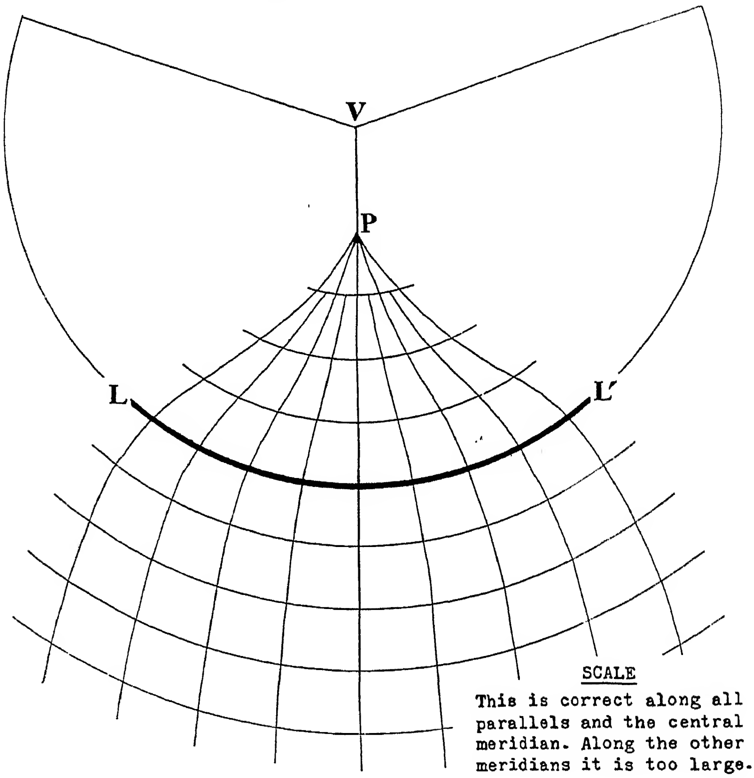


FIGURE 9.

than South America although it is actually about one-tenth the size.

On the simple conical projection with one standard parallel, it is evident that the scale along a meridian cannot be equal to the scale along the parallel at any point because the pole, which is a point on the Earth, is stretched into a line, whereas there is no stretching of the distances between the parallels.

Bonne's Projection. In the simple conical projection with one standard parallel, the scale is correct only along the meridians and

that parallel. In Bonne's projection, the scale is made correct along one central meridian and all the parallels. This is done by dividing each parallel correctly. The remaining meridians are formed by joining the appropriate points of division on the parallels; and, since these meridians are no longer straight lines and therefore do not cut the parallels at right angles, it is clear that the scale along any but the central meridian is not correct. Also the distortion increases as the longitude from the central meridian increases. But the projection gives equal areas, and this fact, allied to its suitability for depicting a wide belt of latitude, has made it popular.

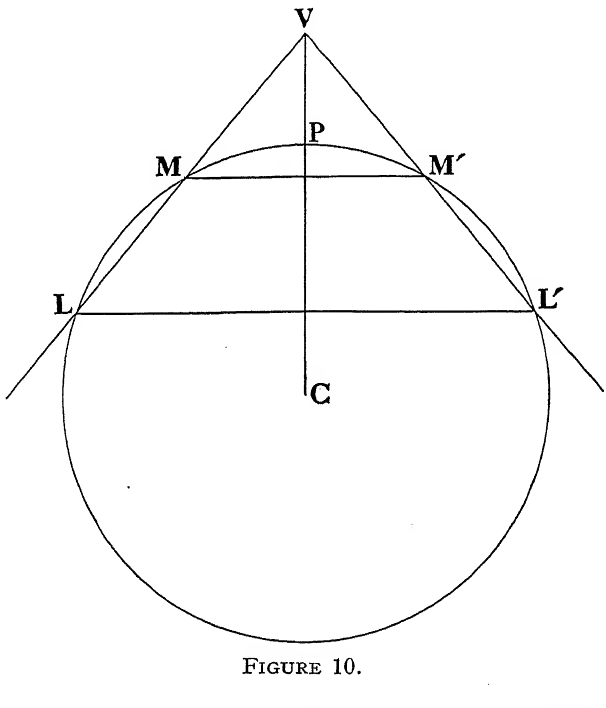


FIGURE 10.

Figure 9 shows the appearance of the graticule when 50°N. is taken as the standard parallel.

Sanson-Flamsteed or Sinusoidal Projection. This is the same as Bonne's projection when the standard parallel is the equator. Its properties are therefore the same as those of Bonne's, and it lends itself for depicting an area like Africa where the equator roughly divides the continent.

Conical Projection with Two Standard Parallels. In this projection two parallels are taken, one towards the top and the other towards the bottom of the area to be shown, and the length between these parallels on the graticule is proportional to the actual arc-distance between them on the Earth. (Figure 10.)

When the graticule is drawn, LL′ and MM′ appear as arcs of concentric circles, and their scale is correct. Since other parallels are inserted so that their distances apart on the map are proportional to their distances apart on the Earth, as in the simple conical projection, they are equally spaced and, again, the graticule (Figure 11) is *not* a geometrical projection.

The introduction of a second standard parallel lessens the distortion if the belt of latitude to be covered is wide, although the

CONICAL PROJECTION WITH TWO STANDARD PARALLELS

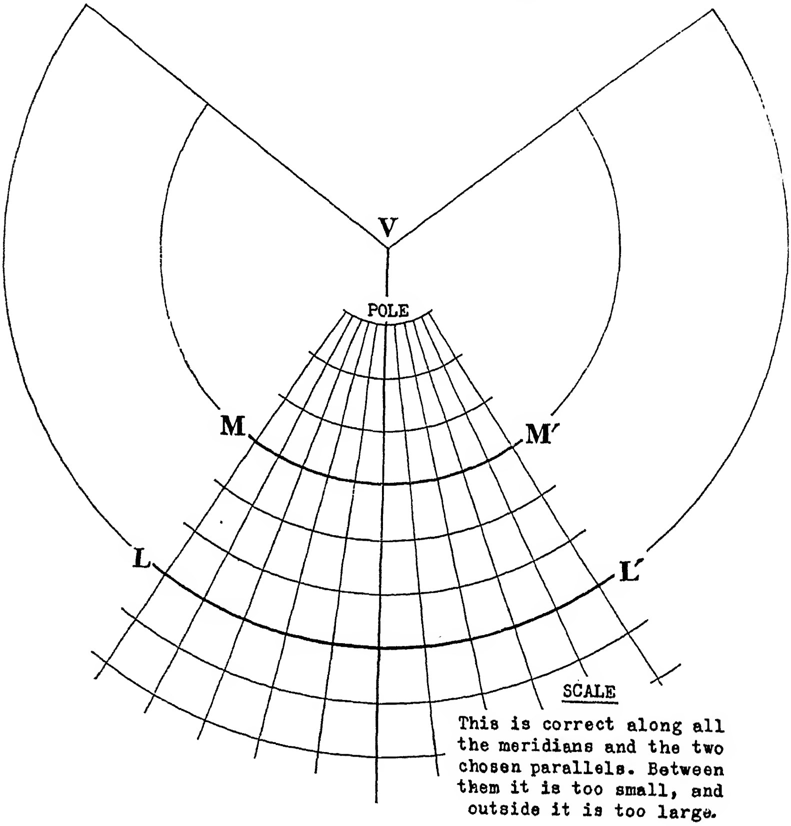


FIGURE 11.

pole still appears as an arc of finite length. The projection thus lends itself for depicting an area like that covered by the British Isles. It was, indeed, used for the original one-in-a-million maps of Great Britain, but has now been superseded by the polyconic projection. It is still much used in atlas maps of small countries.

The Polyconic Projection. In this projection the central meridian is divided correctly, but each parallel is constructed as if it were the standard parallel of a simple conical projection. The

parallels are therefore no longer arcs of concentric circles, but arcs of circles, the radii of which steadily increase as the latitude decreases. Also the meridians, other than the central, are no longer straight lines. The graticule therefore appears as shown in figure 12.

Like the simple conical projection, the polyconic projection is neither orthomorphic nor equal-area, and it is therefore unsuitable for large areas. But it has the great advantage that, if small areas are shown on this projection, each area covering the same

POLYCONIC PROJECTION

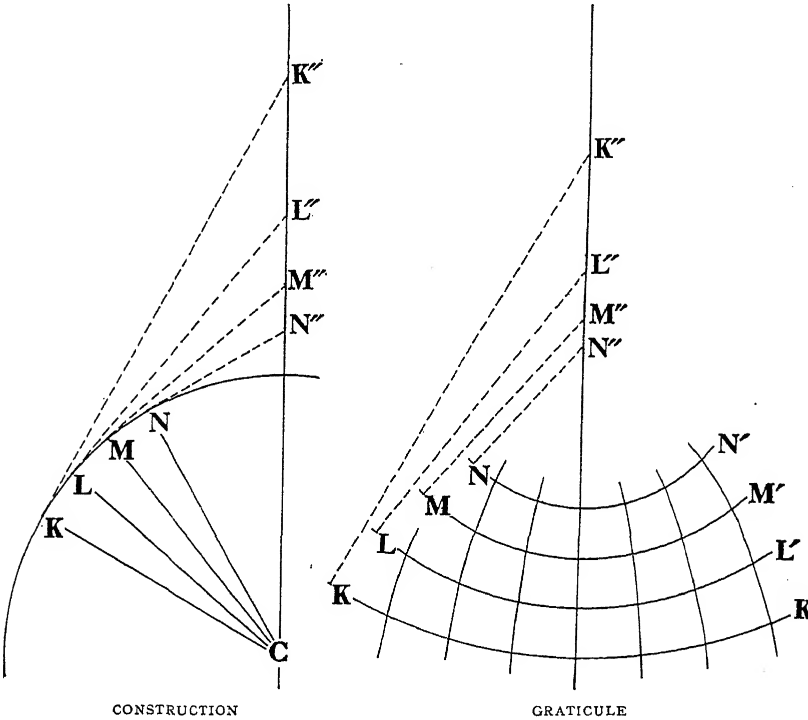


FIGURE 12.

amount of longitude, the sheets on which the graticules are drawn will fit exactly along their northern and southern edges, and also, for ordinary purposes, along their eastern and western edges although the join here is really a ‘ rolling fit ’ since the meridians are curved. The projection thus lends itself for topographical maps that, individually covering relatively small areas, combine to cover a large one. For this reason it has been extensively used in maps showing the United States of America.

The International Map. The projection for the International Map, decided upon by the International Map Committee in 1909,

is a polyconic projection slightly modified to avoid the 'rolling fit' on the eastern and western edges of the component sheets. This is done by making the meridians straight lines instead of curves; that is, by dividing the top and bottom parallels truly and joining the corresponding points of division with straight lines. Also the distance between the parallels is adjusted so that, on sheets covering six degrees of longitude, the meridians two degrees each side of the central meridian are their true lengths.

The scale of this map is 1 : 1,000,000.

CASSINI'S PROJECTION

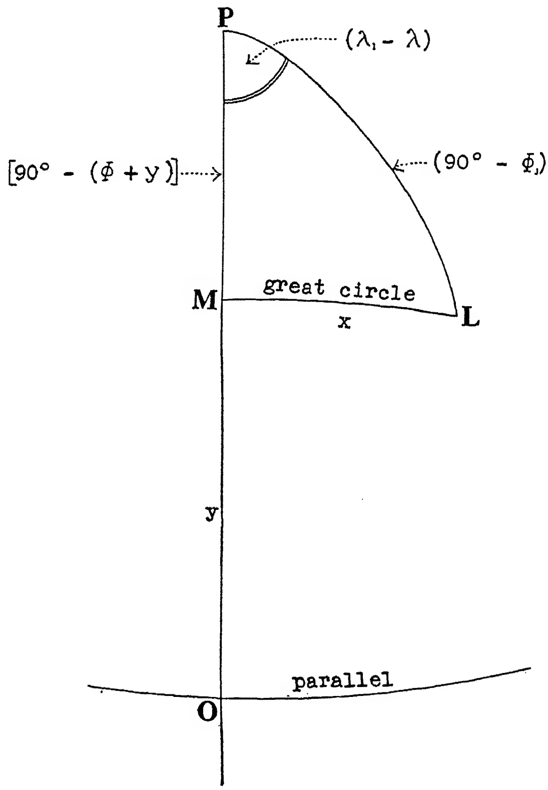


FIGURE 13.

Cassini's Projection. This is a purely conventional projection, differing from the polyconic in that the graticule is built, not sheet by sheet, but in relation to a central meridian and some fixed point on it. The sheets thus fit accurately together.

This projection is used in the British Ordnance Survey.

In figure 13, O is the fixed point on the central meridian and therefore the 'origin' of the map. The latitude of O is ϕ and the longitude λ.

L is any other point of latitude ϕ_1 and longitude λ_1, and LM is the great circle through L that is perpendicular to the central meridian at M. If LM is equal to x and OM to y, then, by Napier's rules :

$$\sin x = \sin (\lambda_1 - \lambda) \cos \phi_1$$

and
$$\cot (\phi + y) = \cos (\lambda_1 - \lambda) \cot \phi_1$$

These two equations give x and y, and the projection is obtained by plotting the values of x and y for various points as rectangular co-ordinates on a plane. For this reason Cassini's method is sometimes known as projection by rectangular co-ordinates.

It should be noted that in this projection only the central meridian is straight. Nevertheless, in the topographical representations of small areas, all meridians can be considered straight, but, since the outer meridians necessarily slope towards the central meridian as the latitude increases, they do not form a rectangular graticule although the projection itself is rectangular.

MOLLWEIDE'S PROJECTION

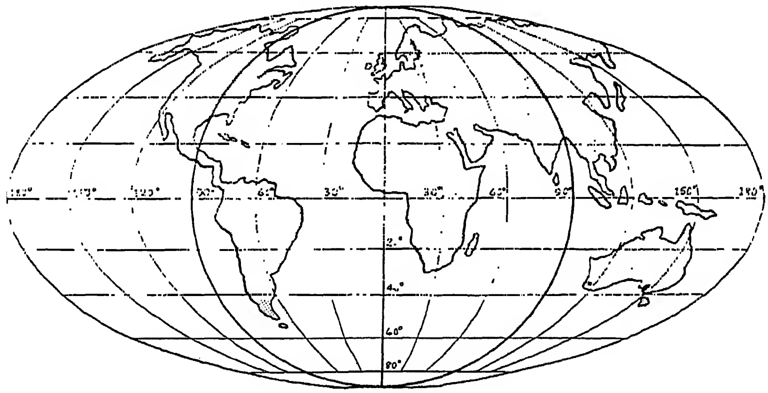


FIGURE 14.

Mollweide's or the Elliptical Projection. This, a conventional projection, is frequently used for World maps showing distributions. All parallels of latitude are straight lines, and all meridians are semi-ellipses. The central meridian and those 90° east and west of it, however, take special forms of the ellipse and are respectively a straight line and semi-circles. The equal-area property is achieved by adjusting the distances between the parallels.

Figure 14 shows the resulting graticule.

Zenithal Projections. Some of these can be derived directly from the conical projections by making n, the constant of the cone, equal to unity. The simple conical projection, for example, becomes the *zenithal equidistant* since all distances measured from the centre are true.

When this centre is the pole, the parallels of latitude are equally-spaced concentric circles, and the meridians are radiating straight lines. When the centre is not the pole, the parallels and meridians are, in general, curved.

The conical projection with two standard parallels has no counterpart in zenithal projections. The conical equal-area and the conical orthomorphic, however, have direct counterparts, the latter giving rise to the stereographic projection which is a perspective projection.

All perspective projections are made by drawing straight lines on to the tangent plane from some chosen point called the centre of projection. If the centre of projection lies on the diameter perpendicular to the tangent plane, several important projections result according to the position of the centre of projection. These include:

(1) the *gnomonic*, which is obtained when the centre of projection is at the centre of the sphere.

(2) the *stereographic*, obtained by placing the centre of projection at the other extremity of the diameter in question.

(3) the *orthographic*, obtained by removing the centre along the diameter produced until it is an infinite distance from the sphere.

The Gnomonic Projection. Since the plane of any great circle passes through the centre of the sphere, it also passes through the centre of projection, and the lines projecting the great circle on to a plane themselves lie in a plane. Two planes cut in a straight line. Therefore the projection of any great circle is a straight line.

The construction of this important graticule is described in detail in Chapter VI.

The Stereographic Projection. Although great circles are no longer projected into straight lines when the centre of projection is moved to the end of the diameter, the area that can be projected is conveniently increased. In the gnomonic method, less than half a hemisphere can be projected, and even then the distortion in the outer parts is considerable. In the stereographic, a complete hemisphere can be projected without undue distortion. Also the resulting projection is orthomorphic.

When the plane of projection is tangential to the sphere at one of the poles, the resulting graticule is known as a polar stereographic, and has the advantage that all parallels of latitude appear as concentric circles.

Figure 15 shows the south polar stereographic projection.

If R is the Earth's radius and ϕ is the latitude of L, then the angle CPL is $(45°-\frac{1}{2}\phi)$, and the radius $P'L'$ of the projected parallel is $2\ R \tan\ (45°-\frac{1}{2}\phi)$. In the polar gnomonic projection the corresponding radius is $R \cot \phi$, and in both projections the meridians appear as straight lines radiating from the pole.

In the polar stereographic projection, the radial is a meridian, and, if the radius $P'L'$ is denoted by r, and the co-latitude of L by χ, the scale along the parallel in the neighbourhood of L' is given by :

$$\frac{r}{R\sin\chi}=\frac{2R\tan\frac{1}{2}\chi}{R\sin\chi}=\sec^2\tfrac{1}{2}\chi$$

Also the scale along the meridian is given by :

$$\frac{dr}{R\,.\,d\chi}=\frac{1}{R}\,.\,2R\sec^2\tfrac{1}{2}\chi\,.\,\frac{1}{2}=\sec^2\tfrac{1}{2}\chi$$

The scale is thus the same in each direction, and the orthomorphic property is established.

STEREOGRAPHIC PROJECTION

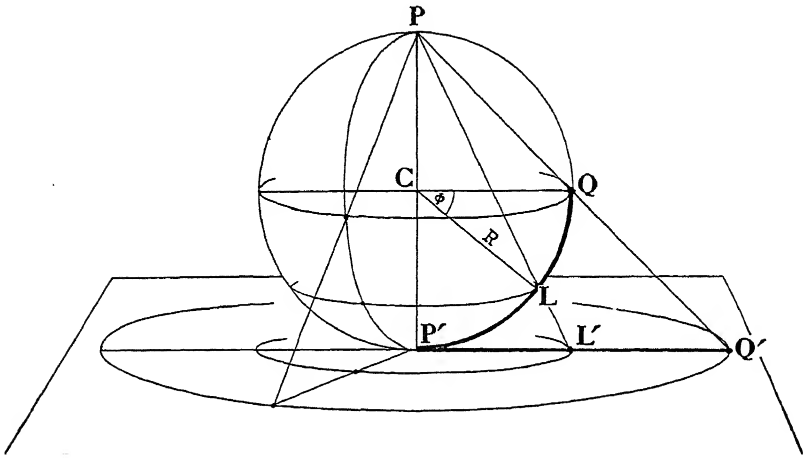


FIGURE 15.

The Orthographic Projection. The appearance of this graticule can be readily imagined because it is the appearance of the meridians and parallels on a globe that is seen from a distance. When the eye is in the equatorial plane, the bounding meridians form a circle, and the others are semi-ellipses unequally spaced. The parallels are straight lines also unequally spaced. The projection, however, is of no importance to the cartographer though it has some interest for astronomers.

Choice of Projections. As stated at the beginning of this chapter, accuracy in one property of a map can be achieved only at the expense of accuracy in another, and a graticule is therefore selected for the purpose of the map. Naval charts, for example, are built to enable the seaman to navigate. To do this, he must know the course to steer. His charts are therefore designed to give this course, and all other considerations are sacrificed. For this reason the Mercator chart shows large areas on a small scale and is

of no particular use to anyone except the navigator. Ordnance survey maps, on the other hand, are essentially topographical maps of large scale, designed to show great detail and preserve bearings over small areas. Cassini's rectangular co-ordinate system therefore suffices. When the whole of Great Britain is shown on this system, the curvature of the meridians is not noticeable on the separate sheets.

The following list gives a summary of the uses to which the projections discussed in this chapter are put :

Area	Purpose of Map	Suitable Projection
The World		
(a) In hemispheres	1. Equal Area	Zenithal and Mollweide.
	2. Orthomorphic	Stereographic.
	3. General	Zenithal equidistant.
(b) In one sheet	1. Equal Area	Cylindrical, Sinusoidal and Mollweide.
	2. Orthomorphic 3. General	Mercator.
Continental Maps		
(a) Asia and N. America	1. Equal Area	Zenithal equal-area and Bonne.
	2. General	Zenithal equidistant.
(b) Europe and Australia	1. Equal Area	Zenithal equal-area and Bonne.
	2. General	Simple conic with two standard parallels.
(c) Africa and S. America	1. Equal Area	Zenithal equal-area, Sinusoidal and Mollweide.
	2. General	Zenithal equidistant and Mercator.
Polar Regions	1. Equal Area 2. Equidistant	Zenithal.
Large Countries in Temperate Latitudes. e.g. U.S.A., Russia and China	1. Equal Area	Zenithal equal-area and Bonne.
	2. General	Simple conic with two standard parallels, and Conical Orthomorphic.
Small Countries not in the Tropics	1. Equal Area	Conic with one standard parallel, and Bonne.
	2. General	Simple conic with one or two standard parallels.
Topographical Maps		Cassini and Modified Polyconic.
Special Maps	1. Navigation	Mercator and Gnomonic.
	2. Directional Wireless	Gnomonic.

Gridded Charts and Maps. There is no fundamental difference between a chart and a map. A chart is simply a graticule on which soundings and marine topography are shown, the use of which is confined principally to navigators. A map is a graticule on which land features are shown. This distinction, however, is not binding. There are, for example, many atlas maps of Oceania.

On charts, where the scale is usually small, it is customary to refer to the position of an object by its latitude and longitude. On maps, where the scale is comparatively large, it is more convenient to adopt a grid system. In this, a pattern of squares, or grid, is imposed upon the map, and a further system of lettering and numbering (known as the Modified British System) is used to specify any particular square. The actual lettering and numbering is adjusted to suit the scale of the map and avoid overcrowding when the scale is small, but, in principle, the system may be summarized thus: an origin is taken and two axes at right-angles, one north–south, the other east–west; lines are drawn parallel to these axes to form large squares which are lettered; each lettered square is sub-divided into smaller squares by further lines which are numbered; the position of a point is then defined by its co-ordinates with reference to the west and south sides of the small square in which it lies.

In order to avoid negative co-ordinates, the origin is always taken well to the south-west, and if necessary a false origin is introduced and positive co-ordinates given with reference to that.

A letter reference is seldom required on a large-scale map; the sub-divisions of the lettered square range from 0 to 100, and the position of the point is given entirely as a numeral.

It is, for example, required to give the position of the point X in figure 16. The south-west corner of the square in which X lies is indicated by co-ordinates measured east and north, the easterly co-ordinate being given by the line 79 and the northerly by the line 51. With reference to these lines the easterly co-ordinate of X is 6 and the northerly 4. The position of X is then given as a combination—796514—of the two easterly components, 79 and 6, and the two northerly, 51 and 4.

Lettering is usually indicated by a key diagram.

On the actual grid it is customary to show certain figures in full; 50, for example, is written $^{9}50$. This merely indicates the number of units between the line and the origin on which the entire grid is built, and it does not affect the system by which a map reference is given. In giving map references, any 'extra' figures should therefore be ignored.

When the scale of the map is so small that the area shown covers several lettered squares, the sub-divisions within a lettered square are reduced from (100×100) to (10×10) and a reference is given by a letter and four figures.

MODIFIED BRITISH SYSTEM (LARGE SCALE)

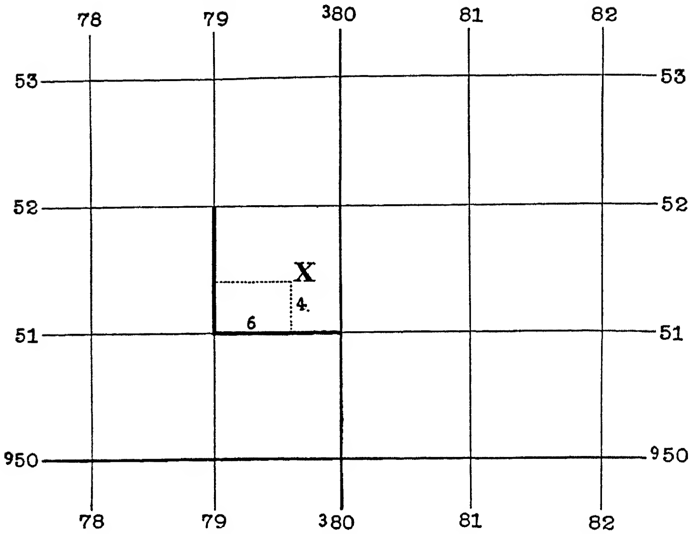


FIGURE 16.

MODIFIED BRITISH SYSTEM (SMALL SCALE)

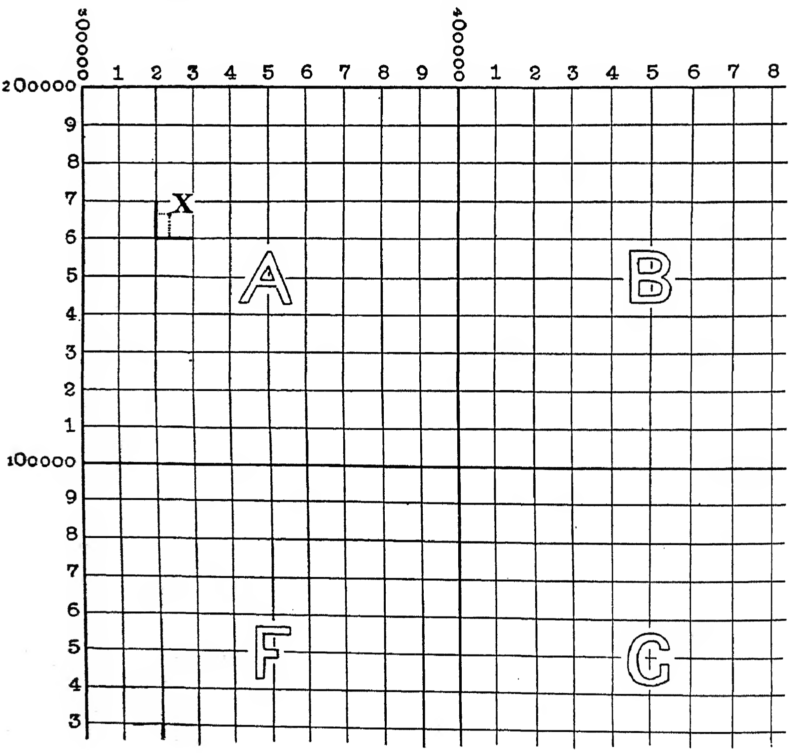


FIGURE 17.

Thus, in figure 17, the co-ordinates of *X* with reference to the square 2,6, are 4,7 ; and the position of *X* is given by A 2467.

Since most topographical maps are drawn on Cassini's or a polyconic projection where the outer meridians slope inwards, and also since the meridian through the origin will seldom pass through the sheet in actual use, there will usually be a slight difference between grid north and true north.

To Transfer a Map Grid to a Chart. There is a theoretical difficulty in transferring a map grid to a chart because the map projection is almost certain to be different from the chart projection. If, as is likely, the chart is drawn on the Mercator projection, a grid transferred from Cassini's projection would be formed by curved lines, but in practice, when a large-scale chart is used, the grid may be drawn with straight lines without appreciable loss of accuracy over the small area adjacent to the land which is required to be gridded.

A gridded map usually gives the geographical positions of the extreme corners of the map, and if these are plotted on the chart, the grid can be inserted according to scale. On a Mercator chart, where the latitude scale exceeds the longitude scale except at the equator, the transferred grid will appear as a series of rectangles, but on a large-scale gnomonic projection where the scale over the small area covered is approximately the same in all directions, the transferred grid will be formed of squares.

The map grid can also be transferred by taking positions common to both chart and map, but it may happen that these common positions will not be in exact geographical agreement because the chart and the map are based on different determinations.

When the geographical positions of the corners of a gridded map are not given, the geographical position of the origin is shown at the bottom of the map, and the grid corners of the map can be calculated.

CHAPTER IV

THE MERCATOR CHART

The Mercator chart is essentially the navigator's chart because any straight line joining two points on it is the rhumb line or line of constant course between them.

The idea of the graticule belongs to Gerard Kremer, the Latin form of whose name is Mercator. Kremer used the graticule in the World map which he published in 1569. The graticule, however, was inaccurately drawn above the parallels of 40°, and there was no mathematical explanation of it. That was not forthcoming until Wright calculated the positions of the parallels and published the results in his *Errors of Navigation Corrected* thirty years later. The chart came into general use among navigators about 1630, but the first complete description of it did not arrive until 1645 when Bond published the logarithmic formula.

The Principle of the Mercator Projection. In Chapter V of Volume I, the Mercator graticule was considered without reference to the family of projections, derived from the cone, to which it belongs. The explanation given in that chapter is based on the navigational purpose of the graticule and broadly follows the lines that Wright adopted when he made his calculations. But the graticule can equally well be considered as a particular member of the general conic family. If so, its characteristics are governed by two considerations: it is orthomorphic, and the constant of the cone is zero. For this reason it is always known among cartographers as a cylindrical orthomorphic projection, and it is a mathematical, not a perspective projection.

The orthomorphic property is achieved by spacing the parallels at increasing intervals as they approach the poles, and this arrangement, coupled with the fact that the meridians and the parallels on any cylindrical projection where the standard parallel is the equator must be straight lines at right-angles, the meridians furthermore being equally spaced, leads to the other property so important to the navigator, namely that rhumb lines also are straight lines. The meridians on a Mercator chart being thus parallel straight lines running north and south, any straight transversal makes a constant angle with them, and there is no distortion of this angle because the orthomorphic property ensures that the correct shape is preserved at all points along the transversal. It is thus the true angle, and, since it is constant, the transversal is a rhumb line.

The problem of the Mercator chart is thus the problem of finding the chart length of any parallel from the equator when the orthomorphic property is to be achieved.

The Longitude Scale on a Mercator Chart. Since the meridians on the Mercator graticule are straight lines at right angles to the equator, the longitude scale is the same everywhere and provides the means of comparing chart lengths. The latitude scale cannot be used because it is continually being stretched as the latitude increases, and the distance of any parallel from the equator must be expressed in units of the longitude scale in order that the parallel may be drawn in its correct position on the graticule.

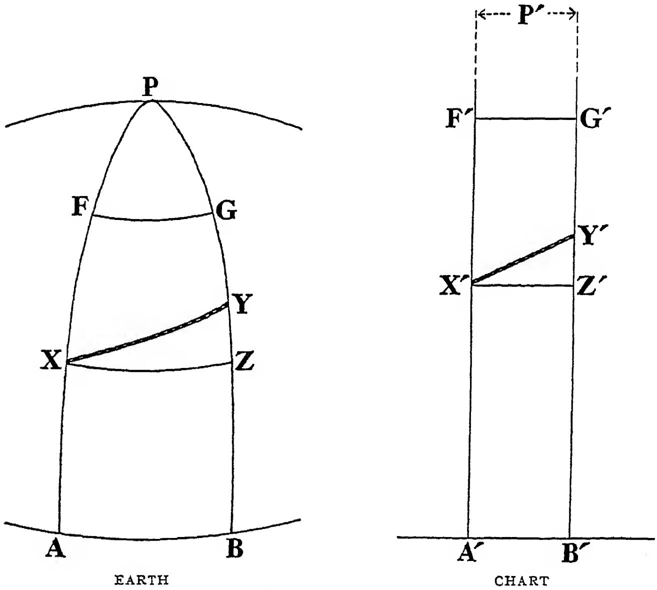


FIGURE 18.

Meridional Parts. In order to differentiate between these chart measurements along a meridian and the geographical distances in nautical miles which they actually represent, the chart measurements are always referred to as *meridional parts.* The meridional parts of any latitude are thus the number of longitude units in the length of a meridian between the parallel of that latitude and the equator, a longitude unit being the length on the chart which represents one minute of arc in longitude.

The formula giving the meridional parts of any latitude can be either built as an integral or derived from the general formula for the cone.

Construction of the Mer-Part Formula. In figure 18, X is any point on the Earth in latitude l, and Y is a neighbouring point

differing from it in latitude by the small amount $\triangle l$. X' and Y' are the corresponding points on the Mercator chart where, since all meridians are straight lines at right-angles to the equator, $A'B'$ is equal to $X'Z'$.

The ratio that the chart length $A'B'$ bears to the geographical distance AB decides the longitude scale of the chart. That is, when AB and $A'B'$ are expressed in the same units, $A'B'$ is some fraction of AB, or, what is the same thing, AB is equal to $k \,.\, A'B'$ where k is some constant. If, for example, AB is one minute of arc and $A'B'$ is one millimetre, one millimetre on the chart is equivalent to one minute of arc or approximately 1,854,000 millimetres on the Earth, and k is 1,854,000. The value of k thus determines the size of the chart. For the actual measurement of meridional parts, however, it is sufficient to know the chart unit that represents one minute of arc along the equator.

In the example quoted, where one millimetre represents one minute, the meridional parts of X' are simply the number of millimetres in $X'A'$.

To calculate this chart length and so determine the number of minutes of arc along the equator to which it is equivalent, consider the distortion that occurs away from the equator.

XZ is the parallel through X, and XY the rhumb line joining X to Y. On the chart $X'Z'$ is the parallel and $X'Y'$ the rhumb line, both lines being straight lines. Then, if all lengths XZ, AB, $A'B'$ . . . are measured in the same units :

$$\begin{aligned} XZ &= AB \cos l \\ &= k \,.\, A'B' \cos l \\ &= k \,.\, X'Z' \cos l \end{aligned}$$

i.e.
$$X'Z' = XZ\left(\frac{1}{k} \sec l\right)$$

Any arc of a parallel, the latitude of which is l, is thus represented on the chart by a line proportional to the actual length of the arc multiplied by $\sec l$, a quantity which is greater than unity. The distance scale along the parallel is therefore stretched.

Again, if Y is taken sufficiently close to X for XYZ to be considered a plane triangle right-angled at Z :

$$\frac{Z'Y'}{ZY} = \frac{X'Z'}{XZ} = \frac{1}{k} \sec l$$

i.e.
$$Z'Y' = ZY\left(\frac{1}{k} \sec l\right)$$

Any small element of a meridian in the neighbourhood of latitude l is thus represented on the chart by a line proportional to the actual length of the element multiplied by $\sec l$, and the distance scale along the meridian is therefore stretched.

The actual distance between Z and Y on the Earth, being $\triangle l$ in circular measure, is $3{,}437{\cdot}8\ \triangle l$ in minutes of arc. Hence:

$$Z'Y'=\frac{1}{k}\,3{,}437{\cdot}8 \sec l \triangle l$$

—in minutes of arc.

But one minute of arc is equal to k millimetres, or whatever the scale units are. Therefore:

$$Z'Y'=\left(\frac{1}{k}\,3{,}437{\cdot}8 \sec l \triangle l\right) k$$

—in millimetres or scale units.

The actual chart length of $Z'Y'$ in millimetres, or whatever the scale units are, is thus:

$$3{,}437{\cdot}8 \sec l \triangle l$$

The chart length of any particular parallel from the equator, measured along a meridian is clearly the sum of all the component elements of which the expression just found is typical. If the latitude of the parallel is ϕ, this sum, in the chosen units, is given by:

$$3{,}437{\cdot}8 \int_0^{\phi} \sec l\, dl$$

That is, the number of meridional parts or longitude units (a longitude unit being the length on the chart that represents one minute of arc in longitude) in the length of a meridian between latitude ϕ and the equator is:

$$3{,}437{\cdot}8 \log_e \tan (45°+\tfrac{1}{2}\phi)$$

Evaluation of the Mer-Part Formula. The actual evaluation of this formula can be accomplished more easily if the logarithm is expressed to base 10. Thus, if y is the number of meridional parts:

$$\begin{aligned} y&=3437{\cdot}8 \log_{10} \tan (45°+\tfrac{1}{2}\phi)\ .\ \log_e 10 \\ &=7915{\cdot}7 \log_{10} \tan (45°+\tfrac{1}{2}\phi) \end{aligned}$$

Suppose the latitude is 40°. Then:

$$\begin{aligned} y&=7915{\cdot}7 \log_{10} \tan 65° \\ &=7915{\cdot}7\times 0{\cdot}33133 && \begin{array}{|l} 3{\cdot}898\ 49 \\ 9{\cdot}520\ 26 \end{array} \\ &=2622{\cdot}7 && \begin{array}{|l} 3{\cdot}418\ 75 \end{array} \end{aligned}$$

Inman's Tables give the values that the formula assumes for every minute of arc as ϕ changes from 0° to 90°.

The Orthomorphic Property. Since the scale along a meridian in the neighbourhood of a point in latitude ϕ is stretched by the same amount ($\sec \phi$) as the scale along the parallel through that point, and the meridians and parallels on the Mercator projection are at right-angles, the projection must be orthomorphic.

Deduction of the Mer-Part Formula. For a conical projection to be orthomorphic, the radius r of any parallel must be given by:

$$r = k\,(\tan \tfrac{1}{2}\chi)^n$$

—where χ is the co-latitude of the parallel, n is the constant of the cone, and k is a constant defining the scale. This relation is obtained by equating the scale along the meridian and the scale along the parallel at any point.

If χ_0 is the co-latitude of the standard parallel, the radius of the standard parallel is given by:

$$r_0 = k\,(\tan \tfrac{1}{2}\chi_0)^n$$

The distance between these parallels is therefore:

$$\begin{aligned} r - r_0 &= k[(\tan \tfrac{1}{2}\chi)^n - (\tan \tfrac{1}{2}\chi_0)^n] \\ &= kn\,[\log_e \tan \tfrac{1}{2}\chi - \log_e \tan \tfrac{1}{2}\chi_0] \end{aligned}$$

—an equality obtained by expanding the right-hand side in its exponential form, and remembering that n ultimately tends to zero.

The value of k follows at once from the fact that $r_0 \cos \chi_0$ is equal to $R \sin \chi_0$, and is given by:

$$k = \frac{R \sin \chi_0}{\cos \chi_0} \cdot \frac{1}{(\tan \tfrac{1}{2}\chi_0)^n}$$

That is, since $\cos \chi_0$ is equal to n:

$$kn = \frac{R \sin \chi_0}{(\tan \tfrac{1}{2}\chi_0)^n}$$

When the cone becomes a cylinder, χ_0 becomes 90°, and kn is equal to R. The value of $(r - r_0)$, which is now the chart length of a parallel in latitude ϕ from the equator, measured along a meridian, is therefore:

$$R \log_e \tan \tfrac{1}{2}\chi$$

i.e. $$3{,}437{\cdot}8 \log_e \tan (45° + \tfrac{1}{2}\phi)$$

Adjustment of the Mer-Part Formula for the Earth's Ellipticity. The table of meridional parts given in *Inman's Tables* is built on the assumption that the Earth is a sphere, and makes no allowance for the ellipsoidal shape. Hence, if greater accuracy is required in finding the meridional parts of any place, the geographical latitude ϕ must be converted into the geocentric latitude θ, and the tables entered with the latter, θ being equal to $(\phi - r)$ where r is $c \sin 2\phi$. (Chapter I.)

If, for example, the geographical latitude is 50°22′, the reduction is 11′·5. The geocentric latitude is therefore 50°10′·5, and the meridional parts of the place are:

$$\begin{aligned} &7915{\cdot}7 \log_{10} \tan (45° + 25°05'{\cdot}25) \\ &= 3490'{\cdot}8 \end{aligned}$$

This figure could also be obtained directly from *Inman's Tables* by entering them for a latitude of 50°10′·5. The figure obtained for a latitude of 50°22′ is 3508′·8, an increase of 18′ on the true value.

In terms of the geographical latitude, the accurate meridional parts of any place can be shown to be:

$$3437{\cdot}8 \log_e \tan\left(45° + \tfrac{1}{2}\phi\right) - 3437{\cdot}8\, c \sin\phi$$

Mercator Sailing. Since the meridional parts of any place are expressed in longitude units, and the meridian along which they are measured is perpendicular to the equator and the parallels, the course between a place F on the equator and a place T in latitude ϕ is given by:

$$\tan(\text{course}) = \frac{\text{d'long}}{\text{mer-part } \phi}$$

If F is not on the equator but in latitude ϕ', the course is given by:

$$\tan(\text{course}) = \frac{\text{d'long}}{\text{mer-part } \phi - \text{mer-part } \phi'}$$

The Equation of a Great Circle on a Mercator Chart. The only great circles that appear as straight lines on a Mercator chart

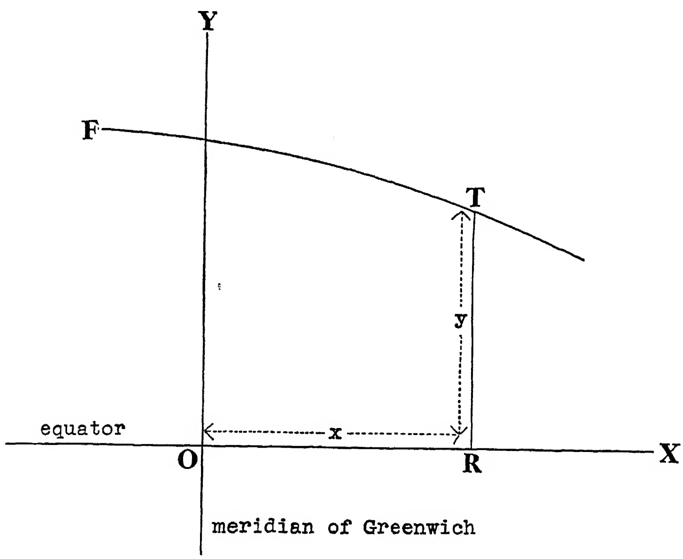


FIGURE 19.

are the meridians and the equator. All others are represented by curves.

In figure 19, FT is any curve on a Mercator chart, and the co-ordinates of any point T on it are referred to OX, the equator,

as the x-axis, and OY, the meridian of Greenwich, as the y-axis. TR is perpendicular to OX and therefore part of a meridian.

If x is the number of minutes of longitude in OR, y, measured in the same units, is the number of meridional parts in latitude ϕ, where ϕ is the latitude of the point on the Earth corresponding to T. It therefore follows that:

$$y = 3437{\cdot}8 \log_e \tan(45° + \tfrac{1}{2}\phi)$$

For convenience, express OR in circular measure. That is, take one radian as the unit instead of one minute of arc. Then, in terms of this unit:

$$y = 3437{\cdot}8 \log_e \tan(45° + \tfrac{1}{2}\phi) \times \frac{\pi}{180.60}$$
$$= \log_e \tan(45° + \tfrac{1}{2}\phi)$$

This equation gives y in terms of ϕ, and ϕ can be found in terms of y. Thus:

$$e^y = \tan(45° + \tfrac{1}{2}\phi)$$
$$= \frac{1 + \tan\tfrac{1}{2}\phi}{1 - \tan\tfrac{1}{2}\phi}$$

From this relation it follows that:

$$e^y = \sec\phi + \tan\phi$$

and

$$e^{-y} = \sec\phi - \tan\phi$$

From these equations, therefore:

$$\sec\phi = \tfrac{1}{2}(e^y + e^{-y})$$
$$\tan\phi = \tfrac{1}{2}(e^y - e^{-y})$$

In figure 20, UQV represents any great circle on the Earth, and POP' the meridian of Greenwich cutting the equator in O. Then, if F is any point on this great circle, the longitude of F is OR, denoted by x in circular measure.

The longitude of Q, the point where the great circle cuts the equator, is OQ, denoted by λ, also in circular measure.

The latitude of F is ϕ, and the angle FQR is denoted by ϵ.

Then, by Napier's rules applied to the triangle FQR:

$$\tan\phi = \tan\epsilon \sin QR$$

i.e.

$$\tan\phi = \tan\epsilon \sin(x - \lambda)$$

This relation holds for all points on the great circle UOV.

Again, if y is the ordinate of the point corresponding to F on the chart:

$$\tan\phi = \tfrac{1}{2}(e^y - e^{-y})$$

Therefore:

$$e^y - e^{-y} = 2\tan\epsilon \sin(x - \lambda)$$

This is the general equation of the curve on a Mercator chart that represents a great circle on the Earth. It contains two

constants, ϵ and λ, and the values of these constants determine the great circle.

A Great Circle Determined by Two Points on it. Suppose F and T are two given points on the great circle UQV, the

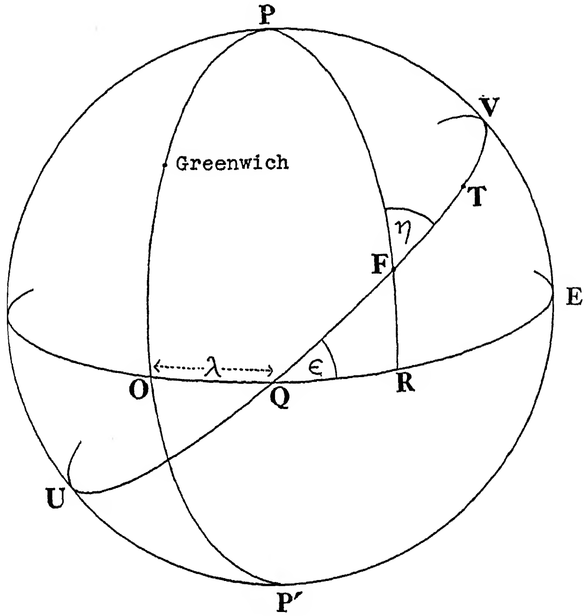


FIGURE 20.

longitudes of these points being x and x', and their latitudes ϕ and ϕ'. Then :

$$\tan\phi = \tan\epsilon \sin(x-\lambda)$$

and
$$\tan\phi' = \tan\epsilon \sin(x'-\lambda)$$

These two equations give ϵ and λ. Thus :

$$\frac{\tan\phi}{\tan\phi'} = \frac{\sin(x-\lambda)}{\sin(x'-\lambda)}$$

i.e
$$\frac{\tan\phi - \tan\phi'}{\tan\phi + \tan\phi'} = \frac{\sin(x-\lambda) - \sin(x'-\lambda)}{\sin(x-\lambda) + \sin(x'-\lambda)}$$

$\therefore$
$$\tan[\tfrac{1}{2}(x+x') - \lambda] = \tan\tfrac{1}{2}(x-x') \times \frac{\sin(\phi+\phi')}{\sin(\phi-\phi')}$$

This equation gives λ.

When λ has been found, ϵ can be calculated from either of the first two equations, and the values of λ and ϵ thus obtained give, when substituted in the general equation, the equation of the curve that represents the great circle through F and T.

It is required to draw on a Mercator chart the curve representing the great circle for which λ is 60°E. and ϵ 50°. The scale is 1 inch to 90° of longitude.

By substituting successive values of the longitude x in :

$$\tan \phi = \tan 50° \sin (x - 60°)$$

—corresponding values of the latitude ϕ are obtained. Suppose these successive longitudes are 60°, 90°, 120°, 150°, 180°, 210°, and 240°. The values of $(x-60°)$ are then 0°, 30°, 60°, 90°, 120°, 150°, and 180°.

Once ϕ has been found, the value of y, being the meridional parts of ϕ, is obtained from *Inman's Tables* and converted into inches according to the scale of one inch to 5,400 minutes of longitude.

The calculation is set out thus :

x	60°, 240°	90°, 210°	120°, 180°	150°
$x-60°$	0°, 180°	30°, 150°	60°, 120°	90°
log sin $(x-60°)$ log tan 50°	— 0·0762	9·6990 0·0762	9·9375 0·0762	0·0000 0·0762
log tan ϕ	—	9·7752	0·0137	0·0762
ϕ	0	30°48′	45°54′	50°
mer-part ϕ	0	1944·0	3106·9	3474·5
y in inches	0	0·360	0·575	0·643

If $(x-\lambda)$ lies between 180° and 360°, $\sin (x-\lambda)$ is negative. Tan ϕ is therefore negative and ϕ is south.

The curve, shown in figure 21, is thus symmetrical about the x-axis.

A Great Circle Determined by One Point on it and the Angle at this Point between the Great Circle and the Meridian. In figure 20, F is the given point, latitude ϕ and longitude x, and the angle PFT (which is equal to the angle QFR) is η.

By Napier's rules applied to the triangle QFR :

$$\cos \epsilon = \sin \eta \cos \phi$$

Therefore, when η and ϕ are known, ϵ can be calculated.

Also, since F is a point on the great circle :

$$\tan \phi = \tan \epsilon \sin (x-\lambda)$$

or

$$\sin (x-\lambda) = \tan \phi \cot \epsilon$$

Hence λ can be found, and, with ϵ, substituted in the general equation to determine the particular curve. This done, the curve is traced as before.

From figure 20, it is clear that the highest latitude reached by the great circle is the latitude of V, the vertex. But EV is ϵ and QE is 90°. The latitude of V is therefore ϵ and the longitude is $(90°+\lambda)$. Both quantities are obtained by making η equal to 90°.

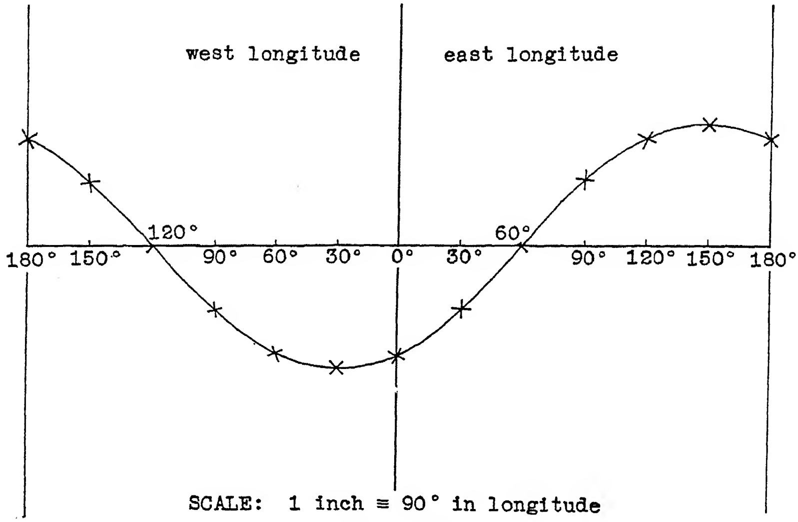


FIGURE 21.

It is required to draw on a Mercator chart the curve representing a great circle which passes through a point in latitude 40°*N*., *longitude* 100°*E*., *and makes an angle of* 58° *with the meridian.*

	$\cos \epsilon = \sin 58° \cos 40°$	9·9284 9·8843
i.e.	$\epsilon = 49°29'$	9·8127
	$\sin (x-\lambda) = \tan 40° \cot 49°29'$	9·9238 9·9318
i.e.	$x-\lambda = 45°49'$	9·8556

But the longitude x is 100°. Therefore λ is 54°11′.

The curve is now traced by the method employed in the previous example.

The latitude of the vertex is 49°29′N.; the longitude 144°11′E.

The Equation of the Curve Representing a Position Circle. A position circle, being a circle drawn on the Earth's surface with the geographical position of the heavenly body as centre, is a small circle. When plotted on a Mercator chart, this curve of position

will no longer be a circle, and the problem is to find the equation of the resulting curve.

Figure 22 shows the relative positions of the pole P, the observer Z, and the geographical position of the heavenly body U, when the true altitude (obtained from a sextant reading) is a, and the declination is d. The latitude of Z is ϕ. Then, if X and x are the easterly longitudes of Z and U, the hour angle of the heavenly body is $(x-X)$.

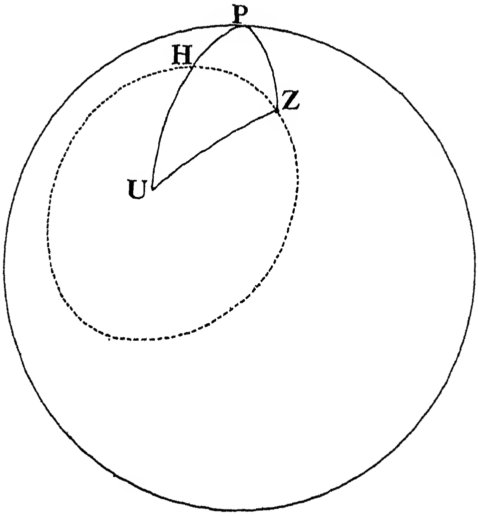


FIGURE 22.

The fundamental formula applied to the spherical triangle PZU gives:

$$\cos UZ = \cos PU \cos PZ + \sin PU \sin PZ \cos UPZ$$

i.e.
$$\sin a = \sin d \sin \phi + \cos d \cos \phi \cos (x-X)$$

or
$$\cos (x-X) = \sin a \sec d \sec \phi - \tan d \tan \phi$$

If the co-ordinates of Z on the chart are x and y, x is given by this equation, and y (page 36) by:

$$\sec \phi = \tfrac{1}{2}(e^{y} + e^{-y})$$

and
$$\tan \phi = \tfrac{1}{2}(e^{y} - e^{-y})$$

Hence, by substitution:

$$2 \cos (x-X) = e^{y}(\sin a \sec d - \tan d) + e^{-y}(\sin a \sec d + \tan d)$$

This is the general equation of the curve on the chart that represents the position circle, and the curve itself is defined by the values of a, d and X. Its equation is simplified when these factors have certain values.

(1) *When the altitude equals the declination.*

When a equals d, the general equation becomes :

$$2 \cos (x-X)=2e^{-y} \tan d$$

i.e.
$$e^{y}=\frac{\tan d}{\cos (x-X)}$$

i.e.
$$y-\log_e \tan d=\log_e \sec (x-X)$$

If Y denotes the left-hand side of this equation, then Y is an ordinate measured, not from the equator, but from the parallel of latitude ϕ_0, where ϕ_0 is defined (page 36) by :

$$\log_e \tan (45^\circ+\tfrac{1}{2}\phi_0)=\log_e \tan d$$

i.e.
$$\phi_0=2d-90^\circ$$

The equation referred to the parallel of ϕ_0 is thus :

$$Y=\log_e \sec(x-X)$$

—and in this form is easily plotted.

(2) *When the altitude and declination are equal but of opposite names.*

When a equals $-d$, the general equation becomes :

$$2 \cos (x-X)=2e^{y} \tan a$$

i.e.
$$y+\log_e \tan a=\log_e \cos(x-X)$$

By moving the origin to the parallel of ϕ_1, where :

$$\phi_1=2a-90^\circ$$

—the equation can be written :

$$Y=-\log_e \sec (x-X)$$

—and this, again, is easily plotted.

(3) *When the declination is zero.*

When d equals 0, the general equation becomes :

$$2 \cos (x-X)=(e^{y}+e^{-y}) \sin a$$

i.e.
$$e^{2y}-2e^{y}\,\frac{\cos (x-X)}{\sin a}+1=0$$

This is a quadratic in e^y. Hence :

$$e^{y}=\frac{\cos (x-X)}{\sin a}\pm\sqrt{\frac{\cos^2 (x-X)}{\sin^2 a}-1}$$
$$=\frac{\cos (x-X)\pm\sqrt{\cos^2 (x-X)-\sin^2 a}}{\sin a}$$

For real values of e^y and therefore of y, $\cos^2 (x-X)$ must not be less than $\sin^2 a$. That is :

$$\cos^2 (x-X) \nless \cos^2 (90^\circ-a)$$

i.e.
$$(x-X) \ngtr (90^\circ-a)$$

Hence the longitudes of points on the curve on the chart must not be more than $(90°-a)$ east or west of the geographical position of the heavenly body. The curve is thus bounded by the meridians that are this distance in longitude east and west of the geographical position.

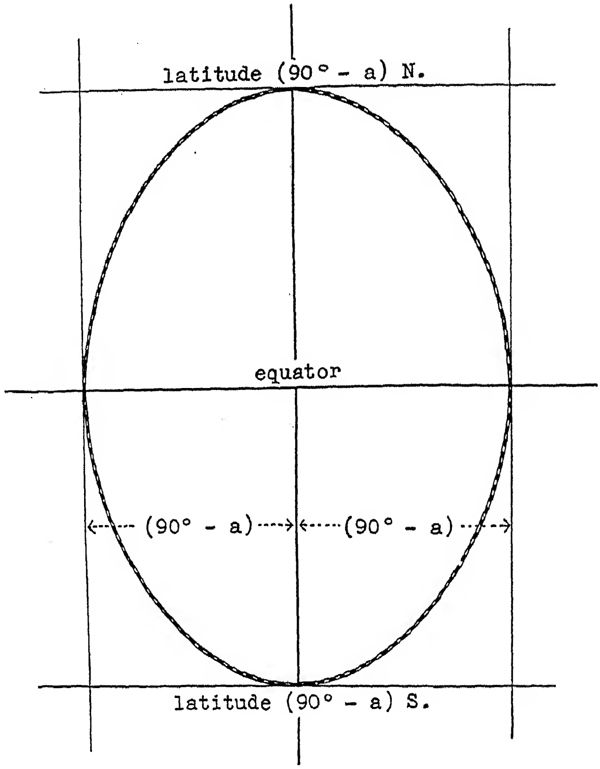


FIGURE 23.

The points where the curve cuts the meridian through the geographical position are found by putting $(x-X)$ equal to zero.

$$e^y = \frac{1 \pm \cos a}{\sin a}$$

i.e. $y = \log_e \cot \tfrac{1}{2}a$ or $\log_e \tan \tfrac{1}{2}a$

If ϕ_2 and ϕ_3 are the latitudes of these two points of intersection:

$$\log_e \cot \tfrac{1}{2}a = \log_e \tan (45° + \tfrac{1}{2}\phi_2)$$

and

$$\log_e \tan \tfrac{1}{2}a = \log_e \tan (45° + \tfrac{1}{2}\phi_3)$$

Hence:

$$90° - \tfrac{1}{2}a = 45° + \tfrac{1}{2}\phi_2$$

and

$$\tfrac{1}{2}a = 45° + \tfrac{1}{2}\phi_3$$

i.e.

$$\phi_2 = 90° - a$$

and

$$\phi_3 = a - 90°$$

Since a is less than 90°, ϕ_3 is $(90°-a)$ south.

Figure 23 shows the curve as it appears on a Mercator chart, and figure 24 shows three typical curves representing position circles for three values of the altitude when the geographical position is in latitude 40°N., longitude 60°W.

Procedure when the Altitude is Large. When the observer is in low latitudes, and the heavenly body is nearly overhead, the position circle may be drawn as a circle on a Mercator chart without appreciable loss of accuracy. The centre of this circle is the geographical position, and the radius is the true zenith distance.

When two observations of this kind are taken, two position circles may be drawn, and the observer's position is at one of their two points of intersection. To determine the point, the observer

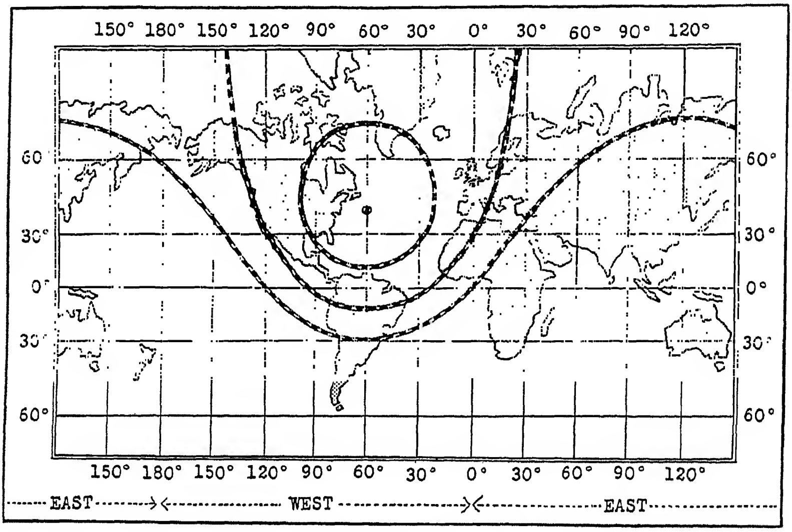


FIGURE 24.

must note whether he is north or south of the heavenly body when it crosses his meridian.

If the observer is in a ship and there is a run between sights, the first position circle must be transferred for the run. This can be done by transferring the geographical position and then drawing the circle.

Example of Transferred Position Circles. *At Z.T.* 1215(—6) *on the 3rd April* 1937, *in D.R. position* 5°40′*N.*, 86°45′*E.*, *the following observations of the Sun were taken :*

	h. m. s.		
deck watch	6 14 10	*sext. alt.*	89°03′·0
	6 16 54		89°15′·2
	6 19 04		88°56′·8

The deck watch was 4^s *slow on G.M.T.; the index error was* $+1'\cdot5$ *and the height of eye* 40 *feet. The ship was steaming* 300°, 18 *knots, and the Sun crossed the observer's meridian to the south.*

				Sun's Declination
Z.T.	1215	3rd April		5°09′·8N.
Zone	−6			0·2
G.D.	0615	3rd April		5°10′·0N.

	h m s		E. h m s
D.W.T.	6 14 10		
Error slow	4		
G.M.T.	6 14 14	3rd April	11 56 30·8
E.	11 56 31		0·2
G.H.A.T.S.	18 10 45		11 56 31·0
Long. E.	5 49 15		
	=87°18′·7		

The Sun's geographical position at the time of the first observation is therefore:

$$\begin{cases} 5°10'\cdot0\text{N.} \\ 87°18'\cdot7\text{E.} \end{cases}$$

		h m s
D.W.T.	1st Obs.	6 14 10
	2nd Obs.	6 16 54
		2 44=41′W.
		∴departure=40′·8W.
D.W.T.	1st Obs.	6 14 10
	3rd Obs.	6 19 04
		4 54=73′·5W.
		∴departure=73′·2W.

The run between 1st and 3rd observations is:

$$\frac{18\times4\cdot9}{60} \text{ or } 1'\cdot5 \text{ at } 300°.$$

The run between 2nd and 3rd observations is:

$$\frac{18\times2\cdot2}{60} \text{ or } 0'\cdot7 \text{ at } 300°.$$

Obs. Alt.	89°03′·0	89°15′·2	88°56′·8
I.E.	+1′·5	+1′·5	+1′·5
	89°04′·5	89°16′·7	88°58′·3
Corr^n	+9′·7	+9′·7	+9′·7
	89°14′·2	89°26′·4	89°08′·0
T.Z.D.	45′·8	33′·6	52′·0

In figure 25, A, B and C are the geographical positions of the Sun at the three deck-watch times of observation, AB being equal to 40′·8 and AC to 73′·2; and a and b are the positions of A and B transferred for runs of 1′·5 and 0′·7 at 300°. The position circles are drawn with centres a, b and C, and radii 45′·8, 33′·6 and 52′. These circles cut in O, the observed position.

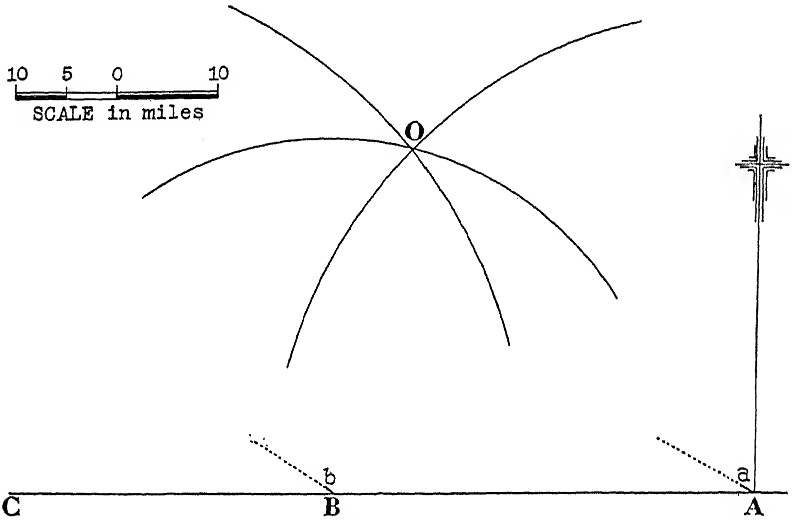


FIGURE 25.

By measurement, the d'lat is 33′·5N., and the departure is 33′·2W., both from A. The d'long is therefore 33′·4W., and the ship's position is :

$$\begin{cases} 5°43'\cdot 5\text{N.} \\ 86°45'\cdot 3\text{E.} \end{cases}$$

Error Introduced by taking the Position Circle as a Circle. The error which is introduced by drawing the position circle as a circle on a Mercator chart in the circumstances of the above example, may be found by comparing the actual curve with the circle.

In figure 26, U is the heavenly body's geographical position and the curve obtained by plotting the position circle is KLM. The circle to which this curve approximates is klm.

The true zenith distance, z, is small. Therefore, if ϕ is the latitude of U :

$$Uk = z \sec \phi \text{ (approximately)}$$

Also :

$$UK = (\text{mer-part } k - \text{mer-part } U)$$

Since the distortion of a Mercator chart increases with the latitude, the error will be greatest at k, and the error at this point is given approximately by :

$$Kk' = (\text{mer-part } k - \text{mer-part } U) - z \sec \phi$$

The latitude of the Sun's geographical position cannot exceed $23\frac{1}{2}°$. For an altitude of $88\frac{1}{2}°$, the maximum error is therefore given by:

$$(\text{mer-part } 25° - \text{mer-part } 23\tfrac{1}{2}°) - 90' \sec 23\tfrac{1}{2}°$$
$$= 0'{\cdot}6 \text{ (approximately)}$$

In low latitudes the position circle of the Sun can thus be drawn as a circle on the Mercator chart without appreciable loss of accuracy

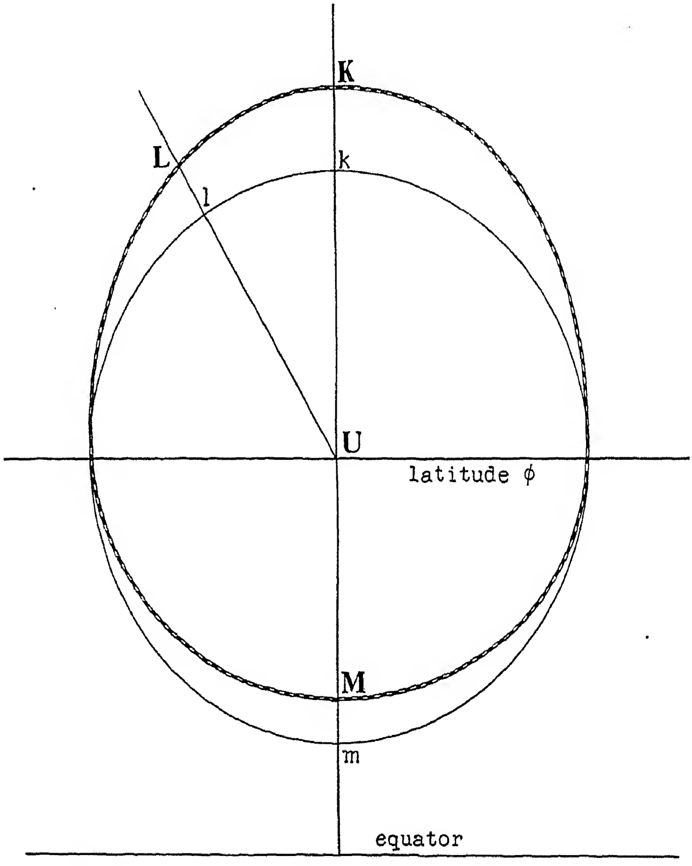


FIGURE 26.

for an altitude greater than $88\frac{1}{2}°$. Also, if the radius of the circle is carefully measured on the latitude scale at a point midway between the geographical and the D.R. positions, the error is still further reduced. The maximum errors for the Sun are then $0'{\cdot}7$, $0'{\cdot}3$ and $0'{\cdot}1$ at altitudes of 82°, 84° and 86°.

CHAPTER V

GREAT-CIRCLE AND COMPOSITE SAILING

The great-circle track between two places F and T is the shortest distance between them. In Chapter IV of Volume II it was shown how this distance and the initial course, or bearing of T from F, can be found by calculation when the latitudes and longitudes of F and T are known. Direct calculation, however, is not always necessary.

Great-Circle Distance from Altitude-Azimuth Tables. If altitude-azimuth tables are available, both distance and initial course can be found from them because the spherical triangle PFT and the astronomical triangle PZX are analogous, as explained in Chapter XIII of Volume II. The difference of longitude between F and T corresponds to the hour angle, the great-circle distance to the zenith distance, and the initial course to the azimuth.

If the latest types of altitude-azimuth tables are used, the calculated altitude is given, not the calculated zenith distance, and the angle obtained must be subtracted from 90°.

For example, the position of F (corresponding to the observer) is 34°31′N., 28°36′W., and the position of T (corresponding to the heavenly body) is 5°13′·5N., 32°50′·6E. The d'long (or hour angle) is then 61°26′·6E.

From the tables for latitude 35°N., declination 5°N., and hour angle 61° :

Altitude	26°27′·7	$\triangle d$ 60	$\triangle h$ 80
$\triangle d$ (for 13′·5)	+8′·1		
$\triangle h$ (for 26′·6)	−21′·3		
Calc. Alt.	26°14′·5		

The great-circle distance between 35°N., 28°36′W., and 5°13′·5N., 32°50′·6E., is therefore 63°45′·5.

From the tables for latitude 34°N., declination 5°N., and hour angle 61° :

Altitude	26°41′·3	$\triangle d$ 59	$\triangle h$ 81
$\triangle d$ (for 13′·5)	+8′·0		
$\triangle h$ (for 26′·6)	−21′·6		
Calc. Alt.	26°27′·7		

The great-circle distance between 34°N., 28°36′W., and 5°13′·5N., 32°50′·6E., is therefore 63°32′·3.

The great-circle distance between F and T is thus approximately:

$$\begin{aligned}&63°32'\cdot3+\frac{31}{60}\,(63°45'\cdot5-63°32'\cdot3)\\&=63°32'\cdot3+6\cdot8\\&=63°39'\cdot1 \text{ or } 3819'\end{aligned}$$

The initial course (or azimuth) can be taken without interpolation as N.103°E.

To Find the Vertex of a Great Circle. If a series of parallels is drawn, it is clear that one parallel will touch the great circle FT at a point V. This point is known as the *vertex* of the great circle, and it is the point on the great circle nearest the pole in the hemisphere under consideration. (Figure 27.)

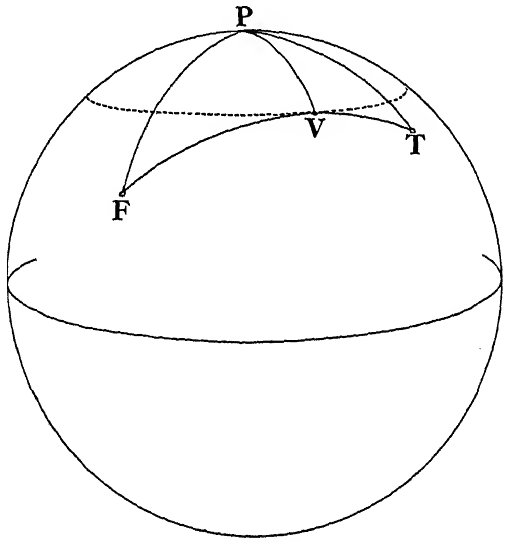


FIGURE 27.

For reasons of safety it is essential that the navigator should know the highest latitude into which a great-circle track will take him. This latitude, which is the latitude of the vertex, can be found by calculation or the use of *Towson's Tables*, but it is most simply found by plotting on a gnomonic chart.

Since the great circle and the parallel touch at V and the meridian PV cuts the parallel at right-angles, it also cuts the great circle at right-angles, and the spherical triangles PFV and PTV are right-angled at V.

The longitude of the vertex can be found at once from the formula, derived from the polar gnomonic triangle:

$$\tan(\text{d'long}_{VT})=\tan(\text{lat.}F)\ \cot(\text{lat.}T)\text{cosec}(\text{d'long}_{FT})-\cot(\text{d'long}_{FT})$$

—and the latitude from:

$$\cot(\text{lat. }V)=\cot(\text{lat. }F)\cos(\text{d'long}_{FV})$$

Otherwise, if the initial course has been found, the position of V can be obtained from Napier's rules. Thus:

$$\cos(\text{lat. V}) = \cos(\text{lat. } F)\sin(\text{initial course})$$

$$\tan(\text{d'long}_{FV}) = \operatorname{cosec}(\text{lat. } F)\cot(\text{initial course})$$

It is required, for example, to find the position of the vertex of the great circle joining F (39°20′S., 110°10′E.) to T (44°30′S., 46°20′W.).

By solving the spherical triangle $P'FT$ (both places are in the southern hemisphere), the initial course is found to be 16°33′·9. Then:

lat. F=39°20′	log cos	9·888 44	log cosec	0·198 03
$P'FV$=16°33′·9	log sin	9·455 00	log cot	0·526 59
	log cos (lat V)	9·343 44	log tan (d'long)	0·724 62
i.e.	lat. V=77°15′·6S.		d'long= 79°19′·4	

The longitude of F is 110°10′E., and V is west of F. The longitude of V is thus 30°50′·6E.

The position of V is $\begin{cases} 77°15'{\cdot}6\text{S.} \\ 30°50'{\cdot}6\text{E.} \end{cases}$

Towson's Tables. The production of suitable gnomonic charts covering all oceans where great circles can be followed with advantage, has largely done away with the utility of these tables which are constructed for the purpose of giving a navigator his great-circle track.

They are based on the solution of the spherical triangle PFV by Napier's rules, which give:

$$\cot PF = \cot PV \cos FPV \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$\tan FV = \sin PV \tan FPV \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$\cos PFV = \cos PV \sin FPV \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (1) shows that if the latitude of the vertex and the d'long between the vertex and a point F on the great circle are known, the latitude of F can be found.

Equation (2) shows that the distance along the great circle from the vertex to F can be found.

Equation (3) shows that the angle PFV, which gives the course at F along the great circle, can be found.

In *Towson's Tables* the latitude of F, the great-circle distance between V and F, and the angle PFV are given for every degree of the latitude of F and every degree, up to 90°, of d'long between V and F. Hence, if the latitudes of V and F are known, the d'long between V and F, the great-circle distance FV and the angle PFV can be found.

The latitude of V, for the great circle joining the two given points F and T, is taken from a diagram by means of the Linear Index, an explanation of which is given in the tables.

When the latitude of V has been found, the d'long between V and F is obtained from the tables. The angles FPV and TPV are thus known. The distances of F and T from V, and the values of the angles PFV and PTV can then be found from the tables by inspection. The labour of calculating the great-circle distance between two places and the courses along the great circle is therefore reduced considerably.

To Plot a Great-Circle Track on a Mercator Chart. The simplest method of plotting a great-circle track on a Mercator chart is that

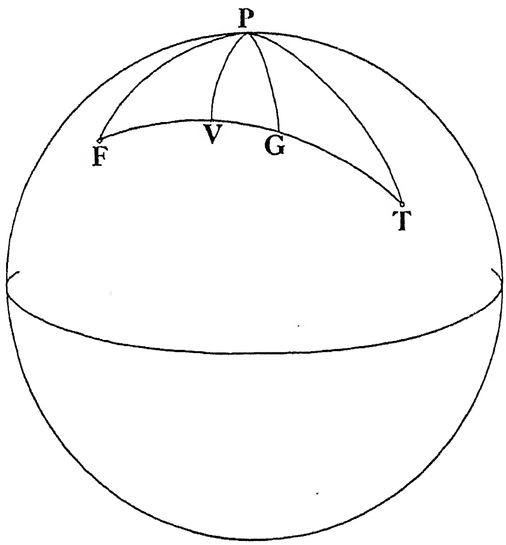


FIGURE 28.

by which points are transferred from a gnomonic chart. (Chapter V, Volume II.) But if a gnomonic chart is not available, the track can be plotted with reference to the vertex, a method which is less laborious than that described on page 38 of this Volume if only a part of the great circle need be drawn.

Consider the position of any point G on the great circle joining F to T, G being fixed by its difference of longitude from V which is taken for convenience as 10°. (Figure 28.)

Since F and T are known points, the latitude of V can be found.

Napier's rules applied to the spherical triangle PVG give :

$$\tan PG = \tan PV \sec VPG$$

or

$$\cot (\text{lat. } G) = \cot (\text{lat. } V) \sec (\text{d'long})$$

The latitude of G is thus known, and the point on the Mercator chart corresponding to G can be plotted. If a series of points is now taken at intervals of 10° in longitude, the latitude of each can be calculated, and corresponding points marked on the chart and joined by a smooth curve.

Towson's Tables make even this small calculation unnecessary because they include d'long as an argument, and the latitudes of G and the other points in the series can be obtained by inspection.

It is, for example, required to plot on a Mercator chart the great circle, the vertex of which was found in the previous example worked on page 49. The given positions are thus:

$$\left\{\begin{array}{l}39°20'\text{S.}\\110°10'\text{E.}\end{array}\right. \qquad T\left\{\begin{array}{l}44°30'\text{S.}\\46°20'\text{W.}\end{array}\right. \qquad V\left\{\begin{array}{l}77°16'\text{S.}\\30°51'\text{E.}\end{array}\right.$$

If G is a point on this great circle, differing in longitude from V by 10°E., the latitude of G is given by:

$$\cot(\text{lat. } G) = \cot 77°16' \sec 10°$$

$$\therefore \quad \log \cot(\text{lat. } G) = 9{\cdot}354\ 05 + 0{\cdot}006\ 65$$
$$= 9{\cdot}360\ 70$$

i.e. $$\text{latitude } G = 77°04'$$

In a similar manner the latitudes of points differing in longitude from V by 20°, 30°, 40° . . . are found and are shown in the following tables:

	Longitude West from Vertex						
	70°	60°	50°	40°	30°	20°	10°
Latitude	56°32′	65°39′	70°37′	73°35′	75°24′	76°35′	77°04′
Longitude.	39°09′W.	29°09′W.	19°09′W.	9°09′W.	0°51′E.	10°51′E.	20°51′E.
	M'	L'	K'	J'	I'	H'	G'

	Longitude East from Vertex						
	10°	20°	30°	40°	50°	60°	70°
Latitude	77°04′	76°35′	75°24′	73°35′	70°37′	65°39′	56°32′
Longitude.	40°51′E.	50°51′E.	60°51′E.	70°51′E.	80°51′E.	90°51′E.	100°51′E.
	G	H	I	J	K	L	M

Figure 29 shows the great-circle curve obtained by joining these points on the Mercator chart.

Once the great circle has been plotted on the chart, the navigator can obtain all information about the courses he must steer to keep as close as possible to the great circle, merely by drawing a series of chords from consecutive and conveniently situated points on it. If *M*, *L*, *K* . . . are these points, he would steer rhumb-line courses along *FM*, *ML*, *LK*. . . . He cannot keep to the great circle itself because, to do that, he would have to alter course continuously.

Great-Circle Sailing by Means of Azimuth Tables. An easy method of sailing along a great circle is afforded by ordinary azimuth tables because the azimuth corresponds to the initial course. The navigator, setting out from *F* to *T*, has merely to look up the solu-

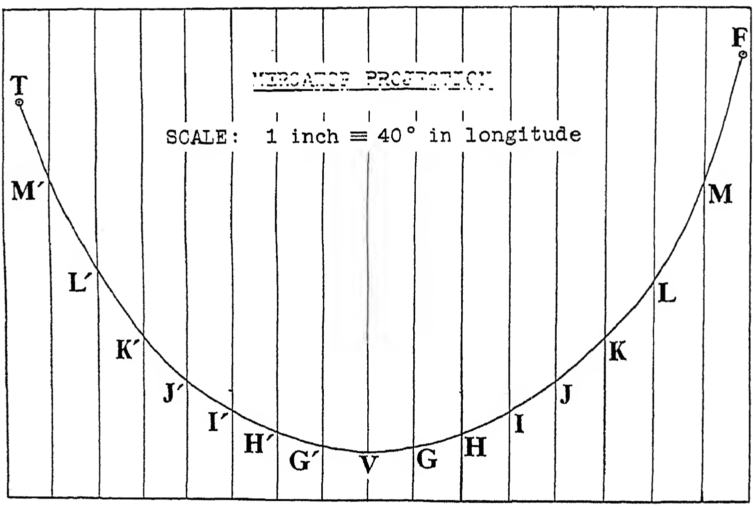


FIGURE 29.

tion of the spherical triangle for the latitudes of *F* and *T* and the d'long, in order to find his initial course.

This course takes him steadily away from the great circle joining *F* and *T*. Also he may experience a set. When, therefore, he fixes his position, he may find himself at *A*. (Figure 30, which is purposely exaggerated.) He has now merely to look up the solution of the spherical triangle for the latitudes of *A* and *T* and the d'long, in order to find the initial course for the great circle joining *A* and *T*. Next day, after fixing his position at *B*, he would find the initial course for the great circle joining *B* and *T*.

This method is often adopted in merchant ships employed on a service that takes them frequently over the same great-circle

route. All calculations are made in the navigator's notebook, and the chart plays only a small part in the actual navigation.

Composite Sailing. When danger is likely to be encountered in high latitudes, the great-circle track must be modified to avoid

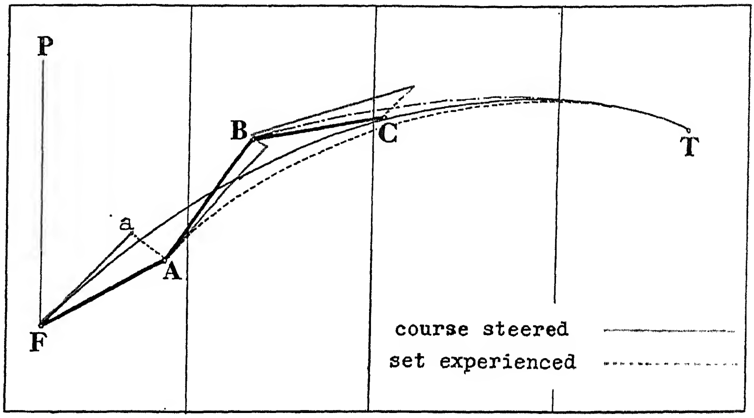


FIGURE 30.

it and yet remain the shortest track open to the navigator. The resulting track is known as a *composite track* because it is formed by two great-circle arcs and an arc of the limiting or 'safe' parallel beyond which the navigator will meet danger.

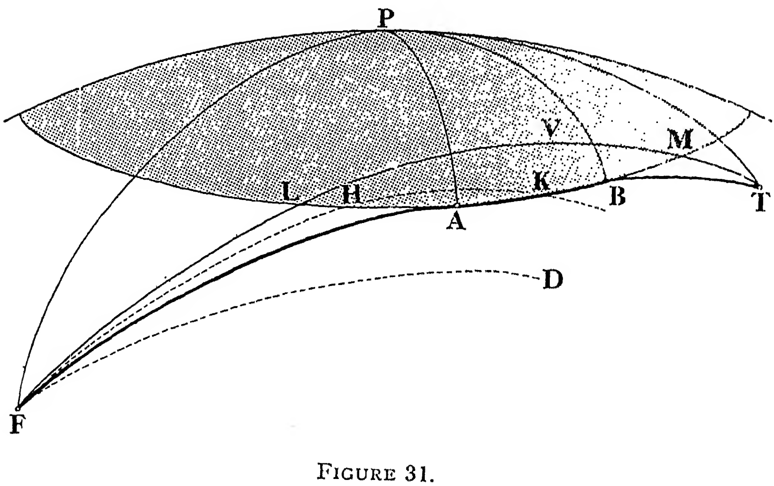


FIGURE 31.

In figure 31, the limiting parallel is $LABM$; the great circle joining F and T is $FLVMT$; and the composite track is $FABT$, in which FA and BT are great-circle arcs touching the parallel at A and B, and AB is part of the parallel itself.

The composite track $FABT$ is the shortest track which the navigator can follow without crossing the limiting parallel for two reasons.

(1) If FH is any great circle from F to the parallel, the distance given by the great-circle arc FH and the arc HA of the parallel is greater than the great-circle arc FA, since the arc HA is greater than the great-circle arc joining H and A, and the great-circle arcs FH and HA are together greater than the great-circle arc FA.

(2) If K is any point between A and B, the great circle joining F and K must cut the parallel before it reaches K. This is so because in the spherical triangle PFA, in which the angle A is a right-angle :

$$\sin PA = \sin FP \sin PFA$$

Therefore, if the angle PFA is increased, PA is increased, F being a fixed point. That is, the latitude of the vertex of the new great circle FD is less than the latitude of A and of K. Hence K must lie on a great circle so that the angle PFK is less than the angle PFA. It is thus impossible to join F to any point between A and B by a great circle without crossing the parallel.

To Calculate the Composite Track. The positions of A and B are quickly found because the angles at A and B are right-angles. Also, along AB the ship is steering a course of 090°, and, if the latitude of this limiting parallel is ϕ :

$$AB = \text{d'long} \cos \phi$$

The fundamental and sine formulæ applied to the right-angled spherical triangle PFA give :

$$\cos PF = \cos PA \cos FA$$

i.e.

$$\cos FA = \frac{\cos PF}{\cos PA} \quad \dots\dots\dots \quad (1)$$

$$\sin FPA = \frac{\sin FA}{\sin PF} \quad \dots\dots\dots \quad (2)$$

Formula (1) gives the length of the great-circle arc FA and formula (2) the difference of longitude between F and A by which the position of A is decided.

Similar formulæ can be obtained from the right-angled spherical triangle PTB.

It is required, for example, to find the distance by the composite track from F (39°20′S., 110°10′E.) to T(44°30′S., 46°20′W.), the limiting latitude being 62°S.

From the spherical triangle $P'FA$:

$P'F = 50°40'$	log cos	9·801 97
$PA = 28°00'$	log cos	9·945 93
	log cos FA	9·856 04

i.e. $FA = 44°07'·3$ or $2647'·3$

$FA = 44°07'·3$	log sin	9·842 73
$P'F = 50°40'$	log sin	9·888 44
	log sin $FP'A$	9·954 29

i.e. d'long $= 64°10'·3$

From the spherical triangle $P'TB$:

$P'T = 45°30'$	log cos	9·845 66
$P'B = 28°00'$	log cos	9·945 93
	log cos TB	9·899 73

i.e. $TB = 37°27'·3$ or $2,247'·3$

$TB = 37°27'·3$	log sin	9·784 00
$P'T = 45°30'$	log sin	9·853 24
	log sin $TP'B$	9·930 76

i.e. d'long $= 58°29'·9$

The difference of longitude between A and B is given by:

$$\angle AP'B = \angle FP'T - (\angle FP'A + \angle TP'B)$$
$$= 33°49'·8 \quad \text{or} \quad 2,029'·8$$

Therefore:

$$AB = 2,029'·8 \cos 62°$$

i.e. $\log AB = 3·307\ 46 + 9·671\ 61$
$= 2·979\ 07$

i.e. $AB = 953'·0$

The total distance by composite track is thus:

$$FA + AB + BT$$
$$= 2647'·3 + 953'·0 + 2247'·3$$
$$= 5847'·6$$

The great-circle distance can be shown to be 5612′. The dist. by composite track is thus 236′ longer than the great-circle dist.

CHAPTER VI

THE GNOMONIC CHART

It was stated in Chapter III that the gnomonic chart is a zenithal projection of the perspective type, the centre of projection being the centre of the Earth, and that, since the planes of all great circles pass through the centre of the Earth, all great circles project into straight lines. The great-circle track between two places is therefore found by drawing a straight line between them, and the navigator is thus able to avoid the calculations explained in the last chapter.

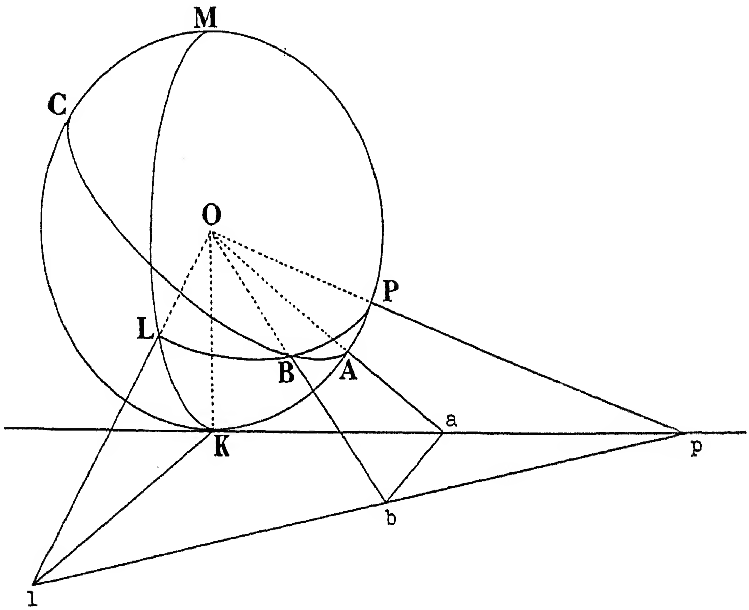


FIGURE 32.

The Principal or Central Meridian. The plane on which the parallels and meridians are projected is a tangent plane, and to avoid distortion the tangent point, K, should be chosen in the centre of the area to be shown. Apart from this, there is no restriction on its position. In figure 32 its latitude is 20°N.

Since the gnomonic projection is a perspective projection, the point on the tangent plane that corresponds to a point on the sphere that represents the Earth, is found by producing the radius at the point until it cuts the tangent plane. Thus p corresponds

to P, the pole, and all points on the meridian PK project into the straight line pK. PK is known as the principal or central meridian.

If B is any point and ABC any great circle through it, the arc AB projects into the straight line ab.

The meridian through B is PBL, and since it is part of a great circle, pbl is also a straight line. The meridians on the gnomonic graticule are thus straight lines radiating from p.

The straight line Kl corresponds to the great-circle arc KL.

If ϕ_K and ϕ_A are the latitudes of K and A, then :

$$KP=90°-\phi_K=\angle KOP$$

and

$$AP=90°-\phi_A=\angle AOP$$

Since Kp lies in a plane tangential to the sphere at K, OK, the radius of length R, is perpendicular to Kp. Therefore :

$$\frac{Kp}{OK}=\tan KOp=\tan KOP$$

i.e.

$$Kp=R\cot\phi_K$$

Also :

$$\angle KOA=\angle KOP-\angle AOP$$
$$=\phi_A-\phi_K$$

and

$$\frac{Ka}{OK}=\tan KOa=\tan KOA$$

i.e.

$$Ka=R\tan(\phi_A-\phi_K)$$

The chart distances of the pole (Kp) and any point on the central meridian (Ka) from the tangent point are thus known, and it is clear from figure 32 that if the latitude of A is greater than that of K, a will lie on the line Kp between K and p. If the latitude of A is less, a will lie beyond K on pK produced.

The Angle Between Two Meridians on the Chart. The difference of longitude between the meridians PBL and PAK in figure 32 is the angle LPK, denoted by λ, and this angle is projected into the angle lpK, denoted by α.

Suppose the great circle ABC is chosen so that it cuts the meridian PK at right-angles. Its projection ab will then be at right-angles to Kp, and, from the plane right-angled triangle pab :

$$ab=ap\tan\alpha$$

Also, if the plane of the great circle KLM is made to cut the central meridian at right-angles, the angle pKl is a right-angle, and from the plane right-angled triangle pKl :

$$Kl=Kp\tan\alpha$$

From the plane right-angled triangles lKO and pKO :

$$Kl=OK\tan KOL$$

and

$$Kp=OK\tan KOP=R\cot\phi_K$$

(2) *When* $\sin\phi$ *is equal to* $\cos\phi_K$. This occurs when ϕ is equal to $(90°-\phi_K)$. The curve is then the parabola given by :

$$x^2-2y\tan\phi_K+1-\tan^2\phi_K=0$$

(3) *When* $\sin\phi$ *is less than* $\cos\phi_K$. This occurs when ϕ is less than $(90°-\phi_K)$. The curve is then a hyperbola, and its equation can be written in the form :

$$x^2 \quad \frac{(y-n)^2}{m^2}=1$$

To Construct a Gnomonic Graticule. When the tangent point is on the equator or at the pole, the graticule admits of simple geometrical construction. When the tangent point is elsewhere, the formulæ just established must be employed.

Figure 34 shows the graticule when the tangent point is in latitude 45°S., longitude 120°W.

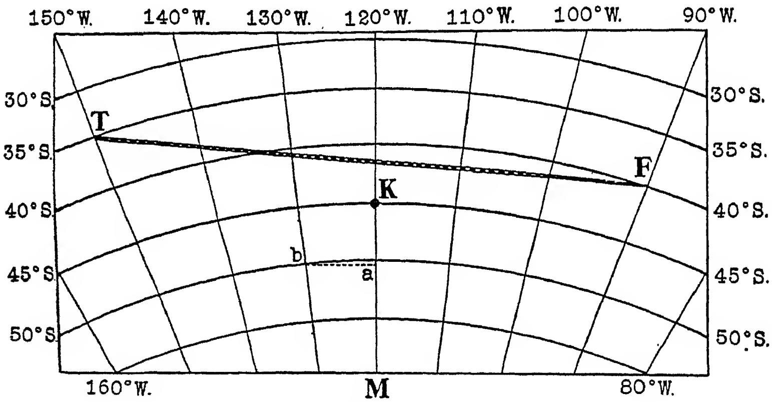


FIGURE 34.

MK is the central meridian, and the other meridians are inclined to it at angles given by :

$$\tan\alpha=\sin\phi_K\tan\lambda$$

—where ϕ_K is 45° and λ has successive values 10°, 20°, 30° . . .

The position of the pole (not shown in the figure) is given by :

$$Kp=OK\cot\phi_K$$

Kp can therefore be marked according to the chosen scale, and the meridians drawn as lines radiating from p at the angles discovered.

Again, if b is the point corresponding to latitude 50°S., longitude 130°W., and ba is the perpendicular from b to MK, the length of Ka in the chosen scale is given by :

$$Ka=\tan(\phi_A-\phi_K)$$

—in which ϕ_A is the latitude of A, the point that a represents on the chart. (Figure 32.)

If ϕ_B is the latitude of B, the point that b represents on the chart, Napier's rules applied to the triangle PBA give:

$$\tan \phi_A = \tan \phi_B \sec \lambda$$

—where λ is the difference of longitude between A and B. This formula gives ϕ_A since ϕ_B is 50° and λ is 10°. Hence Ka can be found. Also, in the chosen units:

$$ab = \tan \lambda \cos \phi_A \sec (\phi_A - \phi_K)$$

i.e.

$$ab = \tan 10° \cos \phi_A \sec (\phi_A - 45°)$$

The point b, corresponding to latitude 50°S., longitude 130°W., can therefore be plotted. Similarly for other points where this

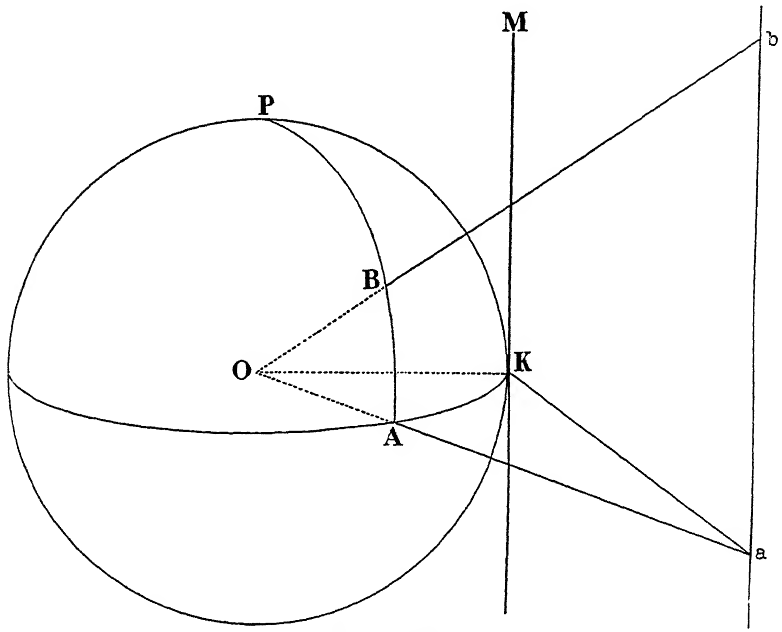


FIGURE 35.

parallel cuts the meridians. These points can then be joined by a smooth curve.

In this way all the parallels can be inserted.

The Equatorial Gnomonic Graticule. When the tangent point is on the equator, ϕ_K is zero, and the general formulæ are simplified considerably. The graticule, however, lends itself to a geometrical construction.

In figure 35, the central meridian is KP, and this is represented on the chart by KM which is at right-angles to OK. The equator KA projects into the straight line Ka at right-angles to KM, and any other meridian, AP, projects into a line at right-angles to Ka and therefore parallel to KM.

The distance between the projected meridian *ab* and the central meridian is given by :

$$Ka = OK \tan KOA$$
$$= R \tan (\text{d'long between } K \text{ and } A)$$

The positions of the meridians can thus be decided.

If B is any point on the meridian AP in latitude ϕ, B projects into b, and ab represents this latitude on the chart.

Figure 36 shows the geometrical construction for finding the position of b.

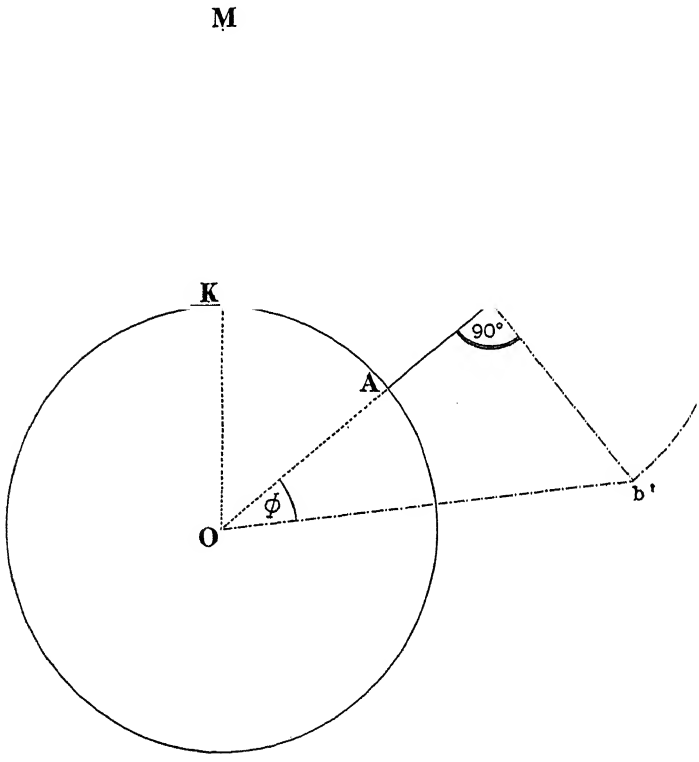


FIGURE 36.

The plane of projection is represented by MKa in the plane of the paper, and Ka is a tangent to the equatorial circle at K. A is fixed on this circle by its exact difference of longitude from K, and it projects into a. If ab' is now drawn at right-angles to Oa, so that the angle aOb' is equal to the latitude of B, the triangle aOb' is equal in all respects to the triangle aOb in figure 35. The position of b can thus be marked merely by making ab equal to ab'.

Other points on the projection of the parallel through B can be found in the same way. Since, however, a graticule is usually drawn for equal intervals of latitude and longitude, the work can

be shortened by drawing radials at the required interval and using them for both d'long and latitude as shown in figure 37.

This same construction can be used for finding the position of the vertex and the latitude of any point on a great circle, the longitude of which is known.

Any great circle projects into a straight line. Also a great circle cuts the equator in two points 180° apart. In figure 38, Q

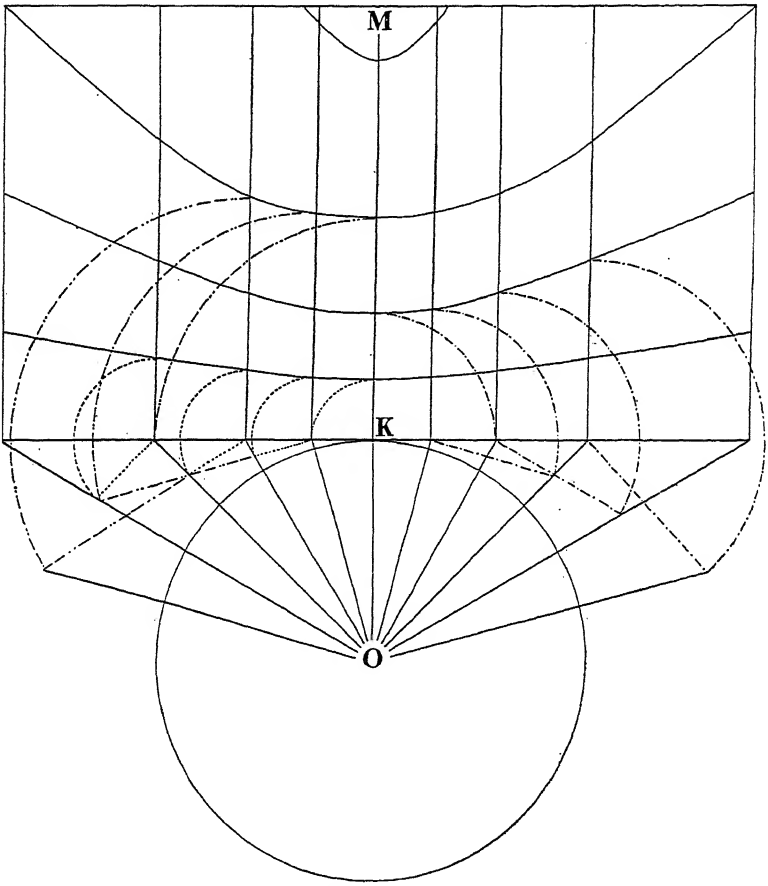


FIGURE 37.

is one of these points, and q its projection. Then, since the longitude of the vertex is 90° from the longitude of Q, the position of the vertex, v, is found merely by making the angle QOU a right-angle.

The angle uOv' measures the latitude of the vertex.

If the latitude of any point x is required, it can be found in the same way; that is, by drawing xy at right-angles to uq and yx' at

right-angles to Oy, and making yx' equal to xy. The angle yOx' then measures the latitude of the point X on the Earth to which x corresponds on the chart.

The Polar Gnomonic Graticule. It was shown on page 58 that when the tangent point is the pole, the angles between the meridians are correctly represented on the chart, and the meridians are therefore straight lines radiating uniformly from the point that represents the pole.

The parallels of latitude project into concentric circles of radius

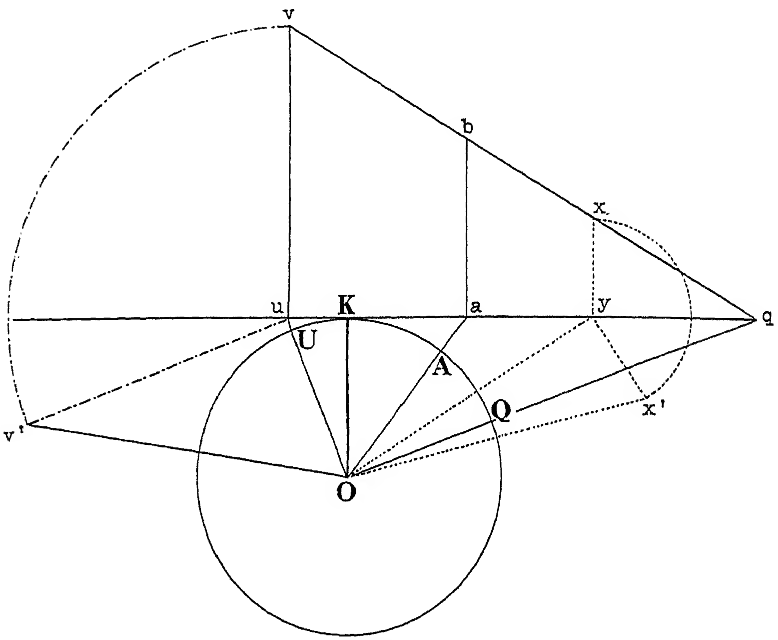


FIGURE 38.

proportional to $R \cot \phi$, a fact which is established at once by substituting 90° for ϕ_K in the general equation :

$$x^2 \sin^2 \phi + y^2(\sin^2 \phi - \cos^2 \phi_K) - 2y \sin \phi_K \cos \phi_K = \sin^2 \phi_K - \sin^2 \phi$$

The general equation then becomes :

$$x^2 \sin^2 \phi + y^2 \sin^2 \phi = 1 - \sin^2 \phi$$

i.e.

$$x^2 + y^2 = \cot^2 \phi$$

Figure 39 shows a simple geometrical construction for the polar gnomonic graticule.

Since cot 0° is infinite, the equatorial circle cannot be drawn. Moreover $\cot \phi$ increases rapidly when ϕ is greater than 70°. It is therefore impossible to draw a complete hemisphere, and the distortion will always be considerable away from the tangent point.

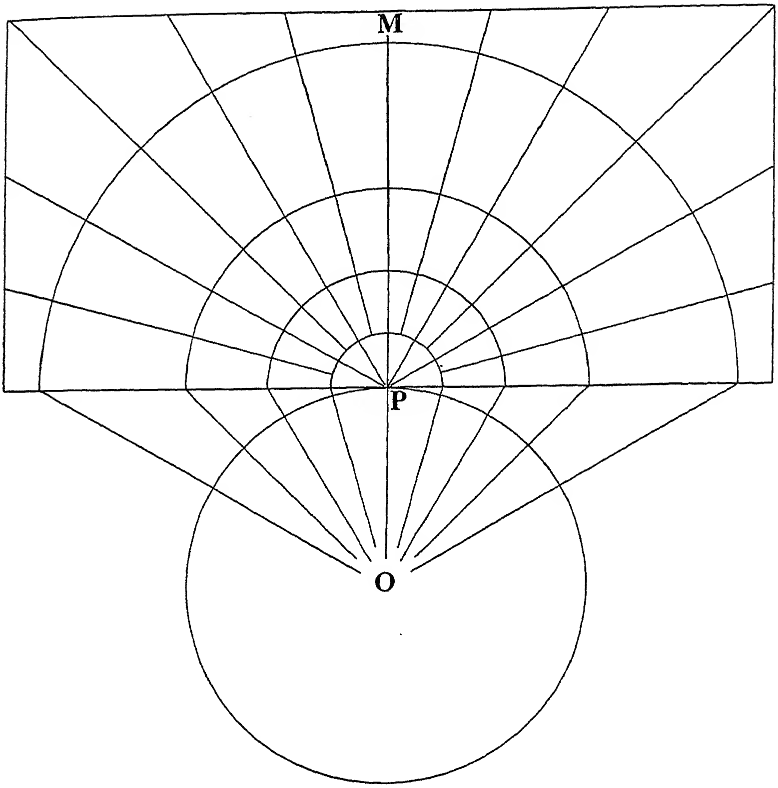


FIGURE 39.

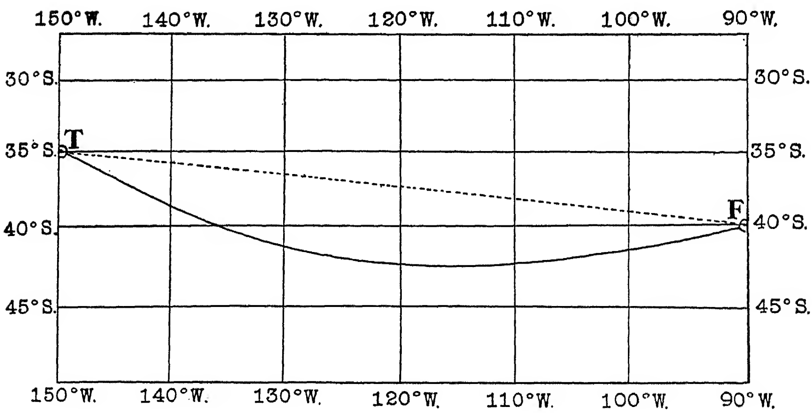


FIGURE 40.

Practical Use of Gnomonic Charts. The distortion of the gnomonic graticule, which is a perspective distortion that gives neither the orthomorphic nor the equal-area property, makes the graticule quite unsuitable for civil purposes, and its purpose is limited entirely to the use that can be made of the fact that, on it, great circles are represented by straight lines.

In figure 34, *FT* is the great-circle track between the points 40°S., 90°W., and 35°S., 150°W. As it appears on the gnomonic chart, it tells the navigator nothing about the course he must steer in order to follow it, because angles, other than bearings from the tangent point, are distorted. The track must therefore be transferred to a Mercator chart, a transference that is easily made by noting the latitudes of the points where the great-circle track cuts the meridians. The result is the smooth curve *FT* in figure 40. The dotted line *FT* shows the rhumb line.

If the composite track is required, it is easily obtained by drawing tangents from the starting and finishing points to the curve that represents the limiting parallel. The track is then given by the two straight lines thus obtained and the part of the curve representing the parallel that lies between the points of contact.

Figures 41a and 41b, in Chapter V of Volume II, show three tracks—rhumb-line, great-circle and composite—between two places, first on a polar gnomonic chart and then, for comparison, on a Mercator chart.

CHAPTER VII

THE PRODUCTION OF ADMIRALTY CHARTS

Charts are produced in the Hydrographic Department of the Admiralty.

The Surveying Service of H.M. Navy is not alone in supplying information for Admiralty charts. Some Dominions maintain their own surveying services. The Mercantile Marine also supplies general information, and harbour boards and similar corporations contribute much local detail. For detail of foreign coasts and waters the Hydrographic Department is dependent on charts and information obtained from the nations concerned.

Canada produces charts of her own coasts.

The Chart Branch of the Hydrographic Department works under a hydrographic surveying officer of captain's or commander's rank; the charts are produced by a staff of cartographers, assisted by draughtsmen. All information received is examined from the standpoint of navigational utility, and a decision is made concerning any action affecting the charts.

A section of the Chart Branch records, analyses and collates all geodetic information. This work involves the appraisement and evaluation of sets of observations; the reconsideration of accepted latitudes and, more especially, longitudes, consequent upon new determinations of key positions; and the effect upon charted work of the alterations that it will be necessary to make.

For Admiralty charts drawn on a mid-latitude natural scale not larger than 1/50,000, the Mercator projection is used. Plans and approach sheets on larger scales, ocean charts for great-circle sailing and polar charts, are drawn on the gnomonic projection. The principles of the construction of these projections are described in Chapters IV and VI.

Construction of a New Chart. In constructing a new chart, the projection is drawn to cover the limits. The main stations of the work are then plotted by calculation from the triangulation or other data available, the latitudes and longitudes being checked by reference to the geodetic-positions section. The new work may form only a portion of the chart, and its triangulation may not, for various reasons, have been directly connected with the older stations. To ensure the correct relation between the new and the old work, considerable balancing and adjustment may be necessary. When, as sometimes happens, definite discrepancies are found, the satis-

factory adaptation is decided only after the most exhaustive search through and collation of all available evidence.

The borders of the sheet, with the graduation, are drawn on a sheet of stout drawing paper, but the remainder of the work is drawn, for the convenience both of the Chart Branch and of the engravers, on tracing paper. If the chart is on a smaller scale than the original, the work is reduced either by photography or by similar squares.

The information supplied by the surveyors is rearranged to conform with the standard Admiralty practice, much care being given to the placing and style of the names and other lettering. The selection of soundings demands considerable experience to avoid overcrowding and at the same time to include all essential information. As the scale of the chart decreases, the difficulty of making a suitable choice is greater because not only is it necessary to show all dangers, shoals and least depths, but the general trend of the bottom must be indicated so that the depths obtained by sounding with the lead shall correspond with the chart depths. The surveyor's units are never converted to give an appearance of greater precision than is warranted; thus, *data in feet may be given on the chart as fathoms and feet, but data in fathoms would never be converted into feet.* Should it be necessary to complete some portion of a sheet by enlarging an original drawing, this fact is invariably shown by engraving all the work on this part of the chart in fine line, known technically as 'hair-line'.

All sea marks and lights are checked by the appropriate information, and the whole drawing is examined with the *Sailing Directions, Notices to Mariners,* and other publications. Views are usually drawn separately, since they are not engraved on the copper plate with the rest of the work, but are etched in with acid by a different worker. Before being sent to the engraver, the whole drawing is examined by the Chief Cartographer in charge of the section concerned, who has also supervised the work.

The correction of a part of a chart frequently gives trouble when the new work is adjusted to the old, especially in the lesser-known parts of the world. When this occurs, long experience of such problems can be the only guide.

When a proof is received back from the engraver, it is compared minutely with the drawing for any errors, omissions, or unfinished work. Measurements and 'laying down' are checked on a proof pulled on dry paper, since the ordinary proof, which is damped in order to take the ink properly, is liable to shrink. Accuracy to one-hundredth of an inch is insisted on. Any work or information that reaches the department after the drawing is made is embodied before the corrected proof is returned to the engraver. Further proofs are examined in order to see that these corrections and amendments, and also the previously uncompleted details, such as tint, hatching of buildings, low-water rocks and views, have been

embodied. The final proof is carefully revised from the original data, and a search is made for any information that, by its inclusion, can improve the chart.

Before publication, the completed proof is referred for checking to the officers responsible for the *Sailing Directions* and *Light Lists*, to the Wrecks Officers and to the Tidal and other Branches of the Department. After examination by the chief officers of the Chart Branch, it goes to the Assistant Hydrographer and, finally, to the Hydrographer for his signature. Any amendments resulting from this inspection are then made, and the chart is ready for printing and issue, a fact which is announced by a *Notice to Mariners*. All the necessary references to other charts are inserted, and smaller scales covering the area are corrected to agree with the new work.

As far as possible, all names are now given on charts as they would be spelt in the country of their origin. When a change is made that differs in essentials from the form appearing in the current edition of the *Sailing Directions*, the older name is retained in 'hair-line' on the chart, in brackets, below the accepted form. The orthography used is that of the Royal Geographical Society's Permanent Committee on Geographical Names.

Reproduction of Charts. The basis of all permanent Admiralty charts is the engraved copper plate. The borders and graduation lines are first marked and cut, and certain meridians and parallels are ruled very faintly on the copper to assist fitting. The work from the drawing is then transferred to the copper plate in one of three ways, to guide the engraver, the choice being made to suit the work being done.

(1) A photographic print of the whole drawing on to a sensitized film can be obtained by direct contact of the reversed tracing.

(2) The work can be marked through the reversed tracing, on to a thin wax coating, with a fine point.

(3) The work can be traced direct from the tracing on to sheets of gelatine with a needle-point, into the cuts of which is rubbed a red powder that, in its turn, is transferred to the wax coating when the gelatine is laid face downwards on it.

Through the print on the film, or the marks on the wax, thus obtained, the engraver scratches the work on to the copper surface with a needle-point, and the film or the wax is then removed. The actual cutting, which appears reversed, is done by the engraver with the drawing in front of him. On new plates, the soundings and much of the writing can now be inserted by an engraving machine, with considerable economy in time and cost.

A copper plate can be corrected locally by beating it from the back until it is flat, and then scraping off the work affected and resurfacing the front. In the ordinary way, copper plates have a

long life, and some still in daily use are over a century old. The surface is protected from wear in printing by an electrolytic deposit of steel, but a much-corrected copper plate may eventually crack in the thin parts. Thin parts of a plate that is otherwise in good condition can be saved from cracking by an electrolytic deposit of copper on the back to bring the beaten part to the original thickness.

If the plate has actually cracked, the part affected is cut out and the hole filled in by deposited copper, the rest of the plate being protected from electrolytic action by wax.

Owing to buckling or other reasons, it may not be possible to patch a cracked plate, because the spring of the plate in the printing press would cause the soft deposited copper to break away from the hard metal of the original plate. When this occurs a duplicate plate is prepared. This can be made by electrolytic facsimile or by an acid etching through a facsimile ground. Since the latter is made on a normally hardened plate, it has a longer life than that of the deposited copper, but this process tends to cause a slight thickening of the work, and for this reason it is not suitable for very fine or crowded detail.

Much temporary and semi-permanent work is now reproduced by lithography, the work being drawn with greasy ink on stone or on the finely granulated surface of a zinc plate. An increasing use is being made of a combination of copper engraving and lithographic drawing for charts that are continually changing in one part, such as those showing harbours and river entrances. The outline, topography and, possibly, the more stable portions of the water-work, are engraved on the copper, the rest being left blank. This is 'transferred' to a lithographic surface by means of an impression pulled with greasy ink on a plaster-surfaced paper. When this work is on the stone surface, the rest of the water work is drawn thereon, and the continuous corrections to the same area of the copper plate is thus avoided.

The lithographic method is unsuitable for repeated amendments, because constant printing results in a thickening of the work.

Printing Charts. The direct printing from the copper plates in a hand-press is now to a great extent superseded by lithography. A transfer from the copper plate is laid down on zinc, and this is placed on a flat-bed direct-printing machine. Once the machine is 'made ready', copies can be run off with great rapidity. It has the additional advantage that the only damping necessary occurs in laying the transfer to the zinc when it is under pressure, thereby causing little or no distortion by shrinking.

The time taken to 'make ready' renders this method extravagant for printing a small number of copies. Thus, for a few copies from a large number of different plates, copper-plate printing is quicker and more economical, but for a large number of copies of a few charts the lithographic method is more efficient. When

colours are used in the printing, the lithographic method is essential because the shrinking in the direct copper proof would effectively destroy the registration of the colours. When a chart has no complete copper basis, the lithographic zinc or stone is never used for printing, but a transfer to a second zinc plate is made in order to preserve the original.

The type of plate used for printing any chart is shown by the letters just inside the extreme bottom right-hand corner of the sheet.

When the printing plate is a transfer from the original plate, the year of the transfer is added. The following table shows the symbols in use :

C	=Copper.	Originals
P	=Duplicate plate made by acid etching. (This is frequently known as a ' Process Plate '.)	Originals
E	=Duplicate plate made by electrotyping.	Originals
Z	=Lithographic zinc.	Originals
S	=Lithographic stone.	Originals
SC= **ZC**=	An original stone or zinc, a portion of the work on which has been transferred from an incomplete copper plate.	
Zc. Zp. Ze. Sc. Sp. Se.	Printing plates or stones made from complete original copper plates.	
Zs. Zz. Ss. Sz.	Printing plates or stones made from complete original lithographic plates or stones.	
Zsc. Zzc. Ssc. Szc.	Printing plates or stones made from composite lithographic originals.	

The exact dimensions of the chart in inches, measured on the copper plate along the innermost line of the graduation, is given in the bottom right-hand corner of the chart, outside the border near the chart number. These figures show if shrinking has occurred during the printing. The day of the year on which the copy was printed is given outside the borders in the top right-hand corner ; thus 62.37 shows that the copy was printed on the 3rd March 1937.

The dating of the plates for new editions, large corrections, and minor amendments is fully explained in the introduction to the various *Sailing Directions*, and also in Chapter II of Volume I of this Manual.

No chart is issued to the public unless it bears the correction and number of the latest *Notice to Mariners* affecting it. For this reason stock copies are corrected by hand daily.

CHAPTER VIII

STAR GALAXIES AND THE SOLAR SYSTEM

To an observer on the Earth, the night sky has the appearance of a backcloth of fixed stars across which a few heavenly bodies describe definite paths. This appearance is independent of the apparent movement imparted to the scene by the Earth's daily rotation about its axis, a movement that results in the rising and setting of all heavenly bodies other than those that are circumpolar. The Moon, for example, may appear one day in the constellation of Sagittarius and a fortnight later in Aquila.

The Sun itself also changes its position against this backcloth, although the change is not so obvious because the Sun's light obscures the backcloth.

Early astronomers, seeking to explain these phenomena, assumed the Earth to be at rest in the centre of the universe, thereby defining the apparent paths of heavenly bodies as a complicated series of epicycles, and this inconvenient theory held until Copernicus killed it at the beginning of the sixteenth century by shifting the static point from the Earth to the Sun, and giving the Earth a daily rotation on its axis. The periods of light and darkness were thus accounted for with commendable simplicity; the backcloth of the stars was seen to be stationary, and the epicycles of the bodies that moved across it resolved themselves into simple orbits about the parent bodies, the planets round the Sun and the Moon round the Earth. The theory neglected any motion of the Sun, and any variation in the Earth's revolution about the Sun, but it was, and still is, adequate as a general explanation of the solar family.

Star Galaxies. The Earth is now seen to be no more than a satellite of a body that is a member of an immeasurably larger family, and it is possible that this stellar family or galaxy is itself a member of an even mightier family. Galaxies of stars and nebulæ, or clouds of gas, are known to exist. One, visible as a fourth magnitude star in Andromeda, reveals itself in the telescope as a colossal aggregation, complete with variable stars and novæ or new stars, and observation has fixed its distance as 270,000 parsecs. A parsec is the distance of a body that has a parallax of one second to an observer on the Earth during the course of a year; that is, 19,160,000,000,000 miles. The distance of the Andromeda galaxy is thus 5,130,000,000,000,000,000 miles, or 880,000 light years, a light year being the distance travelled in one year at 186,000 miles per second.

The *annual parallax* (or simply the *parallax*) of a star is the

angle subtended at the star by the radius of the Earth's orbit. The mean value of this radius, which is 92,900,000 miles, is known as the *astronomical unit.*

The distance of a heavenly body is proportional to its parallax, and a parallax of one second corresponds to 206,265 astronomical units.

The most distant of the bright stars in the galaxy to which the Sun belongs is Canopus with 600 light years. Sirius, the brightest star, is 8·8 light years distant.

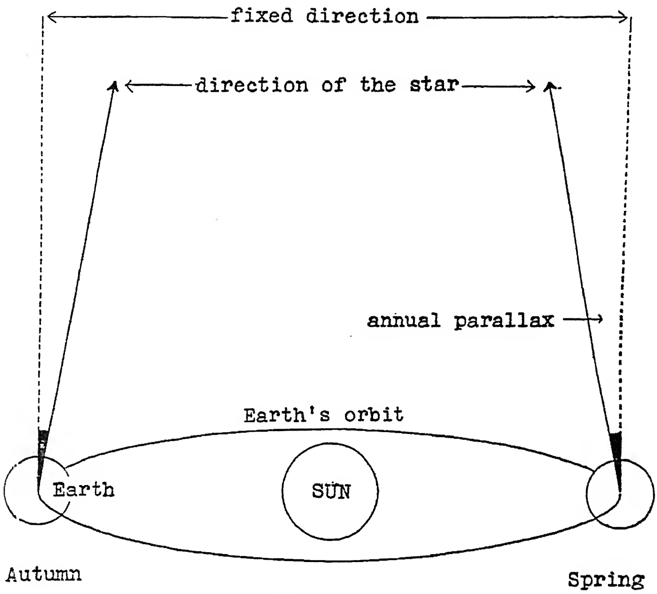


FIGURE 41.

The Galactic Circle or Milky Way. A star galaxy is a definite island of stars in space, fashioned like a flat circular biscuit of large diameter and small thickness. When, therefore, an observer on the Earth looks in a direction at right-angles to the plane of the galaxy containing the Sun, the stars that he sees are few in comparison with the number visible when he looks in a direction in the plane of the galaxy. That number is so vast that the stars themselves appear to lie in a gigantic belt across the heavens, running north and south between Betelgeuse and Procyon and passing close to the Southern Cross.

This belt is known as the *Milky Way* or *Galactic Circle,* and it marks the galactic plane.

In this plane the galaxy revolves as a series of concentric rings, each moving with a different velocity. It is possible to detect this differential motion between the rings, and its analysis shows that the centre of galactic rotation is 7,500 parsecs from the Sun, and

that the Sun completes one revolution about this centre in $2{\cdot}1 \times 10^8$ years. The stars, therefore, are not absolutely fixed on the backcloth. There is a change in relative position, but it is far too slow to influence the problems of navigation on the Earth's surface, and the Sun's orbital velocity has no effect upon them because it is shared by the Earth with the rest of the solar system.

Stellar Magnitudes. It is customary to place stars in categories according to their relative brightness to an observer; that is, according to the amount of light received from them. This amount depends on the distance and the intrinsic brightness of the star.

The apparent brightness of stars can be compared visually. Capella, for example, is obviously brighter than Polaris. This, however, is not to say that Capella is intrinsically brighter than Polaris because Polaris may be more distant.

Figure 42 shows a star S, considered as a point-source of light, emitting a cone of rays which fall on areas intersecting the cone

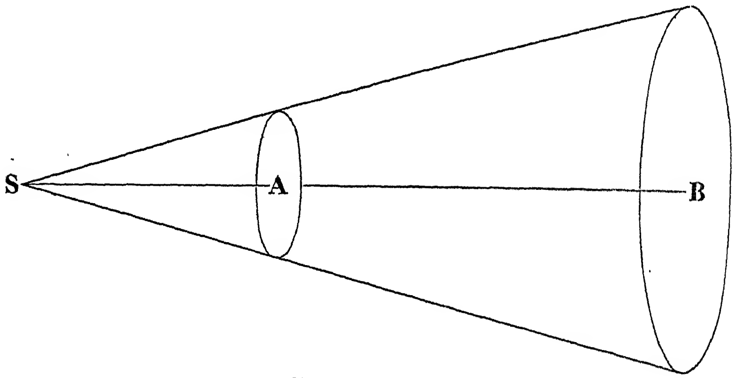


FIGURE 42.

at A and B. These areas are proportional to the squares of SA and SB. Hence, if T is the amount of light received by the area at A or B, the amount of light received by unit area at B is:

$$\frac{T}{\text{area at } B} = \frac{T}{\text{area at } A} \times \frac{SA^2}{SB^2}$$

If the area at A is itself unit area, and the distance SA is unit distance, the amount of light T is then a measure of the intrinsic brightness of the star, and the amount of light received by unit area at B is T/SB^2. This is the apparent brightness of the star to an observer at B, and it is seen to be inversely proportional to the square of the distance between the star and the observer. Thus, if two stars have the same apparent brightness, one may be four times as bright as the other and twice as far away.

Hipparchus and Ptolemy arbitrarily graded the stars visible to the naked eye into six magnitudes. With certain modifications arising from Herschel's discovery in 1830 that a first-magnitude

star gives one hundred times as much light as a sixth, this grading survives to-day. As explained in Chapter XXI of Volume II, the unit defining a magnitude is now 2·51. That is, a first-magnitude star gives 2·51 times more light than a second, and a second-magnitude star 2·51 times more light than a third, and so on. A first-magnitude star thus gives $(2{\cdot}51)^5$ or 100 times more light than a sixth.

With this scale of relative brightness, negative magnitudes are possible.

The magnitude of Sirius, the brightest star in the heavens, is −1·6. That of Venus varies from −3·1 to −4·3.

The Sun's magnitude is −26·7, and the Moon's, at the full, is −12·5. The Sun thus gives $(2{\cdot}51)^{14{\cdot}2}$ or about half a million times as much light as the full Moon.

First-magnitude stars are those with magnitudes less than 1. They are, in order of brightness, Sirius, Canopus, α Centauri, Vega, Capella, Arcturus, Rigel, Procyon, Achernar, β Centauri, Altair and Betelgeuse. Of these, Sirius and Canopus have negative magnitudes, that of Canopus being −0·9.

Only stars with magnitudes less than 6 are visible to the naked eye. These number 4,850, but there are some 1,000,000,000 stars brighter than magnitude 20.

Variable Stars. It will be noticed that several stars given in the *Nautical Almanac* have magnitudes that vary through a considerable range. The magnitude of Algol (β Persei), for example, is shown as varying from 2·3 to 3·5, a change of more than one magnitude; and more striking still as a phenomenon, though of less navigational importance, is Mira Ceti, the magnitude of which varies from 2 to 9·6. The range is thus 7·6 magnitudes, representing a diminution to one-thousandth of the light that the star emits when most brilliant.

Stars that change their magnitudes fall into two categories:

(1) those with magnitudes that undergo periodic changes.
(2) those with magnitudes that undergo irregular changes.

Algol, for example, falls in the first category. It consists of three components with the possibility of a fourth. From time to time one of the fainter components, moving in the same plane as the Earth, crosses in front of and partly eclipses a brighter component. There is a less pronounced diminution in magnitude when the brighter component eclipses the fainter.

Another type of star in the same category is the pulsating star or Cepheid variable, so called after β Cephei, the magnitude of which changes from 3·7 to 4·4 in about a day and a half and then returns to 3·7 during the next four days. There is no explanation of this pulsating effect, but the fact is useful to astronomers in that it enables them to determine stellar distances.

The most striking of the stars that undergo irregular changes

in magnitudes are the novæ. These are not necessarily 'new' stars within the strict meaning of the adjective. As often as not they are old stars that suddenly explode into magnitudes that bear no resemblance to their initial states, and then die away to insignificance. In 1572 Tycho Brahe discovered a nova in Cassiopeia that for a short time became brighter than Venus. More recently, in June 1918, a nova appeared in Aquila and, in the course of four days, increased from the eleventh magnitude until it became the third brightest star with a magnitude of $-0{\cdot}5$. It then decreased five magnitudes in three weeks and, six months later, was no longer visible to the naked eye.

Within this category, and offering no explanation of their behaviour, are a number of stars among which Betelgeuse (α Orionis) and α Cassiopeiæ are the most important. The variation is comparatively small and irregular.

The Constellations. Since an observer on the Earth is conscious only of the angular distance between the stars, as distinct from their actual distances from him, a number of stars may appear to form a cluster although they belong to entirely different rings in the galactic plane. To the observer they appear in the same direction, but a careful examination of their proper motions will probably reveal that the component stars of the constellation are moving independently with different velocities.

One of the few constellations that contain connected groups of stars is the Great Bear, five stars of which (there are seven in all) have almost the same proper motion.

The appearance of those constellations important to navigators is described in detail in Chapter XXI of Volume II.

The Solar System. The solar system is a vast family of planets, planetary satellites, comets, meteors and asteroids, all controlled by the Sun. As the central figure of this family, the Sun is distinguished by its immense size and its radiation of light and heat.

In mass, the Sun is more than seven hundred times larger than all the known members of the family put together.

The most important members of this family are the planets. In order of their distances from the Sun they are Mercury, Venus, Earth and Mars, forming an inner group of comparatively small components; and then Jupiter, Saturn, Uranus, Neptune and Pluto, forming an outer group of bigger components. Between Mars and Jupiter there is a host of tiny asteroids, over a thousand of which have been catalogued.

Unlike the Sun, planets and their satellites are not self-luminous. They reveal their presence by reflecting the light of the Sun. The planets revolve about the Sun in nearly circular orbits, and in planes differing slightly from the plane of the ecliptic in which the Earth revolves about the Sun. The planetary satellites, such as the Moon, revolve about their parent bodies in a similar manner.

In fact, where a planet has a number of satellites—Jupiter and Saturn are known to have nine each—the solar system is repeated in miniature, except for the radiation of light and heat by the parent body. In general, satellites revolve about a planet in the same direction as that in which the planet revolves about the Sun, and in planes differing but little from the planet's orbital plane.

NOTE. The frequent references to plane and planet may suggest an etymological connexion between them, but there is none. Plane is derived from the Latin *plānum*, a flat surface, and planet from the Greek *πλᾰνήτης*, a wanderer.

The following table affords a comparison between the Sun and the planets.

THE SOLAR SYSTEM

	Mean Distance from Sun in millions of miles	Diameter in miles	Time of Orbital Revolution in years	Inclination of Orbit to Ecliptic
Sun	—	864,000		
Mercury ..	36·0	3,008	0·24	7°00′
Venus * ..	67·2	7,576	0·62	3°24′
Earth ..	92·9	7,927	1·00	0°00′
Mars * ..	141·4	4,216	1·88	1°51′
Jupiter * ..	483	88,700	11·86	1°18′
Saturn * ..	886	75,600	29·46	2°29′
Uranus ..	1,782	30,880	84·01	0°46′
Neptune ..	2,793	32,940	164·79	1°47′
Pluto ..	3,666	unknown	249·17	17°09′

Those planets marked with an asterisk are navigational planets sufficiently prominent to be observed with an ordinary sextant.

In detail, the characteristics of the 'inner group' are:

Mercury. The planet Mercury is not much larger than the Moon, and since it is close to the Sun, it is not always visible. When it is near its larger eastern elongation—the point, that is, where the planet reaches a maximum angular distance from the Sun—it sets about an hour and a half after sunset in temperate latitudes, by which time the afterglow has faded sufficiently for the planet to be seen.

There is no atmosphere surrounding Mercury, and the planet itself is dead, like the Moon. For this reason there can be no life on Mercury of the kind known on the Earth.

Venus. Apart from the Sun and the Moon, Venus is the most brilliant of all heavenly bodies. Its magnitude, which on the average is about −3·7, varies with the phase. Since, however, it is never more than 47° from the Sun, it cannot be seen at night in temperate latitudes, and is therefore a 'morning or evening planet'.

In so far as life is concerned, Venus has neither water nor oxygen.

Mars. Mars is considerably smaller than the Earth, a fact which points to an extremely thin atmosphere. The presence of white polar caps suggests ice, and therefore water, but a spectrum analysis gives no sign of oxygen. The navigator may identify the planet by its distinctly red light. Its magnitude varies considerably and is, on the average, about $-0{\cdot}2$.

The ' outer group ' is composed of large planets, surrounded by heavy and poisonous atmospheres, and their great distances from the Sun indicate that their temperatures are a long way below zero. In detail their characteristics are :

Jupiter. Jupiter is remarkable for its satellites which, moving in roughly the same plane as an observer on the Earth, are occasionally eclipsed by the parent body. Before the days of wireless telegraphy, this fact was frequently used in order to find the chronometer error, and it also led to the first measurement of the velocity of light. This was done by observing a periodic eclipse when Jupiter was close to the Earth and again when the Earth was the other side of its orbit and therefore farthest away. On the second occasion the eclipse took place sixteen minutes after the calculated time, an interval accounted for by the extra distance the light had to travel. Since this distance was roughly 186,000,000 miles and the time taken was about 1,000 seconds, the velocity of light is 186,000 miles per second.

Saturn. In addition to its nine satellites, Saturn is encircled by three distinct co-planar rings concentric with it. A fairly powerful telescope, however, is needed if they are to be seen.

These rings are most probably composed of innumerable satellites rotating so rapidly about the parent body that they appear to merge into a complete whole. Since they are seen by the Sun's reflected light, the under side of the rings is dark, and when this side is turned towards the Earth, the rings are invisible.

Uranus. Uranus is the first of the ' modern ' planets. The others, from Mercury to Saturn, were known to the earliest astronomers. Uranus was not discovered till 1781. It is too faint to be seen with the naked eye.

Neptune. The gravitational attraction between the planets gives rise to what are known as perturbations, or departures from the strict Newtonian orbit which a planet would follow under the attraction of the Sun alone. A mathematical examination of the dynamical theory relating to these perturbations led not only to the conclusion that another planet existed, but also to its probable position. In the middle of the last century it was found and named Neptune.

Pluto. A similar study of Neptune's perturbations led, in 1930, to the discovery of an even more distant planet, Pluto, and it is possible that the full extent of the Sun's family is not yet known.

CHAPTER IX

THE SUN'S APPARENT ORBIT

The theory advanced by Copernicus, that the Earth described a circular orbit about the Sun, was later proved to be incorrect in its description of the actual orbit, which is slightly elliptical. The Sun, being the centre of attraction, is situated at one of the foci.

An observer on the Earth is aware of this orbital movement only through the apparent movement of the Sun across the back-cloth of the stars during the year. (This movement is actually deduced because the Sun and the stars are not visible at the same

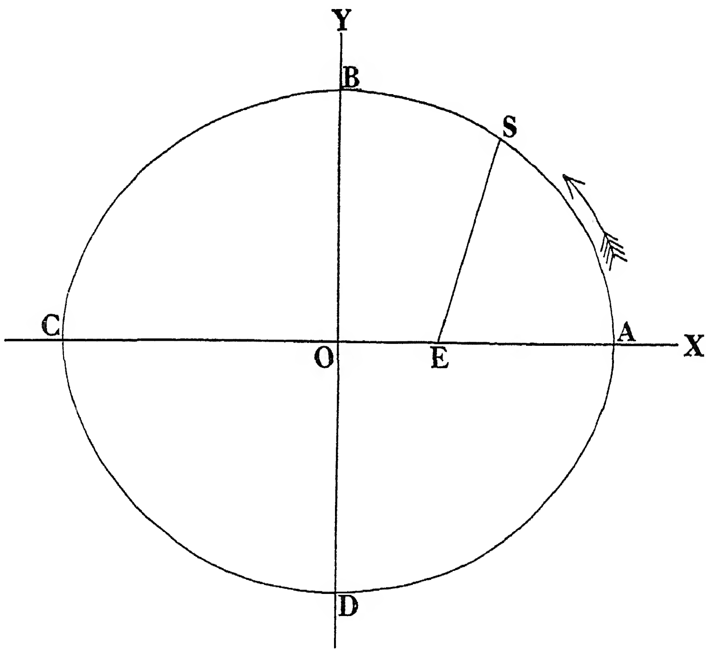


FIGURE 43.

time.) To him the Sun appears to describe an identical ellipse about the Earth, and it is convenient to consider this relative or apparent motion of the Sun instead of the actual motion of the Earth.

Figure 43 shows this relative situation, S being the Sun which describes an ellipse about E, the Earth. The major axis is AC; the minor axis BD; and O, their point of intersection, is the centre of the ellipse.

The Sun is nearest the Earth (in perigee) when it is at A, and farthest away (in apogee) when it is at C. Both these distances can be estimated from the information about the Sun's semi-diameter that is given in the *Nautical Almanac*, because from this information a value of e, the eccentricity of the ellipse, can be deduced, and, if a is the length of the semi-axis major, it is known that:

$$EA=a(1-e)$$
$$EC=a(1+e)$$

To an observer on the Earth, the Sun's semi-diameter in minutes of arc is given by:

$$3{,}438\frac{d}{D}$$

—where d is the semi-diameter of the Sun in miles, and D is the distance between the Earth and the Sun, also in miles. Of these distances, d is constant and D variable. The apparent semi-diameter thus varies inversely as the distance, and the distance of the Sun from the Earth varies inversely as the semi-diameter.

From the *Nautical Almanac* it is seen that the semi-diameter reaches a maximum value of 16′18″ about the 2nd January when the Sun is at the minimum distance given by $a(1-e)$. About the 1st July when the Sun is at the maximum distance given by $a(1+e)$, a minimum value of 15′45″ is reached. Hence, if all distances are expressed in seconds of arc:

$$978=\frac{d}{a(1-e)}\times3438\times60$$

and

$$945=\frac{d}{a(1+e)}\times3438\times60$$

By division:

$$\frac{1+e}{1-e}=\frac{978}{945}$$

$\therefore$

$$e=\frac{1}{60}\text{ (approximately)}$$

Since a is 92,900,000, it follows that EA is about 91,350,000, and EC about 94,450,000.

The length of the semi-axis minor, denoted by b, can also be calculated since:

$$b=a\sqrt{(1-e^2)}$$

When the values of a and e are substituted, b is found to be about 92,890,000. The ellipse thus approximates closely to a circle.

Kepler's Laws. After the death of Copernicus in 1543, Tycho Brahe (1546–1601) devoted himself to obtaining measurements of the relative positions of the Sun and the planets, and Kepler, who studied astronomy under Tycho Brahe, subsequently took these

measurements and from them deduced the laws of planetary motion. They are :

(1) The path of a planet round the Sun is an ellipse, the Sun being situated at one of the foci.

(2) The rate at which the line joining the planet to the Sun sweeps out areas is constant.

(3) The square of the time taken by a planet to complete its orbit is proportional to the cube of the semi-axis major of the orbit.

There are two methods of approach to the mathematical proof of these laws : either the law of gravitational attraction can be accepted and the path of a body moving under its influence shown to be an ellipse, or the path of the body can be accepted as an ellipse and the law of gravitational attraction established.

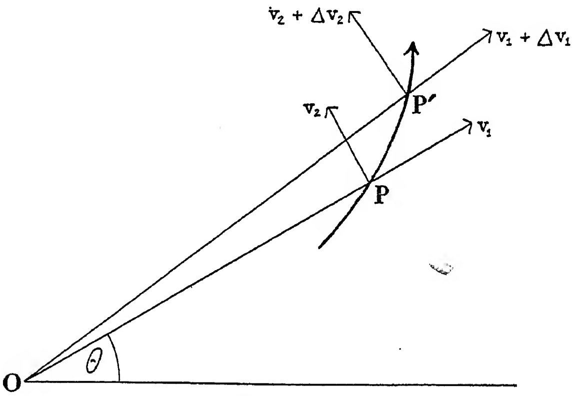


FIGURE 44.

The law of gravitational attraction is that the attractive force between two bodies is inversely proportional to the square of the distance between them. That is, if F denotes the force and r the distance between the bodies :

$$F=\frac{\mu}{r^2}$$

—where μ is a constant depending on the masses of the bodies.

Proof of Kepler's Second Law. In the proof of this law, the only assumption made is that the force under which the body moves is always directed towards a centre of attraction O.

In figure 44, P is a body describing an orbit, and v_1 and v_2 are the components of its velocity V along and at right-angles to the radius vector OP at time t. At time $(t+\Delta t)$, P' is the position of

the body, and $(v_1+\Delta v_1)$ and $(v_2+\Delta v_2)$ are the components of its velocity. Then, in the usual notation:

$$v_1=\mathop{\mathrm{Lt}}_{\Delta t\to 0}\frac{OP'\cos\Delta\theta-OP}{\Delta t}=\frac{dr}{dt}=\dot{r}$$

and $$v_2=\mathop{\mathrm{Lt}}_{\Delta t\to 0}\frac{OP'\sin\Delta\theta}{\Delta t}=r\frac{d\theta}{dt}=r\dot{\theta}$$

Again, if f_1 and f_2 are the radial and transverse accelerations of P at time t:

$$f_1=\mathop{\mathrm{Lt}}_{\Delta t\to 0}\frac{(v_1+\Delta v_1)\cos\Delta\theta-(v_2+\Delta v_2)\sin\Delta\theta-v_1}{\Delta t}$$

$$=\frac{dv_1}{dt}-v_2\frac{d\theta}{dt}$$

$$=\frac{d^2r}{dt^2}-r\left(\frac{d\theta}{dt}\right)^2$$

$$=\ddot{r}-r\dot{\theta}^2$$

and $$f_2=\mathop{\mathrm{Lt}}_{\Delta t\to 0}\frac{(v_2+\Delta v_2)\cos\Delta\theta+(v_1+\Delta v_1)\sin\Delta\theta-v_2}{\Delta t}$$

$$=\frac{dv_2}{dt}+v_1\frac{d\theta}{dt}$$

$$=r\frac{d^2\theta}{dt^2}+2\frac{dr}{dt}\times\frac{d\theta}{dt}$$

$$=r\ddot{\theta}+2\dot{r}\dot{\theta}$$

$$=\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$

The values given to f_1 and f_2 depend on the mass, m, of the body and the force F under which the body is moving. As already stated, F is directed towards O. The equations of motion are therefore:

$$m(\ddot{r}-r\dot{\theta}^2)=-F$$

and $$m\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})=0$$

Of these, the second integrates at once to give:

$$r^2\dot{\theta}=\text{constant}$$

That is, $(r\times v_2)$ is constant. But $(r\times v_2)$ is twice the rate at which areas are swept out by the radius vector. Kepler's second law is thus established.

Proof of Kepler's First Law. The first law can be established more readily by working in terms, not of r and θ, but of r and p,

where p is the perpendicular from the origin on the tangent. (Figure 45.)

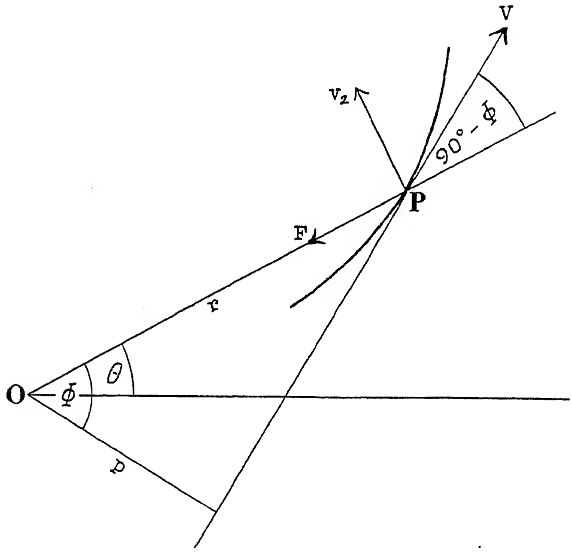


FIGURE 45.

By resolving and equating forces along the tangent at P, it is seen that :

$$-F \sin \phi = mV\frac{dV}{ds}$$

—where $\triangle s$ denotes an element of arc. That is :

$$-F \sin \phi = mV\frac{dV}{dr} \times \frac{dr}{ds}$$

$$= mV\frac{dV}{dr} \times \sin \phi$$

Also, if h is twice the rate of description of areas :

$$h = rv_2 = v_2 p \sec \phi = pV$$

Therefore, by substitution and differentiation :

$$F = -m\frac{h}{p}\frac{d}{dr}\left(\frac{h}{p}\right)$$

$$= m\frac{h^2}{p^3}\frac{dp}{dr}$$

This equation holds for a body describing any orbit. The form it takes depends upon the relation connecting p and r, and this relation in turn depends upon the orbit.

If the body is now assumed to be describing an ellipse, the relation between p and r may be found from the ordinary geometrical properties of the ellipse. Thus, in figure 46 :

$$\frac{p}{r}=\frac{p'}{r'}=\sqrt{\frac{pp'}{rr'}}=\frac{b}{\sqrt{rr'}}$$

and
$$r+r'=2a$$

Therefore, by substitution :

$$\frac{p^2}{r}=\frac{b^2}{(2a-r)}$$

i.e.
$$\frac{b^2}{p^2}=\frac{2a}{r}-1$$

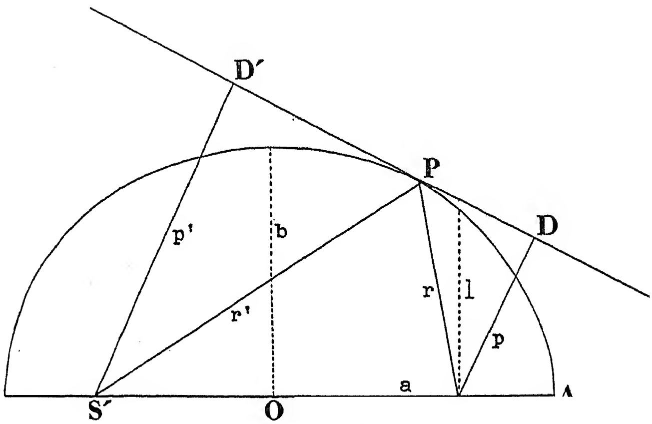


Figure 46.

If l denotes the length of the semi-latus rectum, or half-chord through a focus perpendicular to the major axis, al is equal to b^2, and the equation of the ellipse becomes :

$$\frac{l}{p^2}=\frac{2}{r}-\frac{1}{a}$$

By differentiation :

$$\frac{dp}{dr}=\frac{p^3}{lr^2}$$

Hence, *if a body describes an elliptical orbit about a centre of force, the force itself is given by :*

$$F=m\frac{h^2}{p^3}\frac{dp}{dr}$$

$$=m\frac{h^2}{p^3}\frac{p^3}{lr^2}$$

$$=\frac{\mu}{r^2}$$

—where μ is a constant equal to mh^2/l. That is, the force varies inversely as the square of the distance.

Conversely, *if a body describes an orbit under a force that varies inversely as the square of the distance, that orbit is given by :*

$$\frac{\mu}{r^2}=m\frac{h^2}{p^3}\frac{dp}{dr}$$

$$mh^2\int\frac{1}{p^3}dp=\mu\int\frac{1}{r^2}dr+\text{a constant}$$

$$-\frac{mh^2}{2p^2}=-\frac{\mu}{r}+\text{a constant}$$

i.e.
$$\frac{l}{p^2}=\frac{2}{r}-\text{a constant}$$

The form of this equation shows that the orbit is an ellipse.

To find the value of the constant, consider the situation when p is equal to r. Each is then equal to $a(1-e)$ and :

$$\text{constant}=\frac{2}{a(1-e)}-\frac{l}{a^2(1-e)^2}$$

$$=\frac{2}{a(1-e)}-\frac{b^2}{a^3(1-e)^2}$$

$$=\frac{2}{a(1-e)}-\frac{a^2(1-e^2)}{a^3(1-e)^2}$$

$$=\frac{1}{a}$$

The orbit is therefore an ellipse given, as before, by :

$$\frac{l}{p^2}=\frac{2}{r}-\frac{1}{a}$$

Kepler's first law is thus established.

Proof of Kepler's Third Law. The third law follows at once from the value of h, because the time taken to complete the orbit is simply the area of the ellipse divided by the rate of description of areas. That is :

$$t=\frac{2\pi ab}{h}$$

$$=\frac{2\pi ab}{\sqrt{\mu l}}$$

$$=2\pi ab\sqrt{\frac{a}{\mu b^2}}$$

$$=\frac{2\pi a^{\frac{3}{2}}}{\sqrt{\mu}}$$

i.e.
$$t^2=a^3\times\text{a constant}$$

The square of the time taken by a planet to complete its orbit is thus proportional to the cube of the semi-axis major.

Mean Angular Motion of the Sun. If n denotes the average rate of description of angle by the radius vector:

$$t=\frac{2\pi}{n}$$

When the body is the Earth moving round the Sun, t is equal to a year of approximately $365\frac{1}{4}$ mean solar days. Therefore:

$$n=\frac{2\pi}{365\frac{1}{4}}$$

The quantity n is thus the *mean angular motion* of the Sun in its apparent orbit in radians per day, and it is equivalent to a little less than one degree per day.

The Dynamical Mean Sun. Owing to the tilt of the Earth's axis of spin, the plane of the Earth's orbit is inclined to the plane

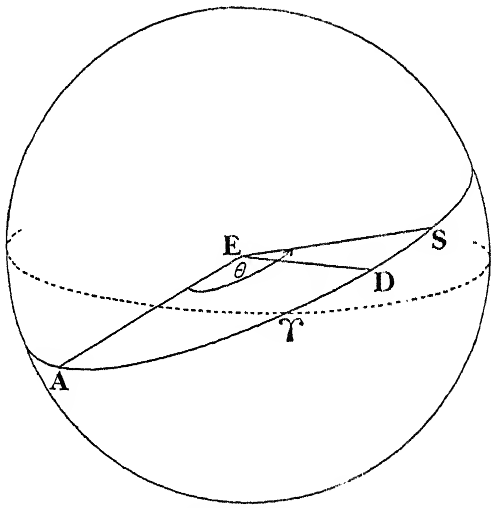


FIGURE 47.

of the celestial equator at an angle of nearly $23\frac{1}{2}°$. The ecliptic, or apparent path of the Sun in the celestial sphere, is therefore inclined at the same angle. By Kepler's second law, the Sun moves at a varying pace round the ecliptic. Its orbital velocity is greatest when it is nearest the Earth. But, as explained in Chapter X of Volume II, the unit of time in common use is a mean solar unit derived from an imaginary body, called the Mean Sun, that moves along the celestial equator at a uniform speed and completes one revolution in the time taken by the True Sun to complete one revolution in the ecliptic. The problem is thus to connect the True Sun and the Mean Sun, and this is done by introducing another imaginary body, the Dynamical Mean Sun, which moves along the ecliptic at a uniform speed, also completing one revolution in the time taken by the True Sun.

In figure 47, A is the position in the celestial sphere of the Dynamical Mean Sun and the True Sun when the Earth is in perihelion about the 2nd January. At this point the True Sun is moving with its greatest apparent velocity.

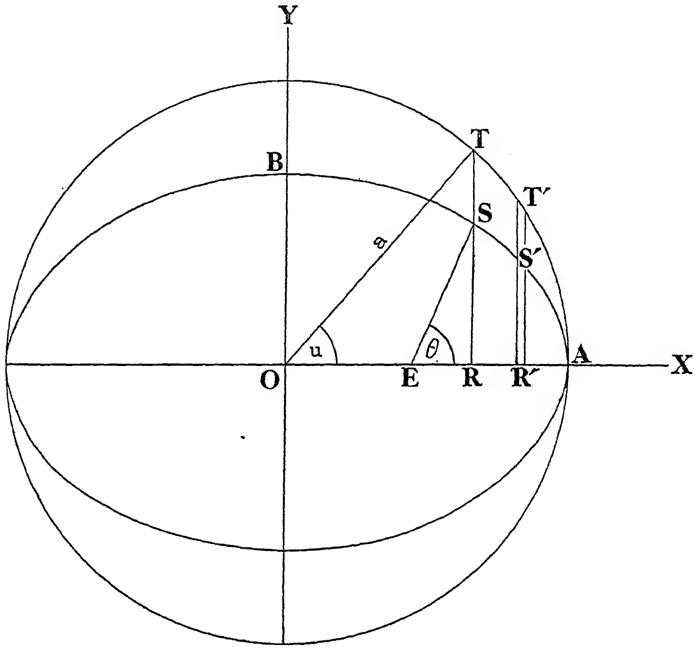


FIGURE 48.

D and S are the positions of the Dynamical Mean Sun and the True Sun after an interval of t days. Then, since D is moving with uniform angular velocity n, the difference between the two positions is given by:

$$DS = \theta - nt$$

—where θ is the angle AES in radians.

To find nt, consider the ellipse shown in figure 48.

S is the apparent position of the True Sun in relation to the focus E. The angle AES is called the *true anomaly*, and the angle AOT the *eccentric anomaly*. Then:

$$\text{area } AES = \text{area } RES + \text{area } ARS$$

To express these areas in terms of u, write:

$$\begin{aligned} \text{area } RES &= \tfrac{1}{2} ER \times RS \\ &= \tfrac{1}{2} RS(OR - OE) \\ &= \tfrac{1}{2} RS(a \cos u - ae) \end{aligned}$$

But the equation of the ellipse is :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

i.e.
$$\frac{a^2 \cos^2 u}{a^2}+\frac{RS^2}{b^2}=1$$

$\therefore$
$$RS=b \sin u$$

Hence, by substitution :

$$\text{area } RES=\tfrac{1}{2}ab \sin u(\cos u-e)$$

The area ARS can be found by considering parallel strips such as $R'S'T'$, and noting that, since RT is equal to $a \sin u$:

$$RS=\frac{b}{a}\times RT$$

It therefore follows that :

$$\begin{aligned}\text{area } ARS&=\frac{b}{a}\text{ area } ART\\ &=\frac{b}{a}\text{ (area } AOT-\text{area } ROT)\\ &=\frac{b}{a}\left(\tfrac{1}{2}a^2u-\tfrac{1}{2}a^2 \sin u \cos u\right)\\ &=\tfrac{1}{2}ab(u-\sin u \cos u)\end{aligned}$$

Hence, by adding the areas RES and ARS :

$$\text{area } AES=\tfrac{1}{2}ab\ [\sin u(\cos u-e)+u-\sin u \cos u]$$

i.e.
$$nt=(u-e \sin u)$$

—where, by Kepler's second law, n is a constant.

When u is equal to 2π radians, nt is also equal to 2π. That is, the period of revolution of S in the ecliptic is $2\pi/n$, a result that agrees with the definition of n on page 86.

The distance DS is thus given by :

$$\theta-(u-e \sin u)$$

or
$$(\theta-u)+e \sin u$$

In order to express θ in terms of u, it is sufficient to let b tend to a and write :

$$\theta-u=\tan(\theta-u)$$

—the value of $\tan\theta$ being given by :

$$\tan\theta=\frac{RS}{ER}=\frac{b \sin u}{a \cos u-ae}=\frac{\sin u}{\cos u-e}$$

This assumption gives :

$$\theta-u=e \sin u$$

—and the distance DS becomes $2e \sin u$, or, since nt is approximately equal to u, $2e \sin nt$. The formula giving DS is thus :

$$DS=\theta-nt=2e \sin nt$$

When nt is equal to 90°, the Dynamical Mean Sun has completed a quarter of its revolution, and :

$$\begin{aligned}\theta-90^\circ&=2e\\&=\frac{1}{30}\\&=2^\circ \text{ or } 8^{m} \text{ (approximately)}\end{aligned}$$

Hence, about the 1st April the True Sun is 8^m ahead of the Dynamical Mean Sun in the ecliptic. That is, it is about 8^m farther east than the Dynamical Mean Sun.

When nt is equal to 180° or 12^h, S and D coincide.

The Equation of Time. The True Sun is unsuitable as a time-keeper for two reasons: its apparent movement is not uniform,

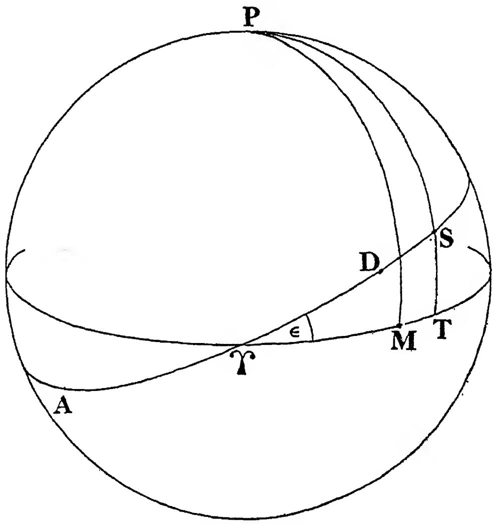


FIGURE 49.

and the plane of its apparent orbit is inclined to the plane of the celestial equator. The Dynamical Mean Sun is also unsuitable for the latter reason, but it affords a connexion with the Astronomical Mean Sun, or as it is usually called, the Mean Sun, which suffers from neither of these disadvantages.

As already explained, the Mean Sun moves in the celestial equator with the same velocity as the Dynamical Mean Sun in the ecliptic. Therefore, if D and M pass through the First Point of Aries together, after some days the relative positions of S, D and M will be as shown in figure 49, where ♈M is equal to ♈D.

The angle between the meridians of the True Sun and the Mean Sun is the equation of time, and is, in effect, the excess of Mean Time over Apparent Time. In figure 49 it is measured by the angle MPT or the arc MT.

In the notation employed in the previous sections of this chapter, AS is equal to θ, and AD to nt. Also :

$$\theta - nt = 2e \sin nt$$

Then, if ϵ is the obliquity or inclination of the ecliptic, it follows from the right-angled spherical triangle $S\Upsilon T$ that :

$$\tan \Upsilon T = \cos \epsilon \tan \Upsilon S$$

i.e.

$$\tan \Upsilon T = \frac{1 - \tan^2 \frac{1}{2}\epsilon}{1 + \tan^2 \frac{1}{2}\epsilon} \times \tan \Upsilon S$$

But ϵ is 23°27′, and $\tan^2 \frac{1}{2}\epsilon$ is therefore a small quantity, so that ΥT can be replaced by $(\Upsilon S - \beta)$ where β is a small quantity depending on $\tan^2 \frac{1}{2}\epsilon$. Then :

$$\frac{\tan \Upsilon S - \beta}{1 + \beta \tan \Upsilon S} = \frac{1 - \tan^2 \frac{1}{2}\epsilon}{1 + \tan^2 \frac{1}{2}\epsilon} \times \tan \Upsilon S$$

i.e.

$$\beta = \tan^2 \tfrac{1}{2}\epsilon \sin 2\Upsilon S$$

If ΥD is written for ΥS, β will still be given with sufficient accuracy. Hence :

$$\Upsilon T = \Upsilon S - \tan^2 \tfrac{1}{2}\epsilon \sin 2\Upsilon D$$

But, since DS is equal to $2e \sin nt$:

$$\Upsilon S = \Upsilon D + 2e \sin nt$$

Therefore :

$$\Upsilon T - \Upsilon D = 2e \sin nt - \tan^2 \tfrac{1}{2}\epsilon \sin 2\Upsilon D$$

i.e.

$$\Upsilon T - \Upsilon M = 2e \sin nt - \tan^2 \tfrac{1}{2}\epsilon \sin 2\Upsilon M$$

This quantity $(\Upsilon T - \Upsilon M)$ is the equation of time, and, in terms of α, the right ascension of the Mean Sun, it is given by :

$$E = 2e \sin nt - \tan^2 \tfrac{1}{2}\epsilon \sin 2\alpha$$
$$= E_1 + E_2$$

—where E_1 is the component depending on the eccentricity of the orbit, and E_2 the component depending on the obliquity of the ecliptic.

In figure 50, E_1 is plotted as a thin continuous curve, and E_2 as a broken curve. They combine to give E, which is shown as a heavy curve. It should be noted that nt is zero when the Sun is in perigee, and E_2 is zero at the vernal equinox. The equation of time itself is zero on the 15th April, 14th June, 1st September and 24th December. Its greatest value is just over $16^m 22^s$.

Precession and Nutation. Volume II, in dealing with problems involving the right ascension and declination of heavenly bodies, avoided certain complications that would have interfered with a simple and sufficient presentation of the subject, by assuming that the planes of the equator and the ecliptic are invariable in direction. Actually they are not. The attractions of the Sun, Moon and the planets on the Earth, which is not a uniform sphere

but an irregular ellipsoid, cause the directions of these planes to alter slowly and so vary the obliquity of the ecliptic and the positions of the First Points of Aries and Libra. In consequence the First Point of Aries does not define a fixed direction relative to the stars, and the right ascensions and declinations of the stars themselves are subject to small variations on this account alone, apart from any that may result from their own proper motions.

These small variations have two components: a progressive variation proportional to the time which is called *precession,* and a periodic variation which is called *nutation.*

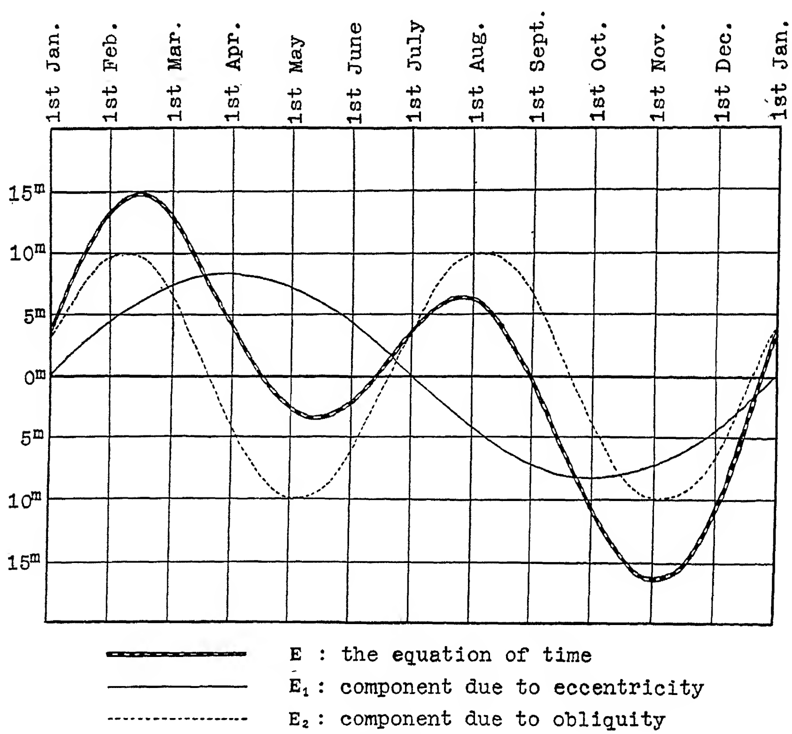


FIGURE 50.

These phenomena of precession and nutation are produced in the ordinary spinning top which, if the end O of its axis is fixed, will move so that the other end describes the circle *LMN* in figure 51. This circular motion is precession. If, furthermore, the end of the axis oscillates about the circular path it describes, as shown by the broken wavy line, this oscillatory motion is nutation. The Earth, being similar to a large spinning top, naturally produces these motions.

The part of the Earth's precession that arises from the influence of the Sun and Moon is called luni-solar precession; that arising

from the planets is called planetary precession. The effect of the planets is to alter the plane of the ecliptic by a small amount.

The Precession of the Equinoxes. So far as the Earth and the Sun are concerned, the plane of the ecliptic may be regarded as a fixed plane, and the pole of the ecliptic—K in figure 52—a fixed point.

The fixed direction OK corresponds to OC in figure 51, and P, the north celestial pole, to L. The north celestial pole thus describes a small circle about the pole of the ecliptic, and ♈ moves backwards along the ecliptic in consequence. There is a corresponding movement of the First Point of Libra, and the two movements are known as the precession of the equinoxes.

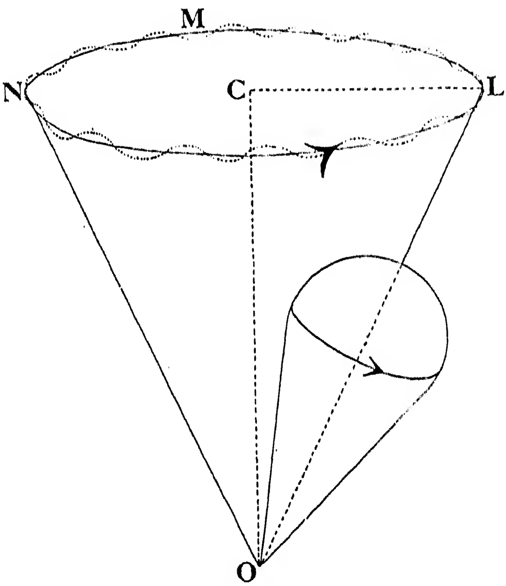


FIGURE 51.

At present the north celestial pole is little over one degree from the second-magnitude star, α Ursae Minoris, which, for the reason of this proximity, serves as a pole star and a quick means of finding the observer's latitude; but it will not remain so. Two thousand years ago the distance was 12°. Twelve thousand years hence the pole will be within a few degrees of Vega.

The time taken for P to describe this small circle about K is about 26,000 years.

The Mean Place of a Star. In deciding the position of a star, it is clearly necessary to have fixed and definite axes from which its co-ordinates can be measured. For this reason, an equator, ecliptic and equinox of a particular year are chosen, and the star's position is given with reference to them over a convenient period.

Star catalogues, for example, use the years 1875·0, 1900·0 1925·0 and others.

Suppose that, in figure 53, $F\Upsilon R$ and $E\Upsilon Q$ are the planes of the

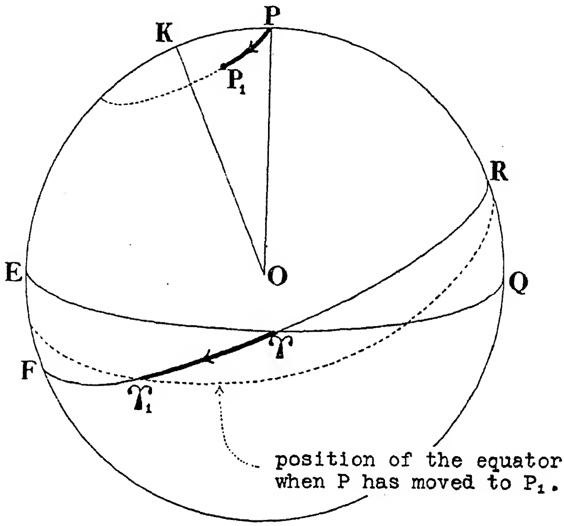


FIGURE 52.

ecliptic and the equator at a chosen date, 1900·0 say. Then, after an interval of t years, P will have moved to P_1 and $F_1\Upsilon_2R_1$ and $E_1\Upsilon_1Q_1$ will be the new positions of the two planes.

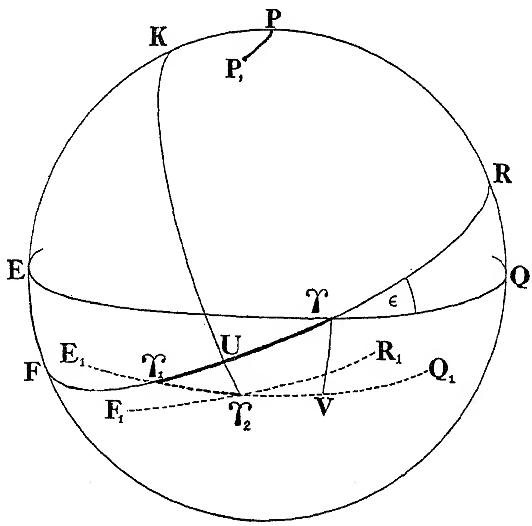


FIGURE 53.

The equator $E_1\Upsilon_1Q_1$ is called the *mean equator* and Υ_2 the *mean equinox* for the date $(1900{\cdot}0+t)$. By giving t its appropriate value,

the mean equator and the mean equinox can be specified for the beginning of any particular year. Referred to the mean equator and the mean equinox at this instant the position of a star is called the *mean place of the star.* This position, however, does not take nutation into account.

Effect of Planetary Precession. When luni-solar precession alone is considered, the effect is to treat the plane of the ecliptic as a fixed plane, and in consequence $\Upsilon\Upsilon_1$ represents the change in the position of the equinox. In one year this movement amounts to $50''\cdot37$. The introduction of planetary precession moves the equinox from Υ_1 to Υ_2, so that $\Upsilon_1\Upsilon_2$ represents the change due to planetary precession. The combined movement in one year amounts to $50''\cdot26$, and is known as the *constant of precession.* The effect of planetary precession is thus seen to be extremely small.

Yearly Change in Right Ascension and Declination. The change in a star's right ascension may be ascertained from the change in the position of the equinox.

If there were a star, the direction of which coincided exactly with Υ at 1900·0, its right ascension and declination would each be exactly zero. At the beginning of $(1900\cdot0+t)$, the star would still be in the direction of Υ, but the equinox itself, if only luni-solar precession were considered, would be at Υ_1, and the right ascension and declination would be $\Upsilon_1 V$ and $V\Upsilon$. Since, over the period of one year, the triangle $V\Upsilon\Upsilon_1$ can be considered plane, it follows that for a star near the equator, the increase in right ascension is approximately $50'\cdot3 \cos 23°27'$ or about $46''$ or 3 seconds of time per annum, and for a star in the neighbourhood of the First Point of Aries the annual change in declination is $50''\cdot3 \sin 23°37'$ or $20''$.

When planetary precession is taken into account, the *general precession* is not $\Upsilon\Upsilon_1$ but ΥU; that is, $\Upsilon\Upsilon_1 - \Upsilon_1 U$, or the luni-solar precession minus the planetary precession in longitude. If p denotes this general precession and ϵ the obliquity of the ecliptic, the annual changes in p and ϵ at a time t years after 1900·0 are given by:

$$p = 50''\cdot2564 + 0''\cdot000222\,t$$

and

$$\epsilon = 23°27'8''\cdot26 - 0''\cdot4684\,t$$

The longitude of Υ_2 is then $360° - p$.

The Year. Before these quantities can be used, the year itself must be defined, and the number of mean solar days in the unit depends upon the origin from which measurements are made. The most important units are:

(1) the sidereal year
(2) the tropical year
(3) the anomalistic year
(4) the civil year
(5) the Besselian year.

The Sidereal Year. If there were a fixed star in the ecliptic, the interval between two successive coincidences of the direction of the Sun's centre with the direction of the star would be a *sidereal year.* It is equivalent to 365·256 mean solar days. There is not, however, a fixed star in the ecliptic to give this fixed direction.

The Tropical Year. By measuring the revolution of the Sun in the ecliptic with reference to the First Point of Aries, the unit obtained is the *tropical year* of 365·2422 mean solar days. It is less than the sidereal year because, during the period of the Sun's apparent revolution, ♈ has moved backwards along the ecliptic to $♈_1$, if luni-solar precession alone is considered. The Sun has therefore to describe 360° less 50″·3, and in these circumstances the tropical year is less than a sidereal year in the ratio of:

$$\frac{(360^\circ - 50''{\cdot}3)}{360^\circ}$$

When other factors are considered, its length is 365·24219 879 mean solar days, but this figure is not constant since the motion of ♈ is slightly accelerated. This results in a decrease at the rate of about 0·6 seconds per century.

The Anomalistic Year. When perigee is substituted for the First Point of Aries, the interval is known as the *anomalistic year.* The anomalistic year may thus be defined as the interval between two successive passages of the Sun through the point at which it is nearest the Earth during the apparent orbit.

The Civil Year. This, being a domestic arrangement, has no astronomical significance. Efforts to combine days into periods suitable to his civil and religious observances have engaged man since he left any record of his existence. The present Gregorian calendar is based on the calendar of Julius Cæsar, which in turn was based on the tropical year.

In the Julian calendar, the civil year was taken as 365 mean solar days with a leap year of 366 every four years. This made the average civil year equal to 365·25 mean solar days, and therefore ·0078 mean solar days or 11 minutes longer than the tropical year.

The slow accumulation of these minutes meant that religious observancies and calendar dates became out of step with the seasons. In 11,000 years, had nothing been done, January would have become a summer month. Pope Gregory XIII therefore revised the calendar in 1582 by suppressing three leaps years in every four centuries. Thus 1700, 1800 and 1900, which in the Julian system would have been leap years, reverted to ordinary years, but 2000 remains a leap year. Hence in every 400 years there are 97 leap years, and the number of days in 400 years is $(303 \times 365) + (97 \times 366)$. From this it follows that the average length of a civil year in the Gregorian calendar is 365·2425 mean solar days.

The further suppression of 11 days, which was necessary in order to bring the calendar dates into step with the seasons, was attended by riots in England where the people, under the impression that their lives were being shortened by that amount, paraded with banners proclaiming: "Give us back our eleven days."

The Besselian Year. This—named after Bessel, the German astronomer—is a fictitious year used in astronomical measurements. Its length is the same as that of the tropical year and, in mean solar days, is:

$$365{\cdot}24219\ 879-0{\cdot}00000\ 614\ T$$

—where T is measured in centuries from the beginning of 1900, but the beginning of the year is defined as the moment when the Sun's mean longitude, freed from aberration, is $280^{\circ}{\cdot}0057$. Aberration is the apparent displacement of a heavenly body's position resulting from the fact that the ratio of the Earth's orbital velocity to the velocity of light is not inappreciable. When the Sun's mean longitude is affected by aberration, its value at the beginning of the Besselian year is 280°. The beginning of the Besselian year is thus independent of the calendar, and therein lies the importance of the Besselian year as a unit in astronomical measurement.

The Apparent Place of a Heavenly Body. This is the position of the heavenly body referred to the true equator and true ecliptic at any instant, precession, nutation, aberration, proper motion and parallax all being allowed for; and because it is the position in which an observer actually sees the heavenly body, apart from the effect of refraction, it is the position tabulated in the abridged edition of the *Nautical Almanac*. In the standard edition, which is not intended for practical navigation, certain information required in theoretical astronomy is given with reference to the mean equator and mean ecliptic at the beginning of the Besselian year.

Besselian Day Numbers. These are numbers derived from the various factors, such as precession and nutation, that decide the position of a heavenly body, and when allied to certain constants for that body, they enable its apparent place to be calculated from its mean place at the beginning of the year in question. Both numbers and constants are given in the standard edition of the *Nautical Almanac*.

Thus, if (α_0, δ_0) are the mean co-ordinates of a star at the beginning of any year, and (α, δ) are the apparent co-ordinates at some specified time during that year, the differences between the co-ordinates are given by:

$$\alpha-\alpha_0=Aa+Bb+Cc+Dd+E+\tau\mu_\alpha$$

and

$$\delta-\delta_0=Aa'+Bb'+Cc'+Dd'+\tau\mu_\delta$$

—where the quantities in capital letters are the Besselian day numbers and the quantities in small letters are the Besselian star constants derived from functions of the star's co-ordinates; μ is the star's annual proper motion and τ the fraction of the year from the beginning of the Besselian year. For heavenly bodies relatively close to the Earth, allowance must also be made for annular parallax.

Another set of numbers, derived from the same factors by different methods but giving the same result, are known as *independent day numbers*.

In the ordinary course of his work, the navigator is not concerned with these day numbers: the apparent positions of all the heavenly bodies of use to him are given in the abridged edition of the *Nautical Almanac* to one decimal of a second in time and a minute in arc, and this degree of accuracy is sufficient for his purpose. If, however, the position of a heavenly body is required to the degree of accuracy demanded by survey work, for example, interpolation must be effected between the quantities taken from the standard edition.

First and Second Differences. When interpolation is effected between two quantities by taking a simple proportion of the difference between them, the assumption is made that the part of the curve between the two points that determine the quantities is a straight line. If it is, the interpolation is accurate. But it will not usually be so, and the interpolation will be inaccurate.

Consider the two curves given by:

		First Diff^n	*Second Diff*^{n}
$x=1$	$y=2$		
		2	
$x=2$	$y=4$		0
		2	
$x=3$	$y=6$		0
		2	
$x=4$	$y=8$		

		First Diff^n	*Second Diff*^{n}	*Third Diff*^{n}
$x=1$	$y=2$			
		2		
$x=2$	$y=4$		1	
		3		0
$x=3$	$y=7$		1	
		4		
$x=4$	$y=11$			

The first is a straight line, as shown in figure 54a. The second is a curve, as shown in figure 54b.

If successive values of y are subtracted, the resulting numbers are known as *first differences*, and it is seen that, for the first curve, they are all equal to 2. This indicates that the y-ordinate is increasing steadily, and that the curve must be a straight line. If these first differences are now subtracted, *second differences* are obtained, and on their values depends the shape of the curve, which is no longer a straight line. Third differences are obtained by subtracting second differences, and so on.

Suppose that the value of y corresponding to x equal to 2·3 is wanted.

In the first curve, a simple proportion of the first difference or a linear interpolation suffices. Thus :

$$y=4+\frac{3}{10}\times 2=4{\cdot}6$$

But in the second curve, a simple proportion of the first difference gives :

$$y=4+\frac{3}{10}\times 3=4{\cdot}9$$

—and this, from the figure, is clearly too large because the curve joining H and K lies below the straight line joining them. The second differences must therefore be taken into account.

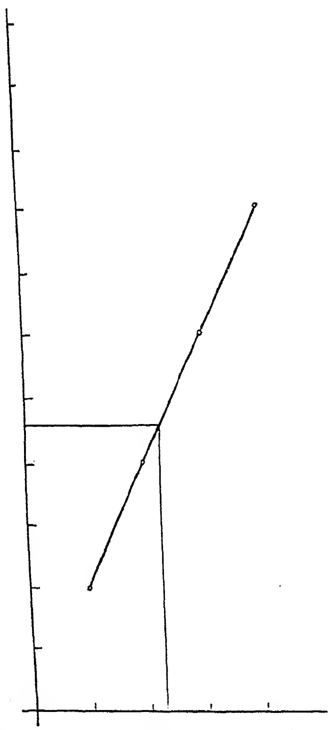


FIGURE 54a.

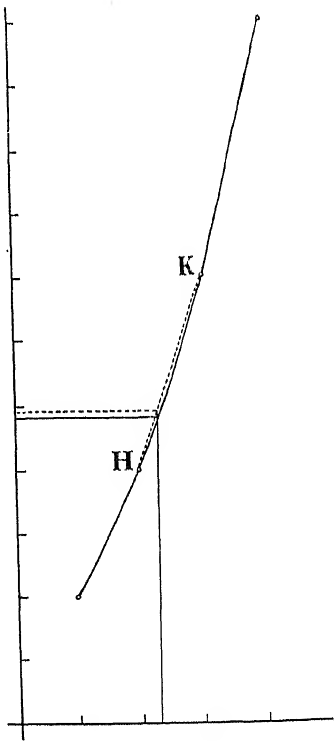


FIGURE 54b.

Besselian Interpolation Coefficients. Allowance for these (and other differences if necessary) is conveniently effected by means of the *Besselian interpolation coefficients* given in the standard edition of the *Nautical Almanac.*

The notation employed to distinguish the various differences is

Function	Differences				
	First	Second	Third	Fourth	Fifth
f_{-2}					
	$\Delta'_{-1\frac{1}{2}}$				
f_{-1}		Δ''_{-1}			
	$\Delta'_{-\frac{1}{2}}$		$\Delta'''_{-\frac{1}{2}}$		
f_0		Δ''_0		Δ^{iv}_0	
	$\Delta'_{\frac{1}{2}}$		$\Delta'''_{\frac{1}{2}}$		$\Delta^{v}_{\frac{1}{2}}$
f_1		Δ''_1		Δ^{iv}_1	
	$\Delta'_{1\frac{1}{2}}$		$\Delta'''_{1\frac{1}{2}}$		
f_2		Δ''_2			
	$\Delta'_{2\frac{1}{2}}$				
f_3					

The quantities f_0, f_1, f_2 and f_3, for example, might correspond to y equal to 2, 4, 7 and 11.

The general Besselian form of the interpolated quantity lying between f_0 and f_1 is then given by :

$$f_n = f_0 + n\Delta'_{\frac{1}{2}} + B''(\Delta''_0 + \Delta''_1) + B'''\Delta'''_{\frac{1}{2}} + B^{iv}(\Delta^{iv}_0 + \Delta^{iv}_1) + \ldots$$

—where n is the fraction of the interval between the two tabular values (·3 in the examples given) and B'', B''', B^{iv} . . . are functions of n. Thus :

$$B'' = \frac{n(n-1)}{2 \,.\, 2\,!}$$

$$B''' = \frac{n(n-1)(n-\frac{1}{2})}{3\,!}$$

and

$$B^{iv} = \frac{(n+1)n(n-1)(n-2)}{2 \,.\, 4\,!}$$

These are the Besselian coefficients, and they are tabulated for various values of n.

As a rule, the navigator called upon to carry out a survey involving astronomical observations, will not have to go beyond second differences. The interpolated quantity is then given by :

$$f_n = f_0 + n\Delta'_{\frac{1}{2}} + B''(\Delta''_0 + \Delta''_1)$$

If this formula is applied to the second curve in the example, B'' is seen, from the tables, to be $-0{\cdot}053$, and the value of y when x is equal to 2·3 is given by :

$$\begin{aligned} & 4 + \frac{3}{10} \times 3 - {\cdot}053(1+1) \\ & = 4{\cdot}9 - {\cdot}106 \\ & = 4{\cdot}8 \end{aligned}$$

This value clearly agrees with figure 54b.

The following example shows the discrepancy that exists between the quantity obtained by a linear interpolation between the tabulated quantities of the abridged edition, and that obtained with the use of the Besselian coefficients applied to the tabulated quantities of the standard edition.

It is required to find the Sun's apparent declination at $17^h37^m28^s$ *G.M.T. on the* 3*rd April* 1937.

From the abridged edition of the *Nautical Almanac* :

G.M.T.	*Declination*	*Diff^n*
16^h	$5°19'\cdot3$N.	
		$2'\cdot0$
18^h	$5°21'\cdot3$N.	

The declination at $17^h37^m28^s$ is therefore :

$$5°19'\cdot3+\frac{1^h37^m28^s}{2^h}\times2'\cdot0$$
$$=5°19'\cdot3+1'\cdot62$$
$$=5°20'\cdot9$$

From the standard edition :

Date	*G.M.T.*	*App. Dec. N.*	*First Diff^n*	*Second Diff^n*	*Third Diff^n*
April 2nd	0^h	$4°40'56''\cdot2$			
			$+1384''\cdot4$		
April 3rd	0^h	$5°04'00''\cdot6$		$-5''\cdot3$	
			$+1379''\cdot1$		$-0''\cdot4$
April 4th	0^h	$5°26'59''\cdot7$		$-5''\cdot7$	
			$+1373''\cdot4$		
April 5th	0^h	$5°49'53''\cdot1$			

The fraction of the interval between the tabulated G.M.T.s on the 3rd and 4th April is given by :

$$n=\frac{17^h37^m28^s}{24^h}$$

—and the value of this, from the table that reduces times to decimals of 24^h, is $\cdot73435$.

The declination at $17^h37^m28^s$, being given to second differences by $f_0+n\Delta'_{\frac{1}{2}}+B''(\Delta''_0+\Delta''_1)$, is therefore :

$$5°04'00''\cdot6+\cdot734(1379''\cdot1)-\cdot049(-5''\cdot3-5''\cdot7)$$
$$=5°04'00''\cdot6+16'52''\cdot26+0''\cdot54$$
$$=5°20'53''\cdot4$$

The discrepancy between the quantities obtained from the two almanacs is thus $0''\cdot6$.

It will be noticed that, in the working of this example, differences are written in the units of the last figure tabulated ($2'\cdot0$, for example, and not $2'$) and that the result of the interpolation is written to the original number of figures in the tabulated quantity although one more figure is retained in the intermediate stages.

Greater accuracy than this in the working is unnecessary because there is always a possible error of half a unit in the last figure tabulated owing to 'rounding off'—·35, for example, could be written as either ·3 or ·4—and the possible error of any interpolated quantity is one unit of the last figure retained. If, for example, y is 1·51 when x is 1, and 4·51 when x is 3, the value of y when x is 2 is clearly $1{\cdot}51+\frac{1}{2}$ (3·00) or 3·01. But if the given values of y are rounded off to the nearest integer, so that y is 2 when x is 1, or 5 when x is 3, the value of y when x is 2 is 3·5 and this, when rounded off, can be written either as 3, which is approximately correct, or as 4, which is approximately 1 unit in error.

The discrepancy obtained in the example just worked is thus reasonable and to be expected, and it is by no means as large as it could be if the quantities involved happened to introduce maximum rounding-off errors.

The Signs of the Differences. In order to lessen the chance of algebraic error, differences should always be obtained, when not given, by subtracting the upper ($\Delta'_{\frac{1}{2}}$ for example) from the lower ($\Delta'_{1\frac{1}{2}}$); that is, by moving down the column. The sign of the difference then decides itself. Thus:

	First Diff^n	*Second Diff^n*	*Third Diff^n*
568			
	$(457-568)=-111$		
457		$-102-(-111)=+9$	
	$(355-457)=-102$		$(8-9)=-1$
355		$-94-(-102)=+8$	
	$(261-355)=-94$		$(7-8)=-1$
261		$-87-(-94)=+7$	
	$(174-261)=-87$		
174			

It will also be noted that:

$$8+9=17=-94-(-111)$$

That is, if 568 is denoted by f_{-1} and the usual notation followed:

$$\Delta''_0+\Delta''_1=\Delta'_{1\frac{1}{2}}-\Delta'_{-\frac{1}{2}}$$

Since first differences are usually tabulated, this affords a check upon second differences.

If, in addition to taking this precaution, the calculations involved in the Bessel formula are always worked *forward* from the tabulated G.M.T. preceding the given instant, the chance of error is still further reduced.

The Seasons. The tropical year (and therefore the civil year) is conveniently divided into four parts, known as *the seasons*, by the passage of the Sun through the First Point of Aries, the solstitial point that marks the Sun's greatest northerly declination, the

First Point of Libra and the solstitial point that marks the Sun's greatest southerly declination.

Spring starts when the Sun is at ♈ about the 22nd March, and finishes when the Sun attains its greatest northerly declination about the 22nd June. On this date summer begins. Summer ends about the 22nd September, when the Sun reaches ♎. Autumn ends about the 22nd December when the Sun reaches the winter solstice. The seasons thus depend on the relative declinations of the Sun.

In considering seasonal changes, it is convenient to divide the Earth's surface into five zones by the parallels of latitude 23°27′ and (90°−23°27′) north and south of the equator. The parallels of 23°27′N. and 23°27′S. are known as the tropics of Cancer and

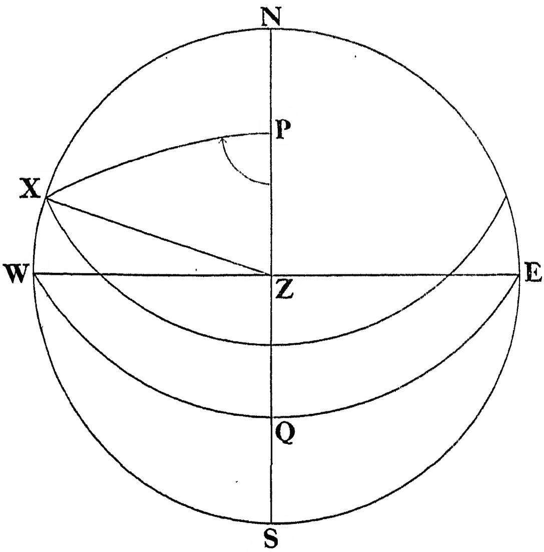


FIGURE 55.

Capricorn, and the belt between them is the *torrid zone.* The parallel of 66°33′N. is the Arctic circle. On its polar side is the *north frigid zone,* and between it and the tropic of Cancer is the *north temperate zone.* Corresponding to these are the *south frigid* and *south temperate zones,* separated by the Antarctic circle.

On the 22nd June at a place in the north temperate zone, the Sun's altitude at noon is greater than at any other time of the year. In consequence the Sun is above the horizon for its greatest number of hours during any day of the year. On the 22nd December, the Sun's altitude and the number of hours above the horizon are least. For a place in the south temperate zone the situation is reversed.

Figure 55, drawn on the plane of the horizon for an observer in north latitude, shows the position of the Sun when setting. If

refraction is neglected, the hour angle at this moment is given by :

$$\cos 90° = \sin l \sin d + \cos l \cos d \cos h$$

—where h, l and d are the usual navigational symbols for hour angle, latitude and declination. That is :

$$\cos h = -\tan l \tan d$$

At a place on the equator, l is 0°. Hence $\cos h$ is 0, and h is 90° or 6^h. The Sun is therefore above the horizon for 12^h, and this period is independent of the declination and, in consequence, the day of the year.

At a place on the Arctic circle, l is 66°33′ and, on the 22nd June, d is 23°27′. Hence :

$$l = 90° - d$$

i.e.
$$\tan l = \cot d$$

It therefore follows that $\cos h$ is minus 1 and h is 180° or 12^h. The Sun thus sets 12^h after it crosses the meridian and rises 12^h before meridian passage. That is, it is above the horizon throughout the day.

In the frigid zone where l is greater than 66°33′, the Sun is above the horizon continuously for a number of days, the number depending on l. The condition that the Sun should be just above the horizon is that h should be 12^h. That is, $\cos h$ should be minus 1. Hence it is necessary that :

$$\tan l \tan d = 1$$

If, for example, l is 70° :

$$\tan 70° = \cot d = \tan (90° - d)$$

i.e.
$$d = 20°$$

In latitude 70°N., therefore, the Sun is continuously above the horizon between the 21st May and the 24th July, on which dates the declination is 20°N.

At the north pole itself, the Sun is continuously above the horizon between the 22nd March and the 22nd September, and below it for the rest of the year. Night and day at the north pole are thus six months each in duration.

Similar considerations apply to places in the southern hemisphere.

The Average Lengths of the Seasons. Since the Sun's apparent motion is not uniform, and also since the First Point of Aries is not a fixed point, the lengths of the seasons are not equal.

These lengths can be determined from the formula established on page 88 :

$$\theta - nt = 2e \sin nt$$

—where, in figure 47, θ is equal to AS and nt to AD. That is :

$$\begin{aligned} \theta - nt &= (A\Upsilon + \Upsilon S) - (A\Upsilon + \Upsilon D) \\ &= \Upsilon S - \Upsilon D \end{aligned}$$

Hence, by substitution, θ being nearly equal to nt:

$$\Upsilon D = \Upsilon S - 2e \sin (\Upsilon S + A\Upsilon)$$

If T is the time in days, measured from the moment the Sun is at Υ, then :

$$\Upsilon D = nT$$

and $$T = \frac{1}{n}[\Upsilon S - 2e \sin (\Upsilon S + A\Upsilon)]$$

Spring starts when ΥS is zero and finishes when ΥS is $\frac{1}{2}\pi$. The number of days in spring is therefore given by :

$$\frac{1}{n}[\tfrac{1}{2}\pi - 2e \sin (\tfrac{1}{2}\pi + A\Upsilon)] + \frac{1}{n} \times 2e \sin A\Upsilon$$

Also $A\Upsilon$ is 80°, according to the Besselian reckoning, and e is 1/60. The expression can thus be evaluated.

In a similar manner the number of days in summer, autumn and winter can be found, and the lengths of the four seasons are seen to be :

Spring	92 days 20 hours
Summer	93 days 14 hours
Autumn	89 days 19 hours
Winter	89 days 0 hours

They are, however, affected by perturbations.

Synodic and Sidereal Periods. The only planetary satellite of importance to the navigator is the Earth's own satellite, the Moon.

The Moon revolves about the Earth at an average distance of 240,000 miles, and the inclination of its orbit to the ecliptic is just over 5°. The direction of its motion is the same as that of the Earth about the Sun.

The period of revolution relative to the stars is about $27\frac{1}{3}$ days, and this is known as the *sidereal period*. Its mean value is 27 days 7 hours 43 minutes, and the greatest variation from this mean value is about 3 hours.

The interval between two successive new moons, that is the period of revolution relative to the Sun, is known as the *synodic period*. It is greater than the sidereal period because the Sun has an apparent eastward movement of about 1° per day relative to the stars. Its mean value is 29 days 12 hours 4·4 minutes, and the greatest variation from this mean value is about 13 hours.

The ordinary month is derived from the synodic period.

Sidereal Time. Sidereal time is discussed fully in Chapter XI of Volume II, where it is defined as the right ascension of the meridian at any moment or the hour angle of the First Point of Aries. A sidereal day is likewise defined as the interval between two successive passages of the First Point of Aries across the observer's meridian.

In these definitions, the true vernal equinox is considered. If the equinox were fixed, a sidereal day would measure exactly one rotation of the Earth about its axis, a quantity that is uniform. But, as shown in this chapter, the equinox is not fixed. Hence a sidereal day does not truly define one rotation of the Earth. Also, the irregular motion of the equinox gives rise to an irregular sidereal day. On the average, a sidereal day is less than the interval corresponding to one rotation of the Earth by $0^{s}\cdot009$.

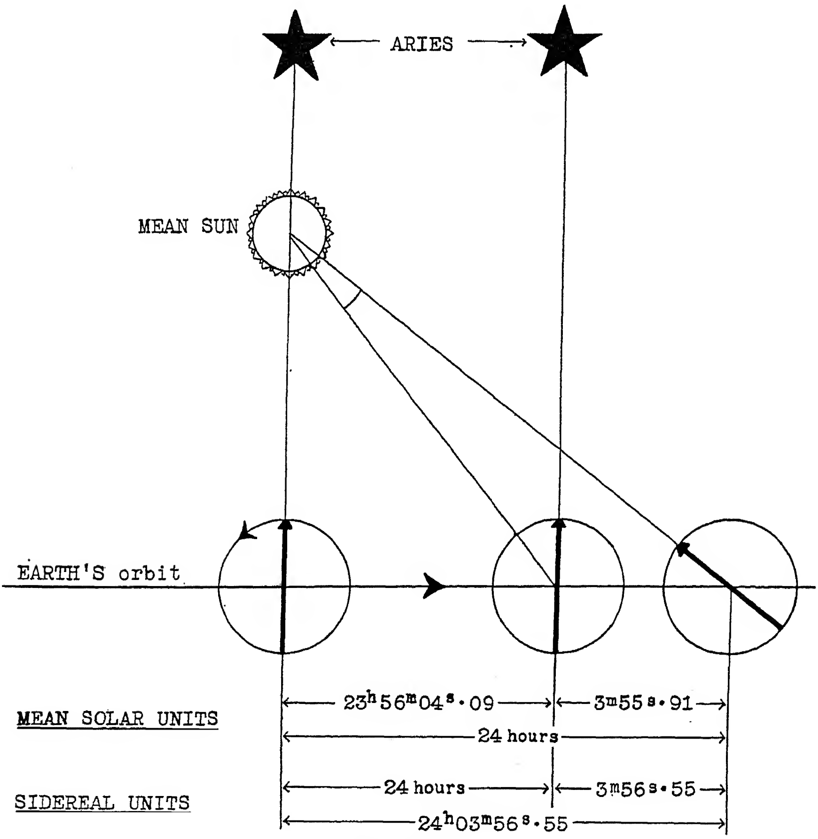


FIGURE 56.

The discrepancy between the definition of a sidereal day as the interval between successive transits of the First Point of Aries across the same meridian, and the definition of a sidereal year as the interval between two successive coincidences of the Sun's direction with the direction, not of the First Point of Aries, but an imaginary fixed star in the ecliptic, results from the difference between navigational and astronomical practice. To the practical navigator, sidereal time is an angle, and, since the positions of the stars which he observes are fixed for him in relation to the meridian

from which he measures this angle, it is only natural that he should, for his own convenience, regard the meridian, and therefore the position of the First Point of Aries, as fixed. He is not concerned with precession because it is too small to affect his practical work at the time, and the *Nautical Almanac* allows for it as it becomes appreciable. The astronomer, however, is concerned with precession, and defines the sidereal day as the interval between two successive passages of an equatorial star without proper motion across the same meridian. His definition of the sidereal year is then seen to be analogous. Sidereal time, however, is necessarily based on the First Point of Aries with its proper motion taken into account because right ascensions are measured from it.

The Difference Between Sidereal and Solar Units. The period of the Earth's axial rotation measures the true sidereal day, and is $23^h\ 56^m\ 04^s{\cdot}100$ in mean solar units, but the precession of the equinox reduces this period by $0^s{\cdot}009$ so that the average length of the sidereal day determined by two transits of the First Point of Aries is $23^h\ 56^m\ 04^s{\cdot}091$ in mean solar units, a period that is equivalent to 24 sidereal hours.

Simple proportion gives the length of the mean solar day in sidereal units. Thus:

$$23^h\ 56^m\ 04^s{\cdot}091 \text{ in mean solar units} \equiv 24 \text{ sidereal hours}$$

$$86{,}164{\cdot}091 \text{ mean solar seconds} \equiv 24.60.60 \text{ sidereal seconds}$$

$$1 \text{ mean solar second} \equiv \frac{86{,}400}{86{,}164{\cdot}091} \text{ sidereal seconds}$$

$$\therefore \quad \begin{array}{c}24.60.60 \text{ mean solar seconds}\\ \text{(or 1 mean solar day)}\end{array} \equiv \frac{86{,}400.24.60.60}{86{,}164{\cdot}091} \text{ sidereal seconds}$$

$$\text{or } 24^h\ 03^m\ 56^s{\cdot}55 \text{ in sidereal units}$$

A mean solar day is therefore longer than a sidereal day by $3^m\ 55^s{\cdot}91$ in mean solar units and a sidereal day is shorter than a mean solar day by $3^m\ 56^s{\cdot}55$ sidereal units.

Figure 56 illustrates the difference between the two sets of units.

CHAPTER X

THE ASTRONOMICAL POSITION LINE

If altitudes and bearings could be measured and laid off on a chart with absolute accuracy, one observation of a heavenly body would give a ship's position. This, however, cannot be done, and at present there is nothing to suggest any modification of the accepted principle that two position lines are necessary in order to decide that position. The problem of finding a ship's position is therefore the problem of finding these position lines.

There are two general methods of solving this problem: the intercept method, and the longitude method.

In the first, the observer's latitude and longitude are assumed; the hour angle is derived from a deck-watch time, and the spherical triangle PZX is solved for ZX and the angle at Z. In the second, the observer's latitude is assumed, ZX is taken as the complement of the observed altitude, and the triangle PZX is solved for the angle at P. Both methods are compared and explained fully in Chapter XIV of Volume II, but in that and subsequent chapters only standard methods of solving the spherical triangle are given. There are others, however, relating particularly to the intercept method, and these, for convenience, may be divided into two classes: those that work from the dead reckoning or estimated position, and those that work from a special assumed position. Each class has its peculiar advantages and disadvantages.

Methods Based on the D.R. Position. These have the advantage that the results obtained are always plotted easily. Also the intercepts obtained are usually short. But the actual working is comparatively long. The most important of these methods are:

(1) the cosine-haversine or Davis method which is adopted by the Royal Navy.
(2) the Aquino 'log and versine' method.
(3) the Yonemura, a Japanese method.
(4) the Ageton. (American Hydrographic Office No. 211.)

Of these, only the Ageton is sharply distinguished in its approach to the problem. The others, being based on the fundamental formula, have common characteristics. That formula, in the usual notation, is:

$$\cos z = \sin l \sin d + \cos l \cos d \cos h$$

In the cosine-haversine method explained in Volume II it is written :

$$\text{hav}\ z = \text{hav}(l \sim d) + \text{hav}\ \theta$$

—where θ is some angle (not actually found) given by :

$$\text{hav}\ \theta = \cos l \cos d\ \text{hav}\ h$$

The Aquino 'Log and Versine' Method. In this method versines are used instead of haversines and the parameter θ is expressed in a different form. Thus :

$$\tfrac{1}{2}(1-\cos\theta) = \tfrac{1}{2}\cos l \cos d(1-\cos h)$$

i.e.

$$\sin^2 \tfrac{1}{2}\theta = \cos l \cos d \sin^2 \tfrac{1}{2}h$$

or

$$\text{cosec}^2 \tfrac{1}{2}\theta = \sec l \sec d\ \text{cosec}^2 \tfrac{1}{2}h$$

The logarithmic part of the solution is therefore worked, with special tables, from the formula :

$$\log \text{cosec}\ \tfrac{1}{2}\theta = \tfrac{1}{2}\log \sec l + \tfrac{1}{2}\log \sec d + \log \text{cosec}\ \tfrac{1}{2}h$$

The Yonemura Method. The same parameter θ is used in this method, but it is, again, expressed differently. Thus :

$$\text{hav}\ \theta = \cos l \cos d\ \text{hav}\ h$$

$$\frac{1}{\text{hav}\ \theta} = \sec l \sec d \left[\frac{1}{\text{hav}\ h}\right]$$

$$\log\left[\frac{1}{\text{hav}\ \theta}\right] = \log \sec l + \log \sec d + \log\left[\frac{1}{\text{hav}\ h}\right]$$

The sight is thus worked on the same lines as the cosine-haversine and Aquino methods, but the functions giving θ and h are reciprocals of the haversine instead of the haversine. If, for example, the logarithmic haversine is 9·38321, the corresponding figure in the Yonemura method is :

$$\begin{aligned} &0 - 9{\cdot}38321 \\ &= 0{\cdot}61679 \end{aligned}$$

The Yonemura method includes tables for finding the azimuth, but the formula used :

$$\frac{\sin (az.)}{\sin (90^\circ - d)} = \frac{\sin h}{\sin (90^\circ - alt.)}$$

i.e.

$$\sin (az.) = \sin h \cos d \sec (alt.)$$

—is ambiguous, the azimuth given being either α or $(180^\circ - \alpha)$.

The Ageton Method. In this method a perpendicular is dropped from X upon the side PZ, and Napier's rules are applied to the right-angled spherical triangles thus formed.

The declination of the foot of this perpendicular—A, in figure 57

—is taken as K, so that AZ is $(K{\sim}l)$, and ZX is expressed in terms of the altitude. Napier's rules then give :

From the time triangle PAX

$$\operatorname{cosec} AX = \operatorname{cosec} h \sec d \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\operatorname{cosec} K = \frac{\operatorname{cosec} d}{\sec AX} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From the altitude triangle ZAX

$$\operatorname{cosec} (alt.) = \sec AX \sec (K{\sim}l) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\operatorname{cosec} (az.) = \frac{\operatorname{cosec} AX}{\sec (alt.)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

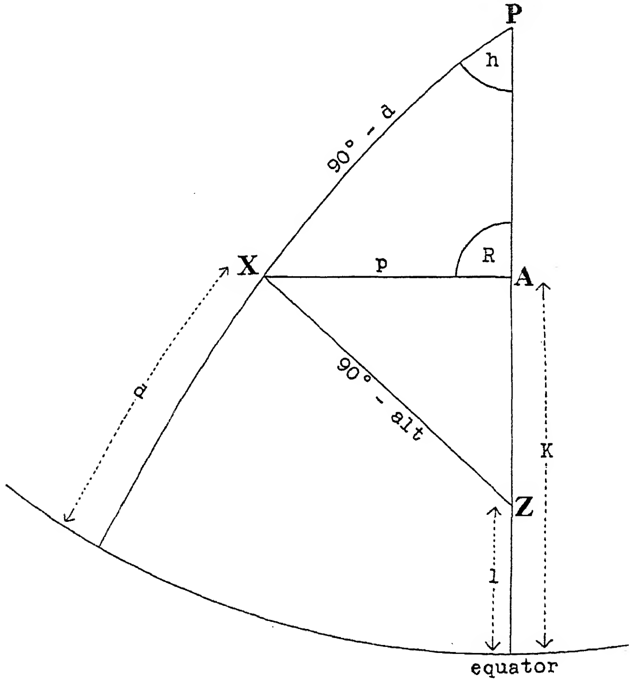


FIGURE 57.

In these equations, h, d and l are known. AX is thus obtained from (1) and K from (2). The quantity $(K{\sim}l)$ is then found, and (3) gives the calculated altitude. If the azimuth is required, (4) can be used, and this gives the method a great advantage over the other three in this group. The tables required are, moreover, simply logarithmic cosecant and secant tables,* called A and B and arranged in adjacent columns. For convenience the actual logarithms are multiplied by 10^5.

* These tables are identical with those given in Table 2 of the *Sea and Air Navigation Tables* described in Chapter XV of Volume II.

The following example, in which the hour angle is assumed to be based on the D.R. longitude, shows the recommended lay-out of the work.

Required the calculated altitude and intercept when the hour angle and declination of a heavenly body are 22°20′·5*E. and* 38°42′·7*N. respectively, the true altitude being* 64°16′·5, *and the D.R. latitude* 21°17′·0*N.*

		ALTITUDE		AZIMUTH	
		Add	*Subtract*	*Add*	*Subtract*
H.A.	22°20′·5E.	A 42007			
	38°42′·7N.	B 10774	A 20384		
		A 52781	B 2000	B 2000	A 52781
K	40°54′·6N.		A 18384		
l	21°17′·0N.				
K∼*l*	19°37′·6			B 2599	
	calc. alt.	64°05′·6		A 4599	B 35963
	true alt.	64°16′·5			A 16818
	intercept	10′·9 towards		azimuth N.42¾°E.	

The great advantage over the other methods that this method enjoys, lies in the quickness with which the azimuth can be found. Also the tables themselves are compact and easy to use. But they are unreliable when the value of *K* lies between 87°30′ and 92°30′, and when the hour angle approaches 90° ; and the rules for their use are not so straightforward as those for, in particular, the cosine-haversine method.

Methods Based on a Special Assumed Position. In these methods a position is usually chosen having an integral number of degrees in its latitude and a longitude that combines with the Greenwich hour angle of the heavenly body to form a local hour angle which is also integral, the position being that nearest the D.R. which fulfils these conditions. The perpendicular is then dropped from *Z* or *X* on to the opposite side, and, as in the Ageton method, Napier's rules are used to solve the right-angled spherical triangles thus formed. A shortening of the arithmetical labour is possible because, when a limit is set to the number of hour angles, solutions of the various equations can be tabulated.

In Chapter XIV of Volume II it is explained that the principle of the intercept method is simply a comparison of distances. The navigator chooses any position in the neighbourhood of the position where he thinks the ship is, and compares the distance between this position and the heavenly body's geographical position, which he obtains by calculation, with the distance between the ship's actual position and that geographical position, which he finds from

a sextant observation. Within limits that will be discussed, the navigator can therefore choose any position convenient to himself.

In figure 58, DJ and $D'J'$ are the intercepts obtained when the same sight is worked from two positions, D and D'. Since the position of G, the geographical position, depends only on the G.M.T. of the observation, GJ and GJ' are radii of the same position circle, and, if G is sufficiently distant for the arc JJ' of this circle to be taken as a straight line, the limits imposed on the positions that D and D' may occupy are governed by the restriction that, for practical purposes, DJ and $D'J'$ must be parallel. This restriction, however, is sufficiently wide for these purposes.

It is apparent from the conditions governing the choice of a special assumed position, that the greatest displacement of this

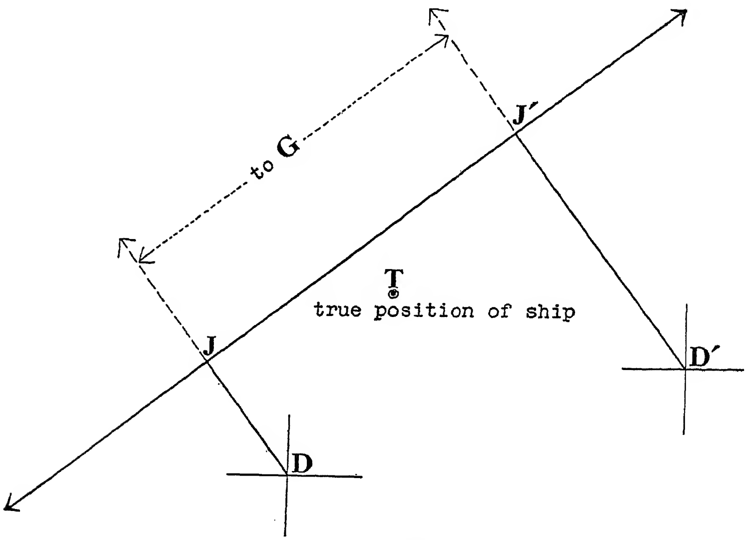


FIGURE 58.

position from the D.R. or E.P. is 30′ (of arc) in latitude and longitude. When this displacement is made from the true position of the ship, the largest error for altitudes up to 60° does not exceed 0′·5 in the length of the intercept, and, in the worst possible circumstances, the probable error in the azimuth does not exceed ½°. Also in modern navigation, the D.R. position is usually close to the true position of the ship. It may thus be accepted that the error introduced by the largest displacement is of no practical importance.

For altitudes in the neighbourhood of 75°, it is possible that the error in the intercept may be 1′ in the most unfavourable conditions. Methods involving a displacement of the D.R. should therefore be used with caution when the altitude is large and the displacement itself approaches its maximum.

Plotting from Special Assumed Positions. Since the longitude of each special position is chosen to combine with the Greenwich hour angle so as to give an integral number of degrees in the local hour angle, and also since no two sights can be taken at exactly the same moment by the same navigator, each sight must be worked from a different position, and this fact introduces a small complication in the plotting. The intercept, moreover, may be large in comparison with that obtained when the sight is worked from the D.R.

In the neighbourhood of the equator when the greatest displacement of 30′ in both d'lat and d'long is made, the distance between the special position and the D.R. is $30\sqrt{2}$ or about 42′. In higher latitudes the distance is less. In 60° (N. or S.), for example, the departure corresponding to a d'long of 30′ is 15′, and the distance between the two positions is $\sqrt{(15)^2+(30)^2}$ or about 33′·5. Intercepts up to 40′ or 50′ may thus be obtained. On the other hand it is possible that the intercept will be less than that obtained when the sight is worked from the D.R. position.

The *Sea and Air Navigation Tables*. These tables, the construction and use of which are explained in Chapter XV of Volume II, result from what is, in effect, a combination of two methods: the Ogura (Japan) and the Dreisonstok (U.S.A.). The columns headed K and A in Table I give the Ogura figures checked and extended to cover latitudes up to 90° instead of the 65° covered by Ogura. The remaining columns in this Table, which relate to the azimuth, are similar to those in Dreisonstok's Table I, but use a complementary angle. In Table II the tabulation is made for every half minute of the argument $(K \sim d)$ instead of every integral minute, which is the tabulation adopted by both Dreisonstok and Ogura.

The Ogura and Dreisonstok Methods. In both these methods, the spherical triangle PZX is divided by a perpendicular from Z on PX, but whereas Ogura works in terms of the declination of F, the foot of this perpendicular, Dreisonstok works in terms of b, the complement of it. (Figure 59a.)

The quantities involved in the calculation of the altitude and the equations obtained by Napier's rules are therefore:

OGURA	DREISONSTOK
Time Triangle	*Time Triangle*
$PF=90°-K$	$PF=b$
$PZ=90°-l$	$PZ=90°-l$
$\cot K=\cos h \cot l$	$\tan b=\cos h \cot l$
$\sin p=\sin h \cos l$	$\sin p=\sin h \cos l$
Altitude Triangle	*Altitude Triangle*
$FX=(K\sim d)$	$FX=90°\sim(d\sim b)$
$\operatorname{cosec}(alt.)=\sec(K\sim d)\sec p$	$\operatorname{cosec}(alt.)=\operatorname{cosec}(d\sim b)\sec p$

The two methods are thus strikingly similar in conception, but the Ogura enjoys a distinct advantage because when the hour angle is less than 6^h east or west of the meridian, within which limits most hour angles lie in practice, the quantity $(K+d)$ can never be greater than 90° whereas the quantity $(b+d)$ can, as figure 59b shows. Also the Ogura tables are based on six-figure logarithms for values of the argument when the logarithm changes slowly, whereas the Dreisonstok tables are based throughout on five-figure logarithms. The effect of this is to introduce a small error into the altitude found from the Dreisonstok tables when the altitude is high or the hour angle is small. The Dreisonstok tables, for example, give the logarithmic cosecant of 81°16′ as 506 and that of 81°17′

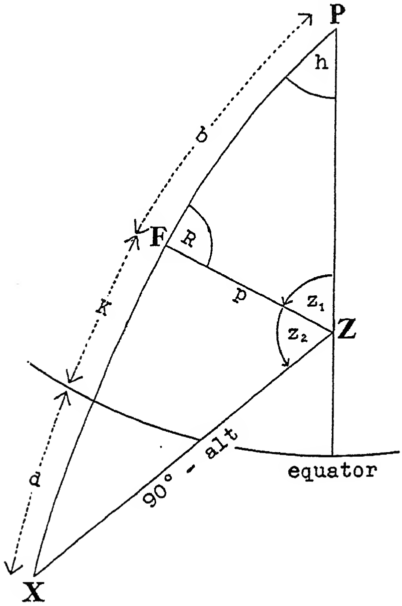


FIGURE 59a.

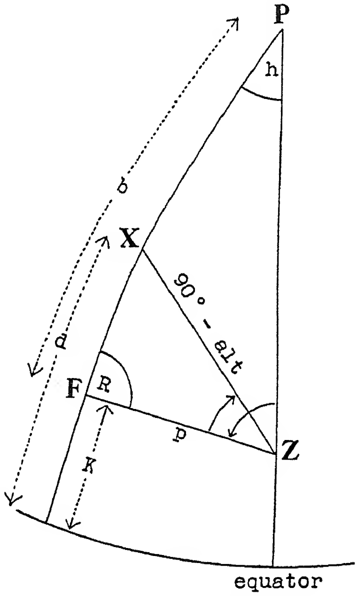


FIGURE 59b.

as 505, whereas the Ogura tables give the corresponding values as 506·5 and 504·5. An entry into the body of these tables with a respondent equal to 505 therefore reveals an error of 0′·2 in the Dreisonstok altitude. This, however, is of little importance in view of the error that may arise from the displacement of the D.R. when the altitude is as high as 81°.

In addition to being incorporated in the *Sea and Air Navigation Tables,* the Ogura altitude tables are used, in conjunction with Rust's azimuth diagram, in the Weem's *Line of Position Book.*

The Ogura tables do not, by themselves, give the azimuth. The Dreisonstok tables do, the equations being :

$\tan z_1 = \operatorname{cosec} l \cot h$ (time triangle)

$\tan z_2 = \cot (d \backsim b) \operatorname{cosec} p$ (altitude triangle)

As shown in the explanation of the *Sea and Air Navigation Tables* in Volume II, the corresponding equations for the extension of the Ogura tables in terms of the complementary function K are :

$$\tan z_1 = \operatorname{cosec} l \cot h \qquad \text{(time triangle)}$$
$$\tan z_2 = \tan (K \sim d) \operatorname{cosec} p \qquad \text{(altitude triangle)}$$

There is thus a similarity in the construction of the two sets of tables that is reflected in the similarity of the procedures adopted when sights are worked from them. The use of b, the complement of K, makes the Dreisonstok rules less straightforward, but apart from this complication the two procedures are the same.

The advantage of the *Sea and Air Navigation Tables* lies in that greater simplicity of rule and in the higher latitudes covered (90° against 65°), and in the lay-out of the tables themselves. Incidentally the tables can, as explained, be used for the Ageton method.

The *Sea and Air Navigation Tables* used with the D.R. Position. Table I of the *Sea and Air Navigation Tables* gives the quantities K, A, D and Z_1, for the arguments latitude and hour angle, the position from which the sight is worked being chosen so that the values of the latitude and hour angle are the integral degrees for which the Table is constructed. If the D.R. position is taken without adjustment, it is most unlikely that these integral values will be obtained, and Table I cannot be used because the interpolation that is now necessary is excessive. The quantities K, A, D and Z_1 can, however, be calculated from Tables II and III for any position.*

The quantities B and C given in Table II are :

$$B = \log \sec (\text{argument}) \times 10^5$$
$$C = \log \operatorname{cosec} (\text{argument}) \times 10^5$$

—and the quantity E given in Table III is :

$$E = \log \tan (\text{argument}) \times 10^3$$

To find K, A, D and Z_1 without using Table I, it is therefore necessary to express them in terms of the secant, cosecant and tangent, and this can be done because, from the Ogura time-triangle (figure 59a) :

$$\operatorname{cosec} p = \sec l \operatorname{cosec} h$$

$$\operatorname{cosec} K = \frac{\operatorname{cosec} l}{\sec p}$$

and

$$\tan Z_1 = \frac{\operatorname{cosec} l}{\tan h}$$

Once K, A, D, and Z_1 have been found from these formulæ, the method is the same as that followed when the sight is worked from a specially chosen position.

* The method of doing this has been devised by Dr. L. J. Comrie.

The following example is that used to illustrate the Ageton method on page 110.

Required the intercept and bearing when the hour angle and declination of a heavenly body are 22°20′·5*E. and* 38°42′·7*N. respectively, the true altitude being* 64°16′·5, *and the D.R. latitude* 21°17′·0 *N.*

l	21°17′·0N.	C	44012	B	3068	$\frac{1}{100}C$	10440
h	22°20′·5			C	42007	E	9614
		Opp.	2911	Sum	45075	Diff.	826
		Diff.	41101				
d	38°42′·7N.						
K	22°50′·3	A	2911	D	451	Z_1	+81°·5
$K{\sim}d$	15°52′·4	B	1689	E	9454		
		$A+B$	4600	$D+E$	9905	Z_2	−38°·8
	calc. alt.	64°05′·5				Azimuth	N.42°·7E.
	true alt.	64°16′·5					
	intercept	11′·0 towards					

From this example it is seen that the steps to be followed are :

(1) On the first line note, in succession, the quantities l, C, B and C with the last two figures rounded off, the quantities B and C being taken from Table II with argument l. This step involves one entry.

(2) On the second line note h in the first column under l ; leave the second column blank ; then write C (from Table II) in the third column and E (from Table III) in the fourth. This step involves two entries, one of which is for the azimuth.

NOTE. If h is greater than 90°, the quantity C is taken from the argument at the foot and right of Table II, and E, if necessary, may be obtained with the argument (180° $-h$).

(3) On the third line form the sum $(B+C)$ and the difference $(\frac{1}{100}C-E)$, and with $(B+C)$ enter either a B or C column in Table II and take out the opposite quantity from the adjacent column, interpolating as required. Enter this quantity in the second column of the calculation. This step involves one entry.

NOTE. If $\frac{1}{100}C$ is less than E, 10,000 must be added to it.

(4) On the fourth line this opposite quantity is subtracted from C, under which it stands, and with this difference in the C column of Table II, K is determined. This step involves one entry.

NOTE. If h is greater than 90°, K is greater than 90° and is read from the foot and right of Table II.

(5) The remainder of the calculation follows the lay-out given in Volume II, but involves only two more entries because A and D are already known, A being the opposite quantity and D the sum $(B+C)$ in the third column with the last two figures rounded off. The total number of entries for finding the altitude is thus six, the same as in the cosine-haversine method. Of the components of the azimuth, Z_1 is found by entering Table III with the difference in the fourth column for argument, and Z_2 comes from the same table with $(D+E)$ as argument.

The *Sea and Air Navigation Tables* contain a full explanation of the method.

The 'Smart and Shearme' or Sine Method. This method, though developed independently and at an earlier date, bears a distinct resemblance to the Dreisonstok altitude tables because it uses a quantity U which is the polar distance of the foot of the perpendicular from Z and therefore exactly analogous to Dreisonstok's quantity b. The same equations therefore hold, namely :

$$\tan U = \cos h \cot l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\sin p = \sin h \cos l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\operatorname{cosec}(alt.) = \operatorname{cosec}(U \sim d) \sec p \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

—but the third is written in the reciprocal form :

$$\sin(alt.) = \sin(U \sim d) \cos p$$

In logarithmic form this is :

$$\log \sin(alt.) = V + \log \sin(U \sim d)$$

—where V is equal to $\log \cos p$. This quantity is found from equation (2) and tabulated with U, which is found from equation (1). The tables are thus constructed on the same lines as the Dreisonstok tables, but in the time argument the hour angle is expressed at intervals of four minutes in time and not at intervals of 1° in arc. This slightly complicates the choosing of the special position from which the sight must be worked. The Dreisonstok and Ogura tables also have the advantage of the simpler figures that result from the use of secants and co-secants instead of sines and cosines.

A typical extract from the 'Smart and Shearme' tables is :

Hour Angle	Lat. 20°		Lat. 21°		Hour Angle
	U	V	U	V	
$1^h 24^m$	68°42′·1	9·973 86	67°38′·9	9·974 22	$22^h 36^m$
1 28	68°34′·0	9·971 27	67°30′·6	9·971 67	22 32
1 32	68°25′·6	9·968 55	67°21′·8	9·969 00	22 28

The following example, which is the same as that worked by the Ageton method on page 110, shows how the tables are used :

Required the calculated altitude and intercept when the hour angle and declination of a heavenly body are $22^h30^m38^s$ and 38°42′·7*N. respectively, the true altitude being* 64°16′·5 *and the D.R. position* 21°17′·0*N*., 38°15′·0*E*.

The hour angle nearest to $22^h30^m38^s$ in the tables is 22^h32^m. The special position from which the sight must be worked and the intercept laid off is thus 21°N., 38°35′·5E.

From the tables, for latitude 21° and hour angle 22^h32^m :

d	38°42′·7N.			
U	67°30′·6N.		V	9·971 67
$U+d$	106°13′·3		log sin $(U+d)$	9·982 36
			sum	9·954 03
∴	calc. alt.	64°06′·0		
	true alt.	64°16′·5		
	intercept	10′·5 towards.		

It is seen that a logarithmic sine table is necessary in addition to the '*U* and *V*' tables. Incidentally the intercept is slightly shorter than that obtained when the sight is worked from the D.R. position although, in this method, there has been an appreciable shift from the D.R. position.

Worked from the Ogura tables, this example reads :

G.H.A.	299°24′·5			
Long. E.	38°35′·5			
L.H.A.	338°W. (or 22°)			
d	38°42′·7N.			
K	22°29′·4N.		A	2833
$K{\sim}d$	16°13′·3		log sec $(K{\sim}d)$	1764
			sum	4597
	calc. alt.	64°06′·0		
	true alt.	64°16′·5		
	intercept	10′·5 towards		

A comparison between this working and the 'Smart and Shearme' working at once shows the simplicity of Ogura's figures and also the advantage of dealing with *K*, which is the declination of the foot of the perpendicular, instead of *U*, which is the complement. When the hour angle lies between 6^h east and 6^h west of the meridian,

the quantity $(K \sim d)$ is found by the ordinary rule for combining two latitudes: opposite names add, same names subtract. But the reverse of this rule holds for the quantity $(U \sim d)$, the plus sign being taken when the latitude and declination have the same names and the heavenly body is within 6^h of the meridian, and the minus sign when the latitude and declination have opposite names or when the body is more than 6^h from the meridian.

The adjustment necessary for finding the quantity $(K \sim d)$ when the heavenly body is more than 6^h from the meridian is explained in Chapter XV of Volume II.

The Supplementary Haversine Formula. This formula makes use of a function that may be conveniently called the *supplementary haversine* of x, or *shav* x. This function is:

$$\begin{aligned}\tfrac{1}{2}(1+\cos x) &= (1-\text{hav } x)\\ &= \text{hav } (180^\circ - x)\\ &= \text{shav } x\end{aligned}$$

If a, b, c and A, B, C are the sides and angles of a spherical triangle, the fundamental formula giving c is:

$$\begin{aligned}\cos c &= \cos a \cos b + \sin a \sin b \cos C\\ &= \cos a \cos b(\cos^2 \tfrac{1}{2}C + \sin^2 \tfrac{1}{2}C) + \sin a \sin b(\cos^2 \tfrac{1}{2}C - \sin^2 \tfrac{1}{2}C)\\ &= \cos (a \sim b) \cos^2 \tfrac{1}{2}C + \cos (a+b) \sin^2 \tfrac{1}{2}C\end{aligned}$$

But, from the definitions of hav x and shav x:

$$\begin{aligned}\text{hav } x &= \tfrac{1}{2}(1-\cos x) = \sin^2 \tfrac{1}{2}x\\ \text{shav } x &= \tfrac{1}{2}(1+\cos x) = \cos^2 \tfrac{1}{2}x\end{aligned}$$

Hence, by substitution:

$$\text{hav } c = \text{hav } (a \sim b) \text{ shav } C + \text{hav } (a+b) \text{ hav } C$$

and $$\text{shav } c = \text{shav } (a \sim b) \text{ shav } C + \text{shav } (a+b) \text{ hav } C$$

To express the first of these formulæ in terms of the latitude, declination and hour angle, the following substitutions must be made:

$$\begin{aligned}a &= \text{polar distance} = 90 \pm d\\ b &= \text{co-latitude} \quad = 90 - l\\ C &= \text{hour angle} \quad = h\\ \text{hav } (a \sim b) &= \text{hav } (l \sim d)\\ \text{hav } (a+b) &= \text{shav } (l+d)\end{aligned}$$

—l and d being given their proper signs throughout.

The formula then becomes:

$$\text{hav } z = \text{hav } (l \sim d) \text{ shav } h + \text{shav } (l+d) \text{ hav } h$$

i.e. $$\text{hav } z = \text{hav } \alpha + \text{hav } \beta$$

It is thus clear that the formula can be evaluated as it stands by means of the ordinary haversine tables. If, for example, the local hour angle of a heavenly body is $19^h 26^m 27^s$, and the latitude

and declination are 21°15′·0N. and 19°30′·3N. (these quantities are taken from the example worked on page 138 of Volume II) the procedure is :

h	19ʰ26ᵐ27ˢ	9·835 16 (log shav)	9·499 46 (log hav)
l	21°15′·0N.		
d	19°30′·3N.		
$l{\sim}d$	1°44′·7	6·365 25 (log hav)	
$l+d$	40°45′·3		9·943 86 (log shav)
		6·200 41	9·443 32
		·000 16 (nat. hav)	·277 54 (nat. hav)
		·277 54	
		·277 70	

The calculated zenith distance is therefore 63°36′·1.

This working, however, can be simplified considerably because an arrangement of the tables is possible in which hav α and hav β are looked up directly with the hour angle and ($l{\sim}d$) and ($l+d$) as arguments.*

The working then reads :

h	19ʰ26ᵐ27ˢ		
l	21°15′·0N.		
d	19°30′·3N.		
$l{\sim}d$	1°44′·7	D	·000 16
$l+d$	40°45′·3	S	·277 54
		S+D	·277 70

i.e. C.Z.D. 63°36′·1

The number of entries into the tables is thus reduced from seven to three.

Tables arranged so that the sight can be worked from the D.R. position, as in the above example, would be unavoidably bulky, but the tabulation can be restricted to integral degrees of latitude and hour angle. The method then takes its place among the best of the short tabular methods. Also it can be extended to give the azimuth.†

* This arrangement of the haversine tables has been devised by Instr. Commander H. A. McDonald, R.N.

† This extension has been investigated by Instr. Commander A. W. Veater, R.N. (Retired).

By the arc-and-angle property of polar triangles, whereby any angle or side of one triangle is equal to the supplement of the corresponding side or angle of the other :

$$\text{shav } C = \text{hav } (A \sim B) \text{ hav } c + \text{hav } (A+B) \text{ shav } c$$
$$\text{hav } C = \text{shav } (A \sim B) \text{ hav } c + \text{shav } (A+B) \text{ shav } c$$

By Gauss :

$$\text{hav } (A \sim B) = \frac{\text{hav } (a \sim b) \text{ shav } C}{\text{hav } c}$$

$$\text{hav } (A+B) = \frac{\text{shav } (a \sim b) \text{ shav } C}{\text{shav } c}$$

In terms of the latitude, declination and hour angle, these become :

$$\text{hav } (Z \sim X) = \frac{\text{hav } (l \sim d) \text{ shav } h}{\text{hav } z}$$

$$\text{hav } (Z+X) = \frac{\text{shav } (l \sim d) \text{ shav } h}{\text{shav } z}$$

The azimuth in the example just worked is found thus :

log hav $(l \sim d)$	6·365 25	log shav $(l \sim d)$	9·999 77
log shav h	9·835 16	log shav h	9·835 16
	6·200 41		9·834 93
g hav z	9·443 55	log shav z	9·858 73
	6·756 86		9·976 20

$Z \sim X = 2°44'·4$ $\qquad$ $Z+X = 153°17'·6$

The azimuth, being half the sum of these quantities, is therefore N.78°E.

It should be noted that in this calculation of the azimuth the value of log hav $(l \sim d)$ shav h has already been found, and that if the formula :

$$\text{shav } z = \text{shav } (l \sim d) \text{ shav } h + \text{shav } (l \sim d) \text{ hav } h$$

is used to check the zenith distance, the finding of the azimuth is further simplified because the value of log shav $(l \sim d)$ shav h is obtained.

The small difficulty that arises when the latitude and declination have the same names and X is greater than Z, can be dealt with by remembering that Z is greater, equal to or less than X according as l is greater, equal to or less than d.

If the latitude and declination have opposite names, Z is always greater than X.

The Aquino Tabular Method. In this method the perpendicular is dropped from X on to PZ, as in the Ageton method. Figure 60 shows the triangles thus formed, the angles and sides being marked in the notation of the Aquino tables. The chief departure from the notation adopted in this Manual is the substitution of t for h, the hour angle, and the use of h to denote the altitude.

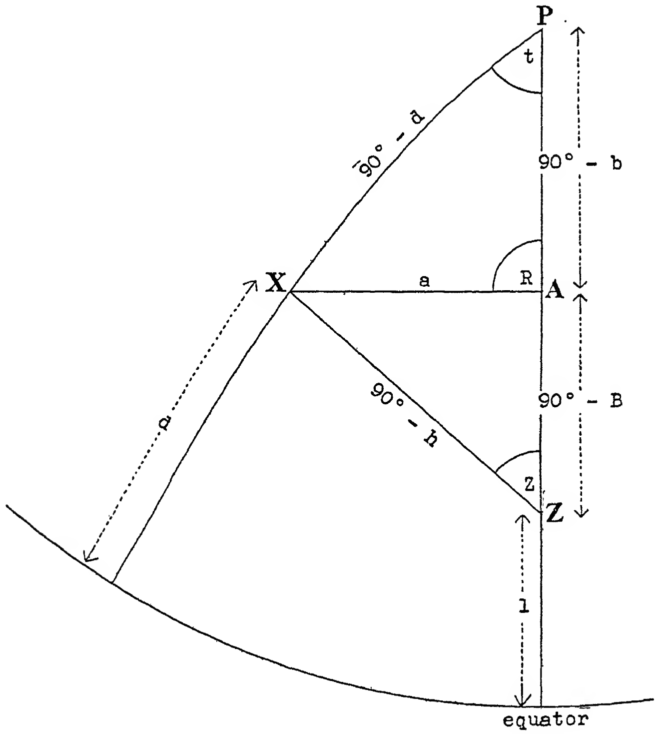


FIGURE 60.

By Napier's rules for the time triangle PAX:

$\sin d = \cos a \sin b \qquad \cot t = \cot a \cos b \quad . \quad . \quad . \quad (1)$

$\sin a = \cos d \sin t \qquad \cot b = \cot d \cos t \quad . \quad . \quad . \quad (2)$

Equations (2) are used for tabulating d and t, for values of a, ranging by intervals of 1° from 0° to 90°.

By Napier's rules for the altitude triangle ZAX:

$\sin h = \cos a \sin B \qquad \cot Z = \cot a \cos B \quad . \quad . \quad . \quad (3)$

$\sin a = \cos h \sin Z \qquad \cot B = \cot h \cos Z \quad . \quad . \quad . \quad (4)$

It will be noticed that if h is put for d, B for b and Z for t in equations (1) and (2), equations (3) and (4) are obtained. The tables involving a, d, b and t are thus the same as those involving a, h, B and Z.

A typical extract from these tables is :

	$a=28°30'$				
b / B	d / h	$\frac{60'}{\triangle}$	t / Z	$\frac{\triangle}{60'}$	c / C
0°					90°
54°	45°19′	1·36	42°44′	0·70	36°
55°	46°03′	1·40	43°26′	0·72	35°
56°	46°46′	1·40	44°09′	0·77	34°
90°					0°
t	a		b		
	$d=28°30'$				

If, for example, a is 28°30′ and b is 55°, then d is 46°03′ and t is 43°26′. Also, if a is 28°30′ and B is 56°, then h is 46°46′ and Z is 44°09′.

Interpolation between these quantities is assisted by tabulations based on the difference $\triangle$.

Thus, when b changes from 54° to 55°—by 60′, that is—d changes from 45°19′ to 46°03′—by 44′, that is. Therefore, when d changes by 1′, b changes by 60′/44, or 1′·36. This change is tabulated in the column headed $60'/\triangle$.

Again, when b is 54°, t is 42°44′, and when b is 55°, t is 43°26′. Hence t increases by 42′ while b increases by 60′, and corresponding to a change of 1′ in b, there is a change of 42′/60 or 0′·70 in t. This quantity is tabulated in the column headed $\triangle/60'$.

In the practical working of a sight, the declination and the hour angle of the heavenly body are known—d and t, that is—and h and Z have to be found. Equations (2) show that, when d and t are given, a and b can be found. The form of equations (2), moreover, is precisely that of equations (1). The tables can therefore be used for finding a and b from d and t. When they are used for this purpose, entry is made from the bottom where a, b, d and t are shown. Thus, if d is 28°30′ and t is 56°, then a is 46°46′ and b is 44°09′.

The following example shows how a sight is worked with the tables :

Required the intercept when the declination and hour angle of Capella are 45°56′*N.* *and* 43°26′ *respectively, the hour angle being worked for the longitude of the D.R. position,* 32°25′*N.*, 146°20′*W.* *The true altitude of Capella is* 54°48′.

The nearest values of d and t in integral degrees are 46° and 43° respectively, corresponding to d equal to 45°56′ and t equal to 43°26′.

By entering the bottom of the table with d equal to 46°, the approximate values of a and b, corresponding to these integral values of d and t, are seen to be:

$$a=28°17' \qquad b=54°46'$$

To the nearest 30′ for a, and the nearest degree for b:

$$a=28°30' \qquad b=55°$$

From the tables for these arguments:

$$d=46°03' \qquad t=43°26'$$

But Capella's declination is 45°56′. Hence, corresponding to this declination, b must lie between 54° and 55° when a is equal to 28°30′.

When b is 54°, d is 45°19′, and the true declination, 45°56′ may be written (45°19′+37′). Hence the value of b corresponding to d equal to 45°56′ is given by:

$$54°+(37\times 1'{\cdot}36)$$
$$=54°50'{\cdot}3$$

Again, since t is 42°44′ when b is 54°, the value of t corresponding to b equal to 54°50′·3 is given by:

$$42°44'+(50'{\cdot}3\times 0'{\cdot}7)$$
$$=43°19'{\cdot}2$$

But the hour angle corresponding to the longitude of the D.R. position is 43°26′. This longitude must be adjusted—that is, a special position must be assumed—so that the hour angle is 43°19′·2. This special position is clearly 6′·8 west of the D.R., and its longitude is 146°26′·8W.

Let C equal $(90°-B)$. This quantity C is tabulated on the right-hand side of each page of the tables. Then, from figure 60:

$$PZ=PA+AZ$$

i.e. $$90°-l=90°-b+C$$

i.e. $$b=l+C$$

Now b is equal to 54°50′·3, and the latitude of the D.R. position is 32°25′. To avoid interpolation, a latitude is chosen so that C is a whole number of degrees. Thus, if the latitude is 32°50′·3, then C is 22° and B is 68°. The chosen position from which the intercept is laid off is therefore:

$$\begin{cases} 32°50'{\cdot}3\text{N.} \\ 146°26'{\cdot}8\text{W.} \end{cases}$$

If the point Z in figure 60 is regarded as the zenith of this special assumed position, then, in the triangle AZX:

$$a=28°30' \qquad B=68°$$

Hence, from the table for a equal to 28°30′ and B equal to 68°:

$$h=54°34' \qquad Z=55°24'$$

The intercept, being the difference between the true altitude 54°48′ and the calculated altitude 54°34′, is therefore 14′ away. Since the observer is in north latitude and Capella is west of the meridian, the azimuth is N.55½°W.

Accuracy of the Aquino Method. Although this method of working a sight is undoubtedly shorter than the cosine-haversine method, once the tables are thoroughly understood, especially as the azimuth is given at the same time as the calculated altitude, the complication of its rules places it at a disadvantage in comparison with some of the other tabular methods. Also the tabulation of quantities to one minute of arc introduces a possible error of 1′ when interpolation is carried out between these quantities, and a comparable error is possible in the altitude found by using them. But in general the resulting error is of no practical importance, and

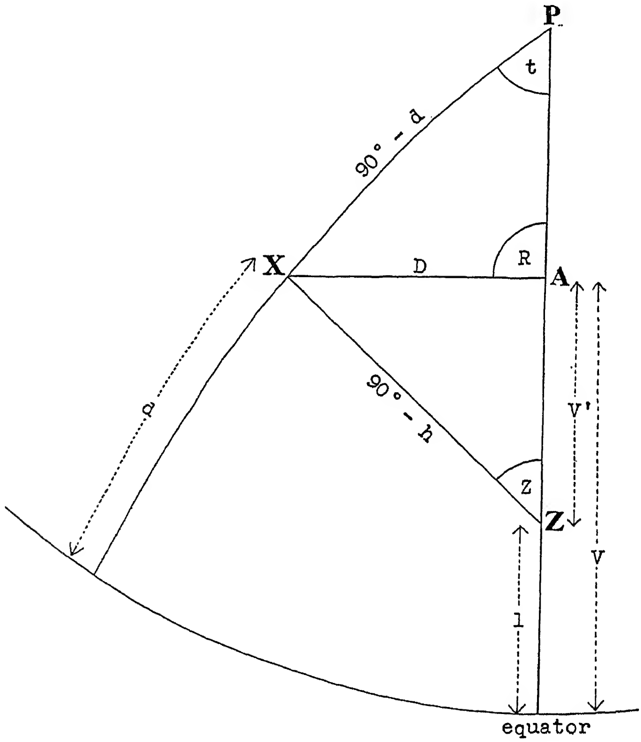


FIGURE 61.

it is incorporated in the further error that may be introduced by rounding off the tabulated values of the altitude to 1′. In the example just worked, the calculated altitude found by the cosine-haversine method and five-figure logarithms is 54°33′·9.

The Pierce Method. In this method, as in the Ageton and Aquino methods, the perpendicular XA is dropped from X upon PZ. The triangles are also solved in terms of the declination of A, as in the Ageton method, but this quantity is tabulated as V, and not as K.

The length of the perpendicular is D, and the hour angle is t. (Figure 61.)

If AZ is denoted by V', it follows that :

$$V' = V \sim l$$

Napier's rules applied to the two triangles give :

$$\sin D = \sin t \cos d$$
$$\sin V = \sin d \sec D$$
$$\sin h = \cos D \cos V'$$
$$\sin Z = \sin D \sec h$$

The tables themselves are divided into two parts ; the first giving V and t' (the hour angle for a special assumed position to be found) for arguments D and d ; the second giving the calculated altitude and azimuth for arguments D and V'. Separate tables are given for each bright star.

Typical extracts from Tables I and II are :

TABLE Ia

CAPELLA dec. : 45°56′

D	t'	V
28°	42°27′·4	54°28′·1
28°30′	43°19′·2	54°50′·8
29°	44°11′·6	55°14′·4

TABLE II—ALTITUDE AND AZIMUTH

V' \ D	22°	
	h	Z
28°	54°57′	125°
29°	54°11′	124°
30°	53°25′	123°

The following example, which is the same as that worked by the Aquino tabular method, shows both the way in which the tables are used and also the similarity of the two methods.

Required the intercept when the true altitude of Capella is 54°48′ *and the Greenwich hour angle at the time of observation is* 189°46′ ; *the D.R. position being* 32°25′*N.*, 146°20′*W.*, *and the declination of Capella* 45°56′*N.*

G.H.A.	189°46′W.
D.R. long.	146°20′W.
t	43°26′

From Table Ia, which is compiled for Capella's declination of 45°56′N., and in this example entered for a value of t' nearest to that of t:

$$t' = 43°19'{\cdot}2$$
$$D = 28°30'$$
$$V = 54°50'{\cdot}8$$

As in the Aquino method, a longitude is now chosen for which the hour angle is 43°19′·2, and a latitude for which V is an integral number of degrees. Thus:

G.H.A.	189°46′·0
t'	43°19′·2
Assumed long.	146°26′·8W.
V	54°50′·8N.
Assumed lat.	32°50′·8N.
V'	22°00′·0

With the arguments D and V', Table II is entered and it is seen that:

$$h = 54°34' \qquad \text{azimuth} = 180° - 124\tfrac{1}{2}° = \text{N.}55\tfrac{1}{2}°\text{W.}$$

The intercept is again 14′ away, and it is laid off from the position 32°50′·8N., 146°26′·8W. The slight discrepancy between this position and that found by the Aquino method results from the cumulative errors inevitable in the use of rounded-off quantities. The tabulation to one decimal of a minute in Table Ia suggests that the Pierce method is the more accurate, but the tabulation of h to one minute and the necessity for interpolation in Table II when D lies between the integral degrees, introduces a possible error of 1′ into the calculated altitude on this count alone.

The altitude calculated from the position 32°50′·8N., 146°26′·8W., by the cosine-haversine method and five-figure logarithms is 54°34′·2.

Ball's Altitude Tables. These tables are of the type described in Chapter XVI of Volume II. They do not, however, provide a simple method of interpolation for both hour angle and declination. Nor do they permit the azimuth to be read with the altitude at a single entry. But, in the absence of other tables, they can be used with a considerable saving of time on the cosine-haversine method.

The absence of any simple method of interpolation for hour angle is not a disadvantage when a single sight is to be worked. The tables are compiled for every integral degree of latitude and

every four minutes of hour angle in time. The special position from which the sight is worked is therefore that position having a latitude which is the nearest integral degree, and a longitude which will combine with the Greenwich hour angle to the nearest four minutes. The altitude is then obtained with a single interpolation for declination.

All sights worked with this single interpolation for declination must be plotted from their special positions, and plotting is complicated accordingly. A second interpolation for hour angle enables the sights to be worked from the D.R. longitude (if they happen to be taken within a few minutes of one another) and the choice of the special position involves only a shift in latitude to the nearest integral degree. When this second interpolation is made, a number of 'simultaneous' sights can therefore be worked and plotted from a single position.

Blackburne's Tables. These tables are compiled for use with the longitude method. They facilitate the finding of the longitude of the point on the position line corresponding to the D.R. latitude by tabulating the hour angle for each degree in latitude, declination and altitude. Thus, for latitude 20°N., declination 18°S., and true altitude 32°, it is seen by inspection that the hour angle is $2^h58^m40^s$. On the same page the change of hour angle in seconds of time is given for changes of one minute of arc in latitude, declination and altitude.

If, for example, the latitude is 20°N., the alteration in hour angle corresponding to a change of one minute in latitude is —3·32 seconds. (This is called the latitude variation.) Hence, for a latitude of 20°20′N., the hour angle is :

$$2^h58^m40^s-(3{\cdot}32\times20)^s$$
$$=2^h57^m33^s{\cdot}6$$

In the same way the alterations in hour angle corresponding to changes in declination and altitude can be found.

Required the hour angle of a heavenly body, the declination and true altitude of which are 16°35′*S., and* 31°16′, *the D.R. position being* 28°14′*N.,* 75°00′*W.*

The tables show that for latitude 28°N., declination 16°S., and true altitude 31°, the hour angle is $2^h42^m23^s{\cdot}6$.

The latitude variation is $-4^s{\cdot}25$, and the correction to the hour angle for a latitude equal to 28°14′N. is therefore $(-4{\cdot}25\times14)$ or $-59^s{\cdot}5$.

The declination variation is $-4^s{\cdot}61$, and the correction to the hour angle for a declination equal to 16°35′S. is therefore $(-4{\cdot}61\times35)$ or $-161^s{\cdot}3$.

The altitude variation is $-6^s{\cdot}21$, and the correction to the hour angle for an altitude equal to 31°16′ is therefore $(-6{\cdot}21\times16)$ or $-99^s{\cdot}4$.

The total correction is thus $-320^s{\cdot}2$, and the hour angle corre-

sponding to latitude 28°14′N., declination 16°35′S., and true altitude 31°16′, is:

$$2^h42^m23^s{\cdot}6-5^m20^s{\cdot}2$$
$$=2^h37^m03^s{\cdot}4$$

The hour angle of the heavenly body at Greenwich is obtained in the ordinary way, and the difference between the hour angle at Greenwich and the hour angle obtained by means of the tables gives the longitude of the point on the position circle, the latitude of which is 28°14′N. When the azimuth has been found, the position line can be plotted.

The tables are constructed for latitudes and declinations between 0° and 30°.

Other Methods of Position-Finding. In addition to the logarithmic and tabular methods of solving the spherical triangle described in this chapter, there are a number of graphical and mechanical devices ranging from star altitude curves and d'Ocagne's Nomogram to the Hagner Position Finder and the Bygrave Slide Rule. Of these the star altitude curves offer the most promising results.

The *Hagner Position Finder* reproduces the spherical triangle mechanically and also, by means of a bubble horizon and a telescope, enables an observer to use the instrument as a sextant. The Willis altitude-azimuth instrument and the spherotrigonometer, used by the *Graf Zeppelin*, are instruments of similar type.

The *Bygrave Slide Rule* consists of three concentric tubes, the innermost carrying a scale of logarithmic tangents and the one next to it a scale of logarithmic cosines, both scales being arranged in spiral form. This construction is made possible by dropping a perpendicular from X on the side PZ of the spherical triangle, as in the Ageton method, and solving the two right-angled triangles thus formed.

The accuracy of graphical and mechanical methods is generally about 1′ in favourable circumstances. If this error occurs in the actual position, it is of little importance, but if it occurs in the intercept, as it usually does, it cannot be ignored because it may lead to a considerably larger error. If, for example, the same error occurs in two intercepts, the displacement of the resulting position is equal to that error multiplied by the cosecant of half the angle of cut. An error of 2′ in each intercept when the position lines cut at 30° thus gives rise to an error as large as 7′·7 in the position, and this initial error of 2′ is to be expected if graphical or mechanical methods are used.

Similar objection can be taken to methods of position-finding that involve the measurement of the rate of change in altitude.

Star Altitude Curves. Within the limits of accuracy imposed by the charts on which the curves are drawn, star altitude curves provide a quick and simple means of finding the position of a ship or aircraft.

The curves themselves are actually arcs of position circles drawn on a Mercator chart.

Figure 62 represents part of a Mercator chart between the parallels of 35°N. and 40°N., bounded by any meridians $7\frac{1}{2}°$ (or 30^m) apart in longitude. The intervening parallels are shown in black and marked as they appear on the chart itself, but the meridians are marked in minutes of local sidereal time, the actual times being governed by the stars selected, which are Capella and Polaris.

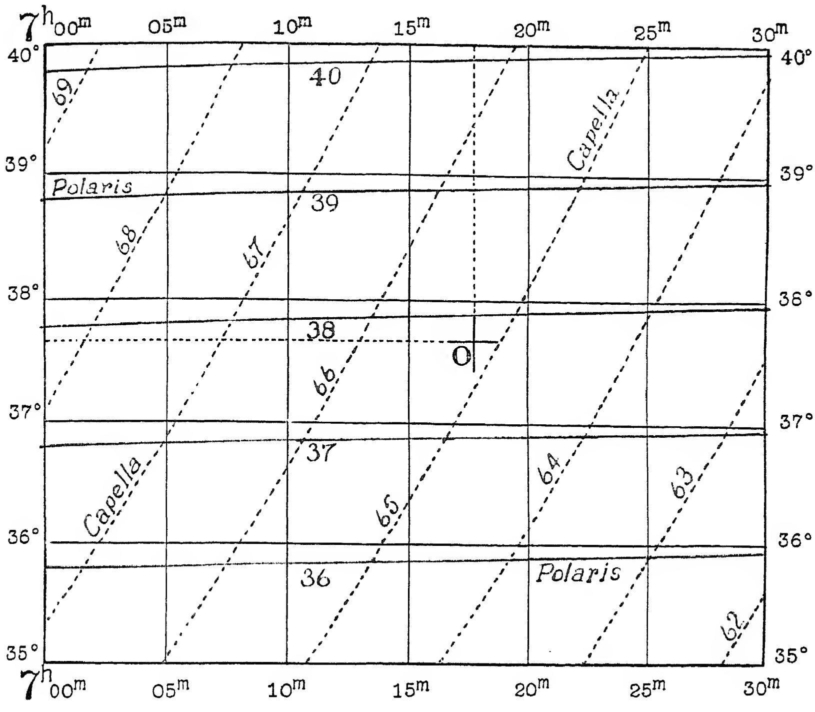


FIGURE 62.

In figure 62, the left-hand meridian is marked 7^h00^m in local sidereal time (that is, the right ascension of this particular meridian is 7^h00^m) and the right-hand one is marked 7^h30^m. Between these limits, for an observer in the belt of latitude concerned, the relevant arcs of the position circles obtained from Polaris appear as lines running almost along, *but not exactly along*, the parallels of latitude, and the arcs of the position circles obtained from Capella appear as lines running approximately from south-west to north-east. The altitudes of the two stars are shown against these lines.

If a navigator takes simultaneous altitudes of Polaris (37°45′) and Capella (65°12′), and notes that the Greenwich sidereal time of the observations is $11^h52^m27^s$, he can obtain his position at once

because it is the point of intersection of the curves that represent these altitudes, marked *O* in figure 62. Its latitude is read from the scale at the side—it is 37°40′N.—and the longitude, which is the difference between Greenwich sidereal time and local sidereal time, is found by subtracting the time measured on the scale along the top ($7^h17^m45^s$) from $11^h52^m27^s$. This difference is $4^h34^m42^s$, and the longitude is therefore 68°45′·5W.

It is clear from this example that the accuracy of the method depends on the accuracy with which the scales can be read ; that is, on the size of the chart. If there is an appreciable interval between the sights, two adjustments are necessary : the first altitude curve must be run on the distance covered by the ship or aircraft during the interval, and it must also be shifted to the right through a distance equivalent to the translation of the curve itself during the interval, this second distance being measured on the horizontal time-scale.

If the bearing is required, it can be found at once because it is at right-angles to the altitude curve.

The disadvantage of this method for surface navigation is that, apart from any difficulty in reading the scale with accuracy, the stars for which the curves are drawn may not be visible when required. Also the curves must be re-drawn every five or six years in order to allow for the changes in right ascension and declination that result from precession if errors greater than about 3′ are to be avoided. The method does, however, lend itself to air navigation where less accuracy is required and opportunity for securing an unrestricted view of the heavens at any moment is greater.

The scales of altitude curves plotted for air navigation are usually corrected for refraction only, the assumption being that a bubble sextant is used. If the ordinary horizon sextant is used, a further correction for dip must be applied.

The *Baker Navigation Machine* is a mechanical adaptation of these curves.

Precomputed Altitude Curves. The use of so-called precomputed altitude curves derived from the Sun and the Moon and the planets, is confined solely to aircraft that intend to follow a definite route. The rapid change in right ascension of these heavenly bodies makes it impossible to do more than construct the curves for the times at which the aircraft is expected to be on them. If, during the flight, an altitude is observed and does not differ from the altitude previously calculated for that instant, the aircraft is either at the estimated position for which the altitude was calculated or on a position line passing through it. If there is a difference between the two altitudes, the aircraft is on a position line parallel to the position line through the estimated position and displaced by the difference between the observed and calculated altitudes.

CHAPTER XI

EX-MERIDIAN AND POLE STAR TABLES

When a heavenly body is on the observer's meridian, it bears either north or south and the position line derived from it lies along a parallel of latitude. When it is near the observer's meridian, observations can be 'reduced to the meridian' by means of the ex-meridian tables described in Chapter XVIII of Volume II. These tables enable the observer to find a small quantity x that, when added to $(l \sim d)$ or $[180° - (l+d)]$, gives the calculated zenith distance.

Explanation of the Ex-Meridian Tables. Figure 63, in which XY is the parallel of declination and ZX is equal to ZY', shows this small quantity x in relation to the latitude and declination.

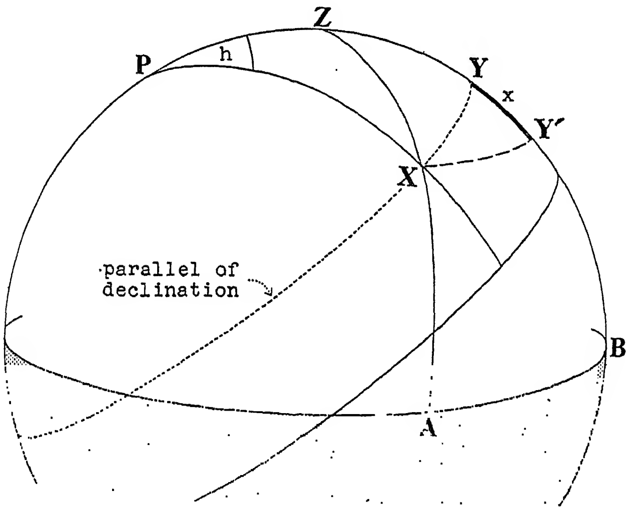


FIGURE 63.

When the hour angle is h, the altitude of the heavenly body is XA. When the heavenly body is on the meridian, its altitude is YB, and clearly YB is greater than XA. The difference between them is the small quantity x. Hence, in figure 63, where l and d are the latitude and declination :

$$\begin{aligned} ZX &= ZY + x \\ &= PY - PZ + x \\ &= (90° - d) - (90° - l) + x \\ &= (l - d) + x \end{aligned}$$

By substituting this value of ZX in the fundamental formula for ZX, it is possible to find an expression for x. Thus :

$$\cos[(l-d)+x]=\sin l\sin d+\cos l\cos d\cos h$$

i.e.
$$\cos[(l-d)+x]=\cos(l-d)-2\cos l\cos d\sin^2\frac{h}{2}$$

The left-hand side of this equation is itself equal to :

$$\cos(l-d)\cos x-\sin(l-d)\sin x$$

i.e.
$$\cos(l-d)\left[1-2\sin^2\frac{x}{2}\right]-\sin(l-d)\sin x$$

Hence, if x is considered to be a small angle expressed in circular measure, so that $\sin x$ is equal to x, it follows that the above expression reduces to :

$$\cos(l-d)\left[1-\frac{x^2}{2}\right]-x\sin(l-d)$$

The fundamental formula thus becomes :

$$\cos(l-d)\left[1-\frac{x^2}{2}\right]-x\sin(l-d)=\cos(l-d)-2\cos l\cos d\sin^2\frac{h}{2}$$

That is, on re-arrangement :

$$x=\frac{2\cos l\cos d}{\sin(l-d)}\operatorname{hav}h-\frac{x^2}{2}\cot(l-d)$$
$$=\quad x_1 \quad - \quad x_2$$

For a first approximation, since x is small and the term in x^2 can be neglected in comparison with the term in $\operatorname{hav} h$, x is equal to x_1 where :

$$x_1=\frac{2\cos l\cos d}{\sin(l-d)}\operatorname{hav}h$$

The second approximation is obtained by substituting this value of x_1 in the expression for x. Thus :

$$x=x_1-\frac{{x_1}^2}{2}\cot(l-d)$$

If x and x_1 are now expressed in minutes of arc instead of radians, this equation becomes :

$$\frac{x}{3{,}438}=\frac{x_1}{3{,}438}-\frac{1}{2}\left(\frac{x_1}{3{,}438}\right)^2\cot(l-d)$$

i.e.
$$x=x_1-\frac{{x_1}^2}{6{,}876}\cot(l-d)$$

The quantity x_2 in minutes, is therefore :

$$\frac{{x_1}^2}{6{,}876}\cot(l-d)$$

—and these two quantities, x_1 and x_2, are called the first and second corrections.

The First Ex-Meridian Correction. Expressed in minutes of arc instead of radians, the first correction is given by :

$$\frac{x_1}{3{,}438}=\frac{2\cos l\cos d}{\sin(l-d)}\text{ hav }h$$

i.e.

$$\frac{x_1}{6{,}876}=C\times H$$

where

$$C=\frac{\cos l\cos d}{\sin(l-d)}$$

and

$$H=\text{hav }h$$

In logarithmic form, this is :

$$\log\left(\frac{x_1}{6{,}876}\right)=\log C+\log H$$

Since C depends only on the latitude and declination, it is easy to construct tables for log C. Also H depends only on the hour angle. The table giving log H is therefore the ordinary table of logarithmic haversines. By adding log C and log H, log $(x_1/6{,}876)$ can thus be found, and a third table giving the values of this expression for various values of x_1 can be constructed so that, when log $(x_1/6{,}876)$ has been found, the value of x_1 can be obtained. Ex-meridian tables 1, 2 and 3 in *Inman's Tables*, are based on this principle.

When the latitude and declination have opposite names, C is given by :

$$\frac{\cos l\cos d}{\sin(l+d)}$$

It is thus evident that there must be separate tables for same and opposite names.

The Second Ex-Meridian Correction. When the latitude and declination have the same names, the second correction is given by :

$$\frac{{x_1}^2}{6{,}876}\cot(l-d)$$

When they have opposite names, it is given by :

$$\frac{{x_1}^2}{6{,}876}\cot(l+d)$$

But the meridian altitude of a heavenly body is either :

$$90^\circ-(l-d) \qquad \text{(for same names)}$$

or

$$90^\circ-(l+d) \qquad \text{(for opposite names)}$$

Hence, in both circumstances :

$$x_2=\frac{{x_1}^2}{6{,}876}\tan\text{ (meridian altitude)}$$

Table 4 of the ex-meridian tables gives x_2 in terms of x_1 and the altitude at the time of the observation. This, in practice, is near

enough to the meridian altitude to avoid error because the correction itself is small when the altitude is below 60° and the heavenly body is close to the meridian. When, however, the correction is not small, the table should be entered with the proper meridian altitude.

The following example shows how the ex-meridian tables are constructed when:

$$h = 53^{m}12^{s}$$
$$l = 61°\text{N.}$$
$$d = 22°\text{N.}$$

In these circumstances C is given by:

$$\frac{\cos 61° \cos 22°}{\sin 39°}$$

$$\begin{array}{lrr} \therefore & \log C = \log \cos 61° & +9{\cdot}685\ 57 \\ & +\log \cos 22° & +9{\cdot}967\ 17 \\ & -\log \sin 39° & -9{\cdot}798\ 87 \\ \hline & & 9{\cdot}853\ 87 \end{array}$$

i.e. $\log C = 9{\cdot}854$ (to three places)

This figure agrees with the value of log C tabulated in Table 1.

As stated, log H is simply a logarithmic haversine, and from the logarithmic haversine tables log hav $53^{m}12^{s}$ is seen to be 8·127. Hence:

$$\log\left(\frac{x_1}{6{,}876}\right) = 9{\cdot}854 + 8{\cdot}127$$
$$= 7{\cdot}981$$

i.e. $\log x_1 - \log 6{,}876 = 7{\cdot}981$

$$\log x_1 = 7{\cdot}981 + \log 6{,}876$$
$$= 7{\cdot}981 + 3{\cdot}837$$
$$= 1{\cdot}818$$

$\therefore$ $x_1 = 65'{\cdot}8$

When the body of Table 3 is entered with 7·981, x_1 is seen to be 65′·8, a value agreeing with that found by calculation.

The meridian altitude is $[90° - (l - d)]$, which is 51°. Then, since x_1 is 65′·8:

$$\begin{array}{lr} \log x_2 = 2 \log 65{\cdot}8 & +3{\cdot}636 \\ +\log \tan 51° & +0{\cdot}092 \\ -\log 6{,}876 & -3{\cdot}837 \\ \hline & 9{\cdot}891 \end{array}$$

i.e. $x_2 = 0'{\cdot}8$

If Table 4 is entered with the meridian altitude equal to 51°, and x_1 equal to 66°, the same value of x_2 is obtained.

The calculated zenith distance is therefore:

$$(l - d) + x_1 - x_2$$
$$= 39° + 65'{\cdot}8 - 0'{\cdot}8$$
$$= 40°05'{\cdot}0$$

Observations Below the Pole. When the hour angle is approximately 12^h, the ex-meridian tables can still be used for calculating the zenith distance. Also the problem is simplified because the latitude and declination must have the same names, and the declination must be greater than $(90°-l)$. If they have opposite names, the heavenly body will not be visible at meridian passage below the pole.

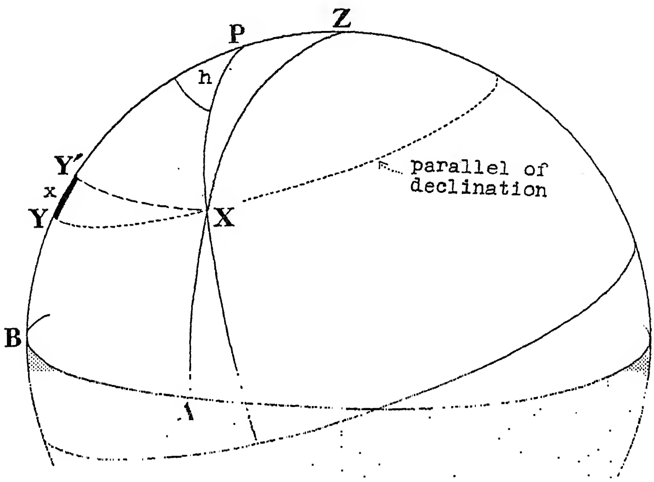


FIGURE 64.

Figure 64 shows the situation when X is distant an amount h from the meridian below the pole. The hour angle is then (12^h-h), and the fundamental formula becomes :

$$\cos ZX = \sin l \sin d + \cos l \cos d \cos (180°-h)$$

i.e.
$$\cos ZX = \sin l \sin d - \cos l \cos d \cos h$$
$$= -\cos (l+d) + 2 \cos l \cos d \text{ hav } h$$

Figure 64 also shows that ZX is equal to $(ZY-x)$, since ZX is less than ZY. Therefore :

$$ZX = PZ + PY - x$$
$$= (90°-l) + (90°-d) - x$$
$$= 180° - (l+d+x)$$

i.e.
$$\cos ZX = -\cos (l+d+x)$$
$$= -\cos (l+d) \left[1 - 2 \sin^2 \frac{x}{2}\right] + \sin (l+d) \sin x$$

When this value of cos ZX is substituted in the fundamental formula :

$$\sin (l+d) \sin x = 2 \cos l \cos d \text{ hav } h - 2 \cos (l+d) \sin^2 \frac{x}{2}$$

And, since x is small:

$$x=\frac{2\cos l\cos d}{\sin(l+d)}\operatorname{hav} h-\frac{x^2}{2}\cot(l+d)$$

That is, as before:

$$x=x_1-x_2$$

The first correction is the same as that derived for opposite names when the body is near the meridian above the pole, and it is tabulated in the last section of Table 1.

The second correction is given by:

$$x_2=-\frac{x_1{}^2}{2}\cot(l+d)$$

With sufficient accuracy for practical purposes:

$$ZX=180°-(l+d)$$

$$\therefore \qquad \cot ZX=-\cot(l+d)$$

$$\text{i.e.} \qquad \cot(l+d)=-\tan(\text{altitude})$$

The second correction is thus:

$$\frac{x_1{}^2}{2}\tan(\text{altitude})$$

For observations below the pole, the sign of the second correction is therefore reversed, and the numerical value obtained from Table 4 must be added to the value obtained from Table 3.

Required the calculated zenith distance of Canopus (declination 52°40′S.) *when the hour angle is* $13^h04^m42^s$, *the observer being in latitude* 54°S.

From the last section of Table 1, for l equal to 54° and d equal to 52°40′, and from Table 2 for h equal to $1^h04^m42^s$:

$$\begin{aligned}\log C&=\ \ 9{\cdot}570\\ \log H&=\ \ 8{\cdot}296\\ \hline &\ \ 17{\cdot}866\end{aligned}$$

From Table 3, the first correction is seen to be 50′·5.

When Canopus is on the meridian below the pole, its zenith distance is:

$$\begin{aligned}&180°-(l+d)\\ &=73°20'\end{aligned}$$

The altitude on the meridian below the pole is therefore 16°40′.
From Table 4:

$$x_2=0'{\cdot}1$$

The total correction is therefore:

$$\begin{aligned}&50'{\cdot}5+0'{\cdot}1\\ &=50'{\cdot}6\end{aligned}$$

The calculated zenith distance is thus :

$$\begin{aligned}&180°-(l+d)-x\\ &=73°20'-50'{\cdot}6\\ &=72°29'{\cdot}4\end{aligned}$$

Calculation of the Azimuth. Since the angle h must be small if the ex-meridian tables are to be used, the angle YZX—figure 63—is small. If this angle is denoted by β, the angle PZX is $(180°-\beta)$, and, by the rule of sines :

$$\frac{\sin(180°-\beta)}{\sin PX}=\frac{\sin h}{\sin ZX}$$

i.e.

$$\sin\beta=\sin h\cos d\sec(\text{altitude})$$

If β is now expressed in degrees, then, since β is small :

$$\sin\beta=\frac{60\beta}{3{,}438} \qquad \text{(approximately)}$$

Also, if h is expressed in minutes of time, then, since one minute of time equals fifteen minutes of arc :

$$\sin h=\frac{15h}{3{,}438} \qquad \text{(approximately)}$$

The value of β is therefore given by :

$$\frac{h}{4}\cos d\sec(\text{altitude})$$

The same formula holds when the heavenly body is below the pole, h being then equal to (12^h-H.A.) or $(\text{H.A.}-12^h)$.

For observations above the pole, the azimuth is equal to $(180°-\beta)$. For observations below the pole, the azimuth is β.

Required the true bearing when the hour angle is 0^h30^m, *the latitude* 48°*N., and the declination* 10°*S.*

From the ex-meridian tables, the calculated zenith distance is seen to be 38°31′. The altitude is therefore approximately 51½°.

From the formula :

$$\begin{aligned}\beta&=7°{\cdot}5\cos 10°\sec 51\tfrac{1}{2}°\\ &=11°{\cdot}9\end{aligned}$$

The true bearing of the heavenly body is therefore 191°·9.

The ordinary method of calculation gives 192°.

The Pole Star Tables. As explained in Chapter XIX of Volume II, the nearness of the Pole Star to the north celestial pole enables an observer to find his position from the Pole Star by means of simple tables.

In figure 65, U is the geographical position of the Pole Star, and PEP' the meridian through the estimated position E. The arc FAG is part of the position circle derived from an observation of the Pole Star, cutting the meridian in A. Then, if the latitude

of A can be found, the position line can be drawn through the point on the chart corresponding to A, at right-angles to the true bearing of the Pole Star.

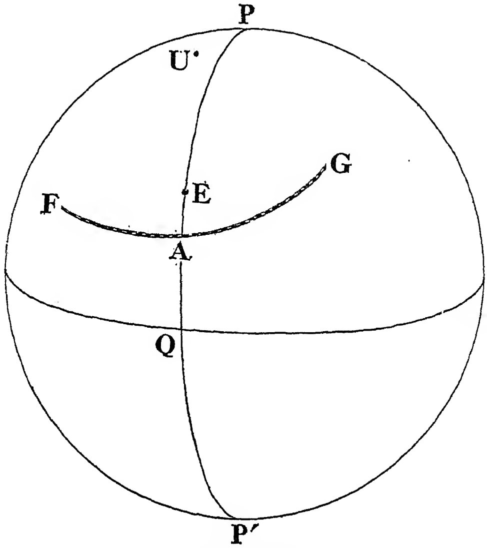


FIGURE 65.

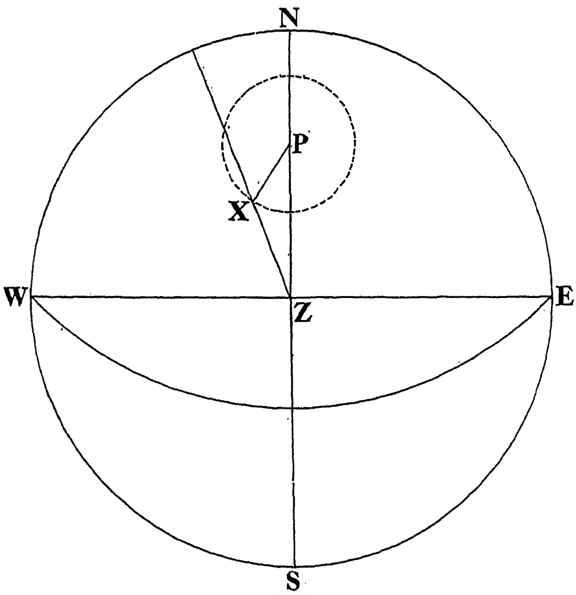


FIGURE 66.

The method of obtaining this position line is the reverse of the longitude method.

In the longitude method, the longitude of the point where the

position line cuts a parallel of latitude is found, the parallel being that passing through the estimated position. In the Pole Star method, the latitude of the point where the position circle cuts a known meridian is found, the meridian being that passing through the estimated position.

In figure 66, X is the position of the Pole Star when the hour angle is h and the altitude is a.

Since the polar distance of the Pole Star is about 1°, and the latitude of a place is the altitude of the pole at that place, it follows that the altitude of the Pole Star cannot differ from the latitude by more than about 1°. The latitude PN may thus be found by applying a small correction y to the altitude of the Pole Star. That is:

$$PN = a - y$$

From the fundamental formula:

$$\cos h = \frac{\cos ZX - \cos PZ \cos PX}{\sin PZ \sin PX}$$

i.e.
$$\cos h = \frac{\sin a - \sin (a-y) \cos p}{\cos (a-y) \sin p}$$

Since y and p are not greater than about 1°:

$$\sin p = p \qquad \sin y = y$$

$$\cos p = 1 - \frac{p^2}{2} \qquad \cos y = 1 - \frac{y^2}{2}$$

Hence, if small quantities of the third order are neglected:

$$\cos h(p \cos a + py \sin a) = \sin a - \sin a\left(1 - \frac{p^2 + y^2}{2}\right) + y \cos a$$

i.e.
$$y = p \cos h + py \cos h \tan a - \frac{p^2 + y^2}{2} \tan a$$

For a first approximation, since all terms except the first on the right-hand side are of the second order:

$$y = p \cos h$$

A second approximation is obtained by substituting this value. Thus:

$$y = p \cos h + \tan a\left[p^2 \cos^2 h - \frac{p^2 \cos^2 h}{2} - \frac{p^2}{2}\right]$$

$$= p \cos h - \tfrac{1}{2} p^2 \sin^2 h \tan a$$

Therefore, when p and y are expressed in minutes of arc:

$$y \sin 1' = p \sin 1' \cos h - \tfrac{1}{2}(p \sin 1')^2 \sin {}^2 h \tan a$$

i.e.
$$y = p \cos h - \tfrac{1}{2} p^2 \sin^2 h \tan a \sin 1'$$

Table I of the Pole Star tables in the abridged edition of the *Nautical Almanac* give the values of $p \cos h$, and Table II gives those of $\frac{1}{2}p^2 \sin^2 h \tan a \sin 1'$.

Since h is equal to (R.A.M.$-$R.A.✳) and the right ascension is almost constant, it is convenient to use the R.A.M. or local sidereal time as an argument.

Table III gives a small correction for the variations of p and the right ascension from the constant values assumed for them in the compilation of Table I.

Pole Star Azimuth Tables. Since the polar distance p is small, the angle PZX is small for latitudes covered by the tables.

The four-part formula applied to the spherical triangle PZX gives :

$$\sin l \cos h = \cos l \cot p - \sin h \cot (az.)$$

i.e.
$$\tan (az.) = \frac{\sin h}{\cos l \cot p - \sin l \cos h}$$
$$= \frac{\sin h \tan p}{\cos l - \sin l \cos h \tan p}$$

But the azimuth itself is small. Hence, if both the azimuth and polar distance are expressed in circular measure :

$$\text{azimuth} = \frac{p \sin h}{\cos l - p \sin l \cos h}$$

Expanded by the Binomial Theorem up to the power of p^2, this becomes :

$$\text{azimuth} = p \sin h \sec l \, (1 + p \tan l \cos h)$$

Or, if the azimuth and polar distance are expressed in minutes of arc :

$$\text{azimuth} \times \sin 1' = p \sin 1' \sin h \sec l \, (1 + p \sin 1' \tan l \cos h)$$

i.e.
$$\text{azimuth} = p \sin h \sec l \, (1 + p \sin 1' \tan l \cos h)$$

As before, (R.A.M.$-$R.A.✳) is written for h so that local sidereal time can be used as the argument when values of the azimuth are tabulated.

The position line is drawn through the point on the chart having the longitude of the estimated position and the latitude found from the observation by the method described. Except in high latitudes, where the position line should be drawn at right-angles to the line of bearing, the azimuth is so small that it may be disregarded and the position line taken as part of the parallel of latitude through the point A in figure 65.

The azimuth, however, must be taken into account if the error of the gyro compass is found from a bearing of the Pole Star.

CHAPTER XII

RATES OF CHANGE IN AZIMUTH AND ALTITUDE

All problems connected with changes in the azimuth and altitude of a heavenly body fall into two classes: those in which the observer is stationary, and those in which he is moving.

Rate of Change in Azimuth (Observer Stationary). In this problem it is convenient to consider the change that occurs during one minute of time; and in figure 67, X is the position of a heavenly body when the hour angle is h, and Y is its position when the hour angle is $(h+1^m)$.

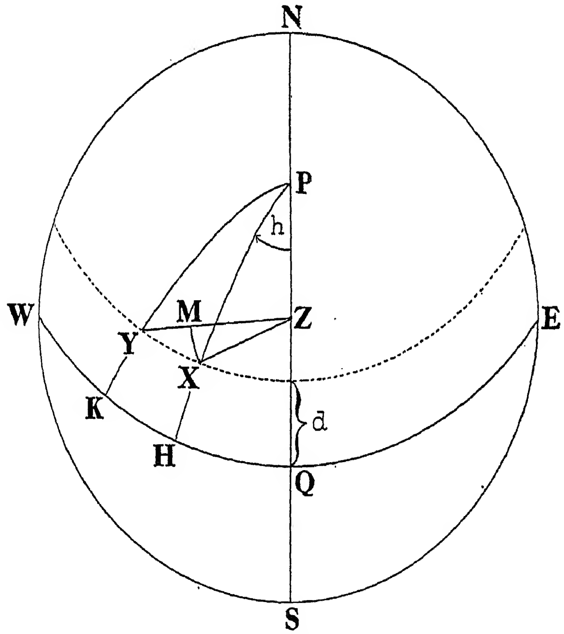


FIGURE 67.

Figure 67 shows that while the hour angle increases by one minute, the azimuth decreases from the angle PZX to the angle PZY.

If ZM is made equal to ZX, YM will represent the change in altitude during the same interval. Also, since XM is perpendicular to ZY, the figure formed by the small-circle arcs XM and XY, and by the great-circle arc MY, may be regarded as a small plane triangle, right-angled at M; and the angle MXY is equal to the

angle PXZ (denoted by β) since ZX is at right-angles to XM and PX is at right-angles to XY. Hence :

$$XM = XY \cos \beta$$

and
$$MY = XY \sin \beta$$

But the length of this small-circle arc XY in terms of the declination is given by :

$$XY = HK \cos d$$

or
$$XY = \widehat{HPK} \cos d$$

In the same way the small-circle arc XM is given by :

$$XM = \widehat{XZM} \sin ZX$$

But XM is equal to $XY \cos \beta$. Therefore :

$$\widehat{XZM} = \frac{XY \cos \beta}{\sin ZX}$$

$$= \frac{\widehat{HPK} \cos d \cos \beta}{\sin ZX}$$

If the change in the azimuth, XZY, is denoted by α, and all angles are expressed in minutes of arc, it follows, since the change in hour angle is one minute in time and therefore 15′ in arc, that :

$$\alpha = \frac{15 \cos d \cos \beta}{\sin ZX}$$

i.e.
$$\alpha = 15 \cos d \cos \beta \sec (alt.) \quad . \quad . \quad . \quad . \quad (1)$$

This formula can be expressed differently since, by the rule of sines :

$$\frac{\sin PX}{\sin ZX} = \frac{\sin PZX}{\sin ZPX}$$

i.e.
$$\frac{\cos d}{\sin ZX} = \frac{\sin (az.)}{\sin h}$$

Hence :
$$\alpha = 15 \sin (az.) \operatorname{cosec} h \cos \beta \quad . \quad . \quad . \quad (2)$$

Alternative Proof. These results can be quickly obtained by the methods of the calculus. For, if $\triangle\alpha$ denotes the small increment in the azimuth that results from a small increment $\triangle h$ in the hour angle :

$$XM = \triangle\alpha \sin ZX$$

and
$$XY = \triangle h \cos d$$

But, as before :

$$XM = XY \cos \beta$$

$$\therefore \quad \frac{\triangle\alpha}{\triangle h} = \frac{\cos d \cos \beta}{\sin ZX}$$

When $\triangle a$ is expressed in minutes of arc and $\triangle h$ in minutes of time, this rate of change in azimuth (or true bearing) is given by :

$$15 \cos d \cos \beta \sec (alt.)$$

or, as above :

$$15 \sin (az.) \operatorname{cosec} h \cos \beta$$

Both formulæ give the number of minutes of arc by which the azimuth changes in one minute of time ; that is, they give the rate of change in azimuth. The first also shows that, for a given declination, this rate is greatest when the altitude is greatest and $\cos \beta$ is equal to unity. These conditions are satisfied when the heavenly body is on the observer's meridian, for the altitude is then greatest and β is either 0° (when the polar distance is greater than the co-latitude) or 180° (when the polar distance is less than the co-latitude).

The greatest rate of change in azimuth is therefore given by :

$$\frac{15' \cos d}{\sin (l \sim d)} \text{ per minute}$$

—the zenith distance when the heavenly body is on the meridian being $(l \sim d)$.

If, for example, the latitude is 60°N. and the declination 30°N., the value of this formula for the greatest rate of change in azimuth is seen, by substitution, to be 26′ per minute—a figure agreeing with that obtained from the ordinary azimuth tables in which the azimuth is given as 180° and 178°15′ for hour angles of 0^h and 0^h4^m respectively. That is, in four minutes the azimuth decreases by 105′, or 26′ in one minute.

Rate of Change at Lower Transit (Observer Stationary). The same formula gives the rate of change in azimuth at the moment of lower transit ; that is when ZX has its greatest value $[(90°-l)+(90°-d)]$ or $[180°-(l+d)]$. Sin ZX is then equal to $\sin (l+d)$ instead of $\sin (l-d)$. Also, at this moment, b is equal to zero so that $\cos b$ is unity. The rate of change in azimuth at lower transit is therefore given by :

$$\frac{15' \cos d}{\sin (l+d)} \text{ per minute}$$

If, for example, the heavenly body is Capella with a declination about 46°N., the latitude still being 60°N., the rate of change in azimuth at lower transit is :

$$\frac{15' \cos 46°}{\sin 106°} \text{ or } 10' \cdot 8 \text{ per minute}$$

At upper transit, the rate is :

$$\frac{15' \cos 46°}{\sin 14°} \text{ or } 43' \text{ per minute}$$

The rate of change at upper transit is thus, for Capella, four times the rate at lower transit.

Limiting Azimuths (Observer Stationary). The general formula for the rate of change in azimuth also shows that the rate is zero when $\cos\beta$ is zero; that is when β is 90°.

Figure 68 shows the spherical triangle PZX right-angled at X. The fundamental formula therefore reduces to:

$$\cos PZ = \cos PX \cos ZX$$

i.e.

$$\cos ZX = \frac{\sin l}{\sin d}$$

Since $\cos ZX$ cannot be greater than unity, $\sin l$ must, as a rule, be less than $\sin d$. That is, the latitude must be less than the

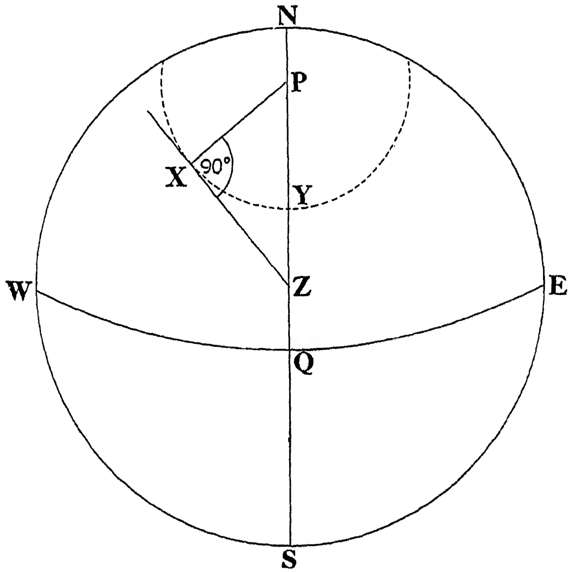


FIGURE 68.

declination, and PZ must be greater than PX. The parallel of declination must therefore intersect the meridian at a point Y between P and Z, and the great circle ZX touches the parallel of declination XY at X. Also, from figure 68, it is clear that the angle PZX is a maximum when the angle PXZ is 90°, and is itself less than 90°.

The formula giving the azimuth at this moment is found by applying the rule of sines to the spherical triangle PZX. Thus:

$$\frac{\sin PZX}{\sin PX} = \frac{\sin 90°}{\sin PZ}$$

i.e.

$$\sin PZX = \frac{\cos d}{\cos l}$$

Since $\sin PZX$ cannot be greater than unity, $\cos d$ must, as a rule, be less than $\cos l$ and, as before, the latitude must be less than the declination.

If, for example, the heavenly body is Capella (declination 46°N.) and the observer is in 30°N., the limiting azimuth is seen to be 53°·5 approximately. The true bearing of Capella must always lie, therefore, in the sector given by 306°·5—360°—053°·5.

Rate of Change in Altitude (Observer Stationary). It was shown in figure 67 that when the hour angle increased by one minute (of time) the zenith distance increased by MY; and it was proved that:

$$MY = XY \sin \beta$$

and

$$XY = \widehat{HPK} \cos d$$

When all angles are expressed in minutes of arc, these combine to give:

$$MY = 15 \sin \beta \cos d \quad . \quad . \quad . \quad . \quad (3)$$

Since MY is the change in the zenith distance corresponding to a change of one minute (of time) in the hour angle, the expression $15 \sin \beta \cos d$ measures the rate of change in the zenith distance and therefore in the altitude.

It can be derived in a more convenient form by applying the rule of sines to the spherical triangle PZX. Thus:

$$\frac{\sin PX}{\sin PZX} = \frac{\sin PZ}{\sin PXZ}$$

i.e.

$$\sin \beta \cos d = \cos l \sin (az.)$$

The change in altitude per minute may therefore be written:

$$15' \cos l \sin (az.) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It is seen from this expression that, when the azimuth is 180° or 0°, that is when the heavenly body is above or below the pole, the rate of change in altitude is zero. Also when the heavenly body is close to the meridian or near its lower transit so that the azimuth differs from 180° or 0° by only a few degrees, $\sin (az.)$ is sufficiently small to make the resulting rate of change in altitude inappreciable. This can be verified by watching the Sun at meridian passage and observing that the change in altitude is scarcely perceptible during an interval of five minutes.

Sin $(az.)$—and therefore the rate of change of altitude—is a maximum when the azimuth is 90°; that is when the heavenly body is on the prime vertical. The latitude and declination must have the same names, otherwise the heavenly body would set before it reaches the prime vertical.

Alternative Proof. This expression for the rate of change in altitude can be obtained by direct differentiation of the fundamental

formula. If z, c and p denote the zenith distance, the co-latitude and the polar distance, and if h is the hour-angle :

$$\cos z = \cos p \cos c + \sin p \sin c \cos h$$

This, differentiated with respect to h, gives :

$$\sin z \frac{dz}{dh} = \sin p \sin c \sin h$$

But, from the rule of sines :

$$\sin (az.) = \frac{\sin p \sin h}{\sin z}$$

Therefore by substitution :

$$\frac{dz}{dh} = \cos l \sin (az.)$$

If $\triangle z$ is a small increment in the zenith distance corresponding to a small increment $\triangle h$ in the hour angle, it follows that :

$$\triangle z = \triangle h \cos l \sin (az.)$$

Hence, if $\triangle h$ is taken as the increase in one minute of time, and all angles are expressed in minutes of arc, the change in altitude per minute is given, as before, by :

$$15' \cos l \sin (az.)$$

Altitude Curves (Observer Stationary). It is clear that if the position of a heavenly body does not change in the celestial sphere during the course of a day, an observer will see that heavenly body rise in the east, reach its maximum altitude at upper transit and set in the west, the azimuths at rising and setting being numerically the same. Represented graphically by the plotting of altitude against hour angle, its apparent path is shown as a smooth curve symmetrical about the ordinate of 0^h.

If, for example, the heavenly body is the Sun and its declination is assumed to be constant at 1°N., the meridian altitude to an observer stationary in 55°N. is [90°−(55°−1°)] or 36°, and the altitudes for hour angles up to 24^m (obtained from the ex-meridian tables) are given by :

Hour Angle			*Altitude*	*Change in* 4^m
h m		h m	° ′	′
0 00	or	24 00	36 00	
				0·4
0 04	or	23 56	35 59·6	
				1·0
0 08	or	23 52	35 58·6	
				1·9
0 12	or	23 48	35 56·7	
				2·6
0 16	or	23 44	35 54·1	
				3·3
0 20	or	23 40	35 50·8	
				4·1
0 24	or	23 36	35 46·7	

The total change in altitude between hour angles of 0^h00^m and 0^h24^m is thus $13'{\cdot}3$, and the resulting curve is shown by figure 69.

The altitude at any hour angle between 23^h36^m and 0^h24^m can be found by inspection. Thus, at $0^h13^m30^s$ it is $35°55'{\cdot}7$.

Effect of a Change in Declination on the Altitude. The change in the Sun's declination can be appreciable. On the 23rd March

ALTITUDE CURVE RESULTING FROM THE EARTH'S ROTATION

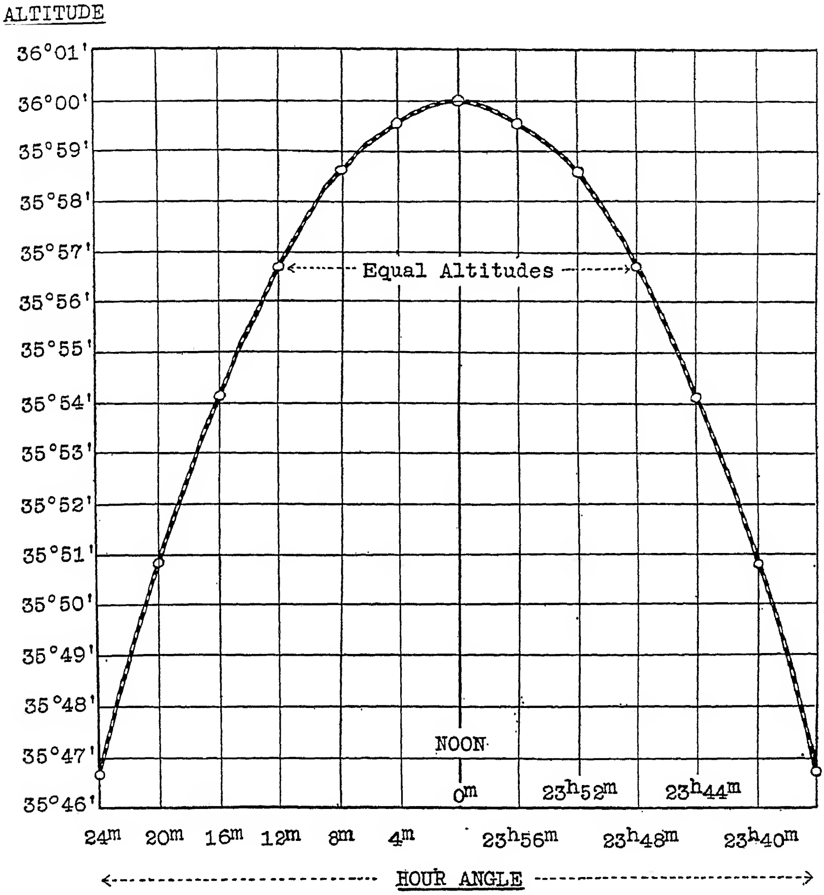


FIGURE 69.

it is about $1'$ per hour, and the effect of this change is to destroy the symmetry of the altitude curve just obtained, because the declination when the Sun is east of the meridian is not the same—less in the circumstances quoted—as when the Sun is west of it.

At upper transit, an increase of $1'$ in the declination results in an increase of $1'$ in the altitude, but when the Sun is not on the meridian, the increase in altitude is less, being equal to $1' \times \cos PXZ$.

In general, the change in altitude resulting from a change in declination is given by:

change in declination $\times \cos PXZ$

When the angle PXZ is 90°, as in figure 68, there is thus no change in altitude. Only when $\cos PXZ$ approximates to unity does the change in altitude equal the change in declination.

Figure 70 shows the resulting curve, greatly exaggerated, in relation to the altitude curve obtained when the declination is taken as constant.

In this figure the dotted line represents the altitude curve when the declination is constant and equal to 1°N., as in figure 69; and the heavy line represents the altitude curve when the declination

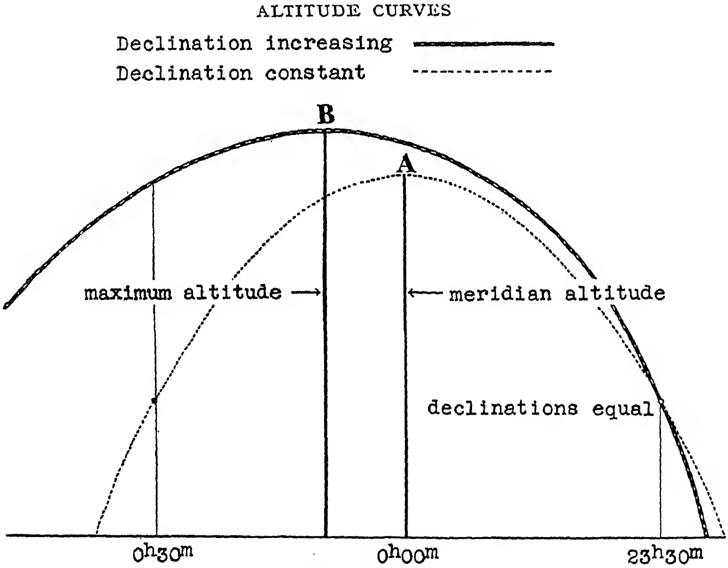


FIGURE 70.

increases steadily and is 1°N. only at 23^h30^m. Before then it is less than 1°; afterwards it is greater.

There is thus a difference between the time of meridian altitude, which occurs when the Sun is at A, and the time of maximum altitude, which occurs when the Sun is at B.

Effect of the Observer's Movement. If the observer is in a moving ship, two further adjustments of the altitude curve are necessary, the first arising from the east–west component of the ship's speed, and the second from the north–south component.

The effect of the east–west component is an easterly or westerly translation of the observer's meridian. The Sun itself moves westward at the rate of 15° or 900′ of longitude per hour. If, then, the ship is on a course that changes her longitude westward at 27′ per hour, for example, the observer's meridian is moving westward

at 27′ per hour and, relative to this meridian, the Sun is changing its longitude at the rate of (900′−27′) or 873′ per hour.

In calculating the four-minute changes in altitude tabulated on page 146, only the effect of the Earth's rotation was considered. The Sun was regarded as fixed in the celestial sphere, and it changed its longitude at the rate of 900′ per hour westward. If this rate is now taken as 873′ per hour westward, it is clear that the four-minute changes in altitude are decreased. For the same range of hour angle they are:

$$\begin{aligned} 4'{\cdot}1\times\frac{873}{900} &= 4'{\cdot}0 \\ 3'{\cdot}3\times\quad &= 3'{\cdot}2 \\ 2'{\cdot}6\times\quad &= 2'{\cdot}5 \\ 1'{\cdot}9\times\quad &= 1'{\cdot}8 \\ 1'{\cdot}0\times\quad &= 1'{\cdot}0 \\ 0'{\cdot}4\times\quad &= 0'{\cdot}4 \end{aligned}$$

The total change in altitude in 24^m is thus $12'{\cdot}9$, not $13'{\cdot}3$ as it was when the observer was stationary, and the altitude when the hour angle is 0^h24^m or 23^h36^m is $35°47'{\cdot}1$, not $35°46'{\cdot}7$.

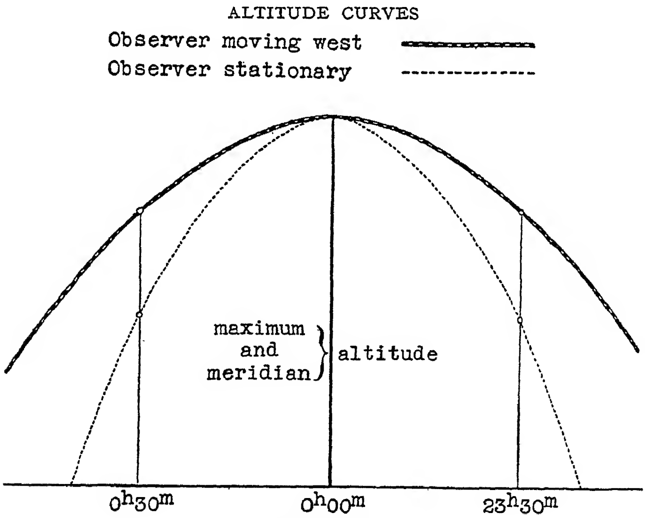


FIGURE 71.

The curve showing the change of altitude, in the neighbourhood of the meridian, that results from the Earth's rotation and the ship's westerly motion is thus not appreciably different from the curve that results from the Earth's rotation itself.

The actual difference is shown, much exaggerated, in figure 71 which is drawn on the assumption that meridian passage occurs for both the stationary observer and the moving observer at the same instant. Clearly, in these circumstances, the Sun sets to the the observer moving westward later than it does to the stationary observer.

The effect of the north–south component is comparable with a change in declination and leads to a difference between the times of meridian altitude and maximum altitude, meridian altitude occurring before maximum if the ship is steaming towards the Sun, and after maximum altitude if she is steaming away from the Sun. This fact may be remembered from the mnemonic :

M*er* before M*a*x : Ship n*ea*ring Sun.

Combined Effect of Ship's Movement and Change in Declination. If, for example, the Sun's northerly change of declination is 1′ per hour, and the ship is not only changing her longitude westward at 27′ per hour but also moving south and changing her latitude at 12′·7 per hour, the ship is approaching the Sun's geographical position at (12′·7+1′) or 13′·7 per hour when the Sun is on the meridian. At other times the rate will be less for reasons explained in the section describing the effect of a change in declination, but, within certain limits which will be discussed, the maximum rate of 13′·7 can be assumed without loss of accuracy in practical work.

Thus, in four minutes the altitude increases by 0′·9, so that four minutes after meridian passage the altitude is 0′·9 greater than it would have been if the ship had no southerly motion and the change in declination were neglected.

In figure 72, the dotted line represents the altitude curve when the Earth's rotation and the westerly movement of the ship are taken into account, and the point D marks the altitude four minutes after the meridian passage. The point Q marking the altitude at this moment when the southerly movement and change of declination are taken into account, is therefore found by adding 0′·9 to the altitude of D.

Twenty-four minutes after meridian passage, the altitude is greater by :

$$\frac{24}{60} \times 13'{\cdot}7$$
$$= 5'{\cdot}5$$

—and this amount must be added to the altitude at A to give the point X.

For reasons explained, the resulting curve through R and X is not strictly symmetrical about the line of maximum altitude, but it may be assumed to be symmetrical over the accepted interval in hour angle. The point R is thus seen to lie below E by 0′·9. This amount can also be arrived at by calculation since, in the four minutes preceding meridian passage the position of R is decided by the change in altitude resulting from both the southerly and westerly components of the ship's movement, the change in declination and the Earth's rotation, whereas the position of E is decided only by the westerly component and the Earth's rotation. R is thus below E by an amount equal to the change in altitude resulting from the southerly component of the ship's movement and the change in declination; that is 0′·9.

ALTITUDE CURVES

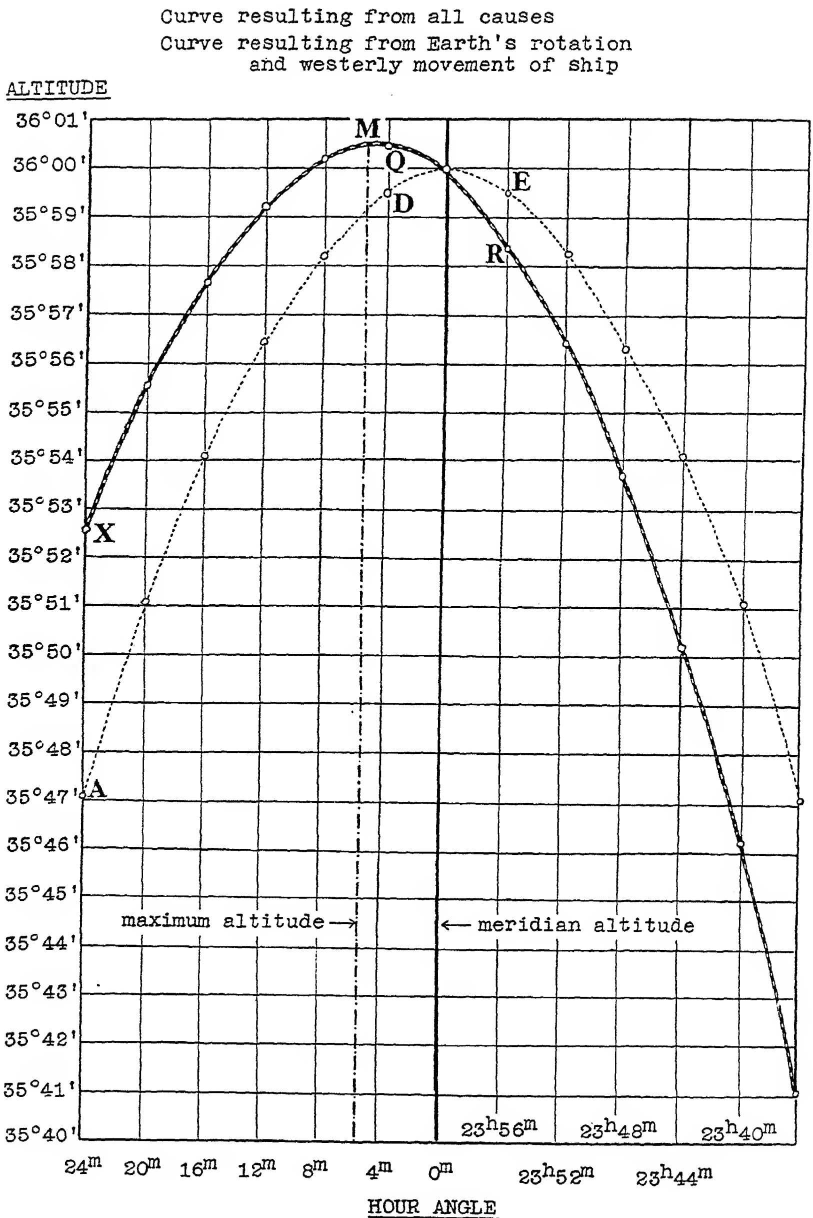


FIGURE 72.

A comparison of the two curves shows that maximum altitude occurs about five minutes after meridian altitude.

Formula Giving the Interval Between Meridian Passage and Maximum Altitude. It is apparent that, in the circumstances described above, maximum altitude occurs at the moment when the rate of increase in altitude resulting from one factor or set of factors is exactly equal to the rate of decrease in altitude resulting from another factor or set of factors.

In figure 73, Z is the observer's zenith when the Sun is on the meridian at X, and X' is the Sun's position at maximum altitude. The observer's zenith is then Z', the angle PZZ' being (360° — course). The angle $PZ'X'$ is denoted by α, and the latitude of Z' is l. Then, since the north–south component of the ship's movement during

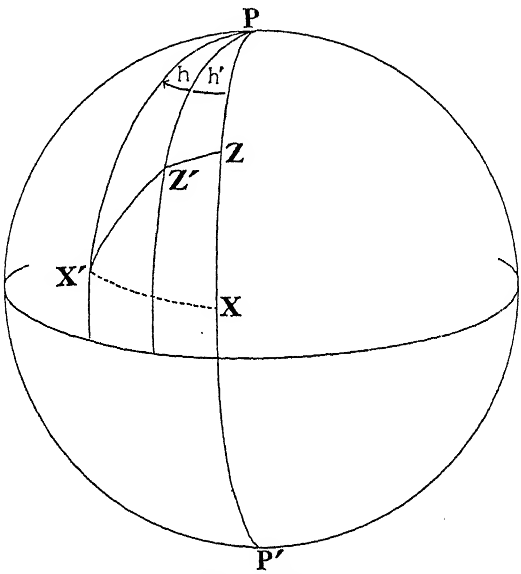


FIGURE 73.

the short interval between meridian and maximum altitude is small compared with the distance travelled by the Sun—that is, ZZ' is negligible in comparison with XX'—the latitude of Z may be taken as the latitude of Z'.

From the spherical triangle $PZ'X'$, in which h denotes the hour angle of the Sun at the ship when the altitude is a maximum :

$$\sin \alpha = \frac{\sin h \cos d}{\sin Z'X'} \qquad \ldots \ldots \ldots \quad (5)$$

Let x denote the ship's rate of change in longitude, and y the rate of north–south change in latitude combined with the rate of change in declination, both x and y being expressed in minutes of arc per hour. If the north–south component of the ship's movement and the change in declination have opposite names, y is the

sum of the rates of change in latitude and declination ; if they have the same names, it is the difference.

When the ship is stationary, the rate of change in altitude resulting from the Earth's rotation is—see page 145—given by :

$$15 \cos l \sin \alpha \quad \text{in minutes of arc per minute of time}$$

i.e. $$900 \cos l \sin \alpha \quad \text{in minutes of arc per hour}$$

When the ship's change in longitude is taken into account, this rate of change becomes :

$$(900 \mp x) \cos l \sin \alpha$$

—the upper sign being taken when the ship is moving west, and the lower when she is moving east.

Maximum altitude occurs when the two rates of change in altitude resulting from :

(1) the Earth's rotation combined with the ship's rate of change in longitude.

(2) the ship's north–south speed combined with the Sun's rate of change in declination.

—are equal but of opposite sign. Hence :

$$y = (900 \mp x) \cos l \sin \alpha$$

i.e. $$\sin \alpha = \frac{y \sec l}{900 \mp x}$$

Therefore, when this value of sin α is substituted in equation (5) and $(l \pm d)$ is written for $Z'X'$:

$$\sin h = \frac{y \sin (l \pm d)}{(900 \mp x) \cos l \cos d}$$

$$= \frac{y\,(\tan l \pm \tan d)}{900\left[1 \mp \frac{x}{900}\right]}$$

But h is small, and if it is expressed in seconds of time, four seconds of time being equal to one minute of arc :

$$\sin h = \frac{h}{4 \times 3{,}438}$$

Also, since $\frac{x}{900}$ is small :

$$\frac{1}{1 \mp \frac{x}{900}} = 1 \pm \frac{x}{900}$$

Hence : $$h = \frac{4 \times 3{,}438}{900}\, y \left[1 \pm \frac{x}{900}\right] (\tan l \pm \tan d)$$

—the upper signs being taken when the ship is moving west and the latitude and declination have opposite names, and the lower when she is moving east and the latitude and declination have the same names.

In figure 73, the interval between meridian passage and maxi-

mum altitude is represented by the angle ZPZ'. If this angle is denoted by h' and the angle ZPX' by H, then :

$$H=h\pm h'$$

But h' is the d'long through which the ship moves in the interval H. Therefore :

$$\frac{h'}{H}=\frac{\text{rate at which the ship is changing her longitude}}{\text{rate at which the Sun is changing its longitude}}$$

$$=\frac{x}{900}$$

i.e. $$h'=\frac{Hx}{900}$$

Hence : $$h=H\mp h'=H\left[1\mp\frac{x}{900}\right]$$

i.e. $$=h\left[1\pm\frac{x}{900}\right]$$

—the upper sign being taken when the ship is moving west and the lower when she is moving east.

By substitution, therefore :

$$H=\frac{4\times3{,}438}{900}y\left[1\pm\frac{x}{900}\right]^2(\tan l\pm\tan d)$$

i.e. $$H=15{\cdot}28\,y\left[1\pm\frac{2x}{900}\right](\tan l\pm\tan d)$$

—the upper signs being taken when the ship is moving west and the latitude and declination have opposite names, and the lower when she is moving east and the latitude and declination have the same names.

The quantities $15{\cdot}28\tan l$ and $15{\cdot}28\tan d$ can be found from the following table, and the value of $15{\cdot}28(\tan l\pm\tan d)$ obtained by addition or subtraction.

TABLE GIVING $15{\cdot}28$ TAN l AND $15{\cdot}28$ TAN d.

l or d		l or d		l or d		l or d	
°		°		°		°	
0	0·0	16	4·4	31	9·2	46	15·8
1	0·3	17	4·7	32	9·5	47	16·4
2	0·5	18	5·0	33	9·9	48	17·0
3	0·8	19	5·3	34	10·3	49	17·6
4	1·1	20	5·6	35	10·7	50	18·2
5	1·3	21	5·9	36	11·1	51	18·9
6	1·6	22	6·2	37	11·5	52	19·5
7	1·9	23	6·5	38	11·9	53	20·3
8	2·1	24	6·8	39	12·4	54	21·0
9	2·4	25	7·1	40	12·8	55	21·8
10	2·7	26	7·4	41	13·3	56	22·7
11	3·0	27	7·8	42	13·8	57	23·5
12	3·2	28	8·1	43	14·3	58	24·5
13	3·5	29	8·5	44	14·8	59	25·4
14	3·8	30	8·8	45	15·3	60	26·5
15	4·1	—	—	—	—	—	—

Required the interval between meridian passage and maximum altitude in 40°N., 60°W. on the 3rd April 1937 when the observer is in a ship steaming 230° at 16 knots.

By traverse table for a run of one hour :

$$\begin{aligned} \text{d'lat} &= 10'{\cdot}3\text{S.} \\ \text{dep.} &= 12'{\cdot}3\text{W.} \\ \therefore \qquad \text{d'long} &= 16'{\cdot}1\text{W.} \end{aligned}$$

Meridian passage in 60°W. occurs at about 16^h00^m G.M.T., and at this hour the Sun's declination is seen, from the *Nautical Almanac*, to be 5°19′N., increasing by 1′ per hour. Therefore x is equal to 16·1 and y to 11·3, and :

$$\begin{aligned} & 15{\cdot}28\,(\tan l - \tan d) \\ &= 12{\cdot}8 - 1{\cdot}4 \\ &= 11{\cdot}4 \end{aligned}$$

The interval between meridian passage and maximum altitude is thus :

$$\begin{aligned} & 11{\cdot}3\left(1+\frac{2\times 16{\cdot}1}{900}\right)11{\cdot}4 \\ &= 133^s \\ &= 2^m13^s \end{aligned}$$

Alternative Proof. The formula giving the interval between meridian passage and maximum altitude can be obtained by direct differentiation of the fundamental formula with respect to the time t. Thus, in the usual notation :

$$\cos z = \cos p \cos c + \sin p \sin c \cos h$$

$$\begin{aligned} \therefore \qquad -\sin z\frac{dz}{dt} &= (\cos p \sin c \cos h - \sin p \cos c)\frac{dp}{dt} \\ &\quad + (\sin p \cos c \cos h - \sin c \cos p)\frac{dc}{dt} \\ &\quad - \sin p \sin c \sin h\frac{dh}{dt} \quad \ldots\ldots\ldots \quad (6) \end{aligned}$$

When the altitude is a maximum, z is a minimum, so that $\frac{dz}{dt}$ is zero.

Also h is small. Therefore no appreciable error will occur if $\cos h$ is taken as unity, and the polar distance and co-latitude at meridian passage are used. Formula (6) then reduces to :

$$\sin h\,\frac{dh}{dt} = \frac{\sin\,(p-c)}{\sin p \sin c}\left[\frac{dc}{dt} - \frac{dp}{dt}\right]$$

But $\frac{dh}{dt}$ is the rate at which the Sun's hour angle changes.

Therefore : $$\frac{dh}{dt} = 900 \pm x$$

Also $\left[\frac{dc}{dt}-\frac{dp}{dt}\right]$ is the rate of change of $(c-p)$.

Therefore:
$$\frac{dc}{dt}-\frac{dp}{dt}=y$$

Formula (6) thus becomes:
$$\sin h=\frac{y \sin (l-d)}{(900\pm x) \cos l \cos d}$$

The remainder of the work is the same as that on page 153.

Longitude by Equal Altitudes. If T_1 and T_2 are the times shown by the chronometer when a heavenly body (of constant declination) has the same altitudes before and after passage across a stationary observer's meridian, it is evident from figure 69 that meridian passage must take place at a time $\frac{1}{2}(T_1+T_2)$ by the chronometer. Hence the G.M.T of meridian passage can be found and from this the longitude of the ship at meridian passage, since the longitude is simply the difference between this G.M.T. and the L.M.T. obtained by subtracting the astronomical quantity E from an hour angle of $0^h0^m0^s$.

When the movement of the ship and the change of declination are taken into account, the mean of the chronometer times, $\frac{1}{2}(T_1+T_2)$ is approximately the time of maximum altitude. It would be the exact time if the curve in figure 72 were symmetrical, but the error introduced by regarding it as symmetrical has no practical importance if the altitudes are taken within about half an hour of the time of meridian passage. Then:
$$H=15{\cdot}28y\left(1\pm\frac{2x}{900}\right)[\tan l\pm\tan d]$$
—and this quantity, when applied to $\frac{1}{2}(T_1+T_2)$, gives the time of meridian passage and therefore the ship's longitude.

In order that the times of equal altitudes may be observed accurately, the altitude of the heavenly body must be changing appreciably when the observation is taken. The heavenly body must not, therefore, be too close to the meridian. The formula giving the rate of change in altitude—15′ cos l sin ($az.$) per minute—shows that this rate is only 0′·1 per second in latitude 30° when the azimuth is 150°. In higher latitudes, or when the heavenly body is nearer the meridian, it is less. The limiting azimuth, consistent with accuracy in sight-taking, is thus about 160°.

If the limiting time-interval from the meridian is now taken as 40^m, the limiting rate of change in azimuth can be found from formula (2). This rate in minutes of arc per minute of time is:
$$15 \sin (az.) \operatorname{cosec} h \cos \beta$$
—and, since β is approximately 0, the rate in the limiting circumstances is therefore:
$$15 \sin 160° \operatorname{cosec} 40^m$$
—or at least 30° per hour.

The necessity for achieving this rate of change in azimuth imposes a limit on the altitude because, by formula (1), this rate of change in minutes of arc per minute of time is also :

$$15 \cos d \cos \beta \sec (alt.)$$

Again $\cos \beta$ may be taken as unity, and if the heavenly body observed is the Sun, as it usually is in practice, d must lie between 0° and 23°. When d is 0°, the altitude must be 60° in order to produce a rate of change in azimuth of 30° per hour. If d is 23°, it must be $62\frac{1}{2}$°. Equal altitudes of the Sun should therefore be taken when the Sun is not less than 20° from the meridian in azimuth and not more than 40^m in hour angle, and its altitude is not less than 65°. The method is therefore particularly suitable for use in low latitudes.

The value of the method lies in the briefness of the interval that elapses between the observations and, since the altitudes are equal, in the fact that refraction and dip will be the same for both observations if the height of eye is not altered. There is, moreover, no need to correct the sextant altitudes if the same sextant is used. It is the instant of observation, not the actual altitude, that must be recorded. Also the error that results from inaccurate reckoning between observations when a position line has to be transferred, is for the most part avoided.

Finally, if a meridian altitude is taken, the position of the ship is obtained.

At Z.T. 1230 (—5) *on the 3rd April* 1937, *the D.R. position was* 8°12′*N.*, 69°16′*E.*, *course* 325° ; *speed* 18 *knots. The deck watch was* 16^s *fast on G.M.T., and equal altitudes of the Sun were observed at* $6^h58^m53^s$ *and* $7^h54^m57^s$ *by the deck watch.*

Z.T.	1230	3rd April	From the *Nautical Almanac*
Zone	—5		
			E
G.D.	0730	3rd April	$11^h56^m31^s{\cdot}8$
	h m s		
T_1	6 58 53		Sun's Declination
T_2	7 54 57		5°11′·2N.
			increasing at 0′·95 per hour
$\frac{1}{2}(T_1+T_2)$	7 26 55		
Error	—16		
G.M.T.	7 26 39	3rd April	

From the traverse table for a run of one hour :

d'lat=14′·7N.

dep.=10′·3W.

∴ d'long=10′·4W.

northerly speed in latitude=14·7 knots

northerly speed in declination= 0·95

∴ relative speed y=13·75 knots, opening

From the table on page 154 :

$$15{\cdot}28 \tan 8°12' = 2{\cdot}1$$
$$15{\cdot}28 \tan 5°11' = 1{\cdot}3$$
$$\therefore \quad y \times 15{\cdot}28\,(\tan l - \tan d) = 13{\cdot}75 \times 0{\cdot}8$$
$$= 11{\cdot}0$$

The ship's speed in minutes of westerly longitude per hour is 10·4. Hence :

$$H = 11\left(1 + \frac{2 \times 10{\cdot}4}{900}\right)$$
$$= 11{\cdot}25$$

The interval between the times of meridian passage and maximum altitude is thus approximately 11 seconds.

In this example, maximum altitude occurs *before* the time of meridian passage.

	h m s
G.M.T. of maximum altitude	7 26 39
Interval to meridian passage	0 00 11
G.M.T. of meridian passage	7 26 50
H.A.T.S. at meridian passage	0 00 00
E	11 56 32
L.M.T. of meridian passage	12 3 28
G.M.T. of meridian passage	7 26 50
Longitude at meridian passage	4 36 38 *east*

At $7^{h}26^{m}50^{s}$ the ship is therefore in longitude 69°09′·5E.

CHAPTER XIII

ERRORS IN POSITION LINES

It is convenient to study the errors that occur in position lines, and the further errors that they introduce into the positions obtained, under the headings where the position lines themselves lie—terrestrial and astronomical. There are, however, errors common to both categories.

Instrumental Errors. Whenever a sextant is used, whether for measuring an altitude or a horizontal angle, there is a possibility of an error which may not be negligible. Index error is easily found, reduced if necessary, and allowed for, but the failure to eliminate *perpendicular error, side error* and *collimation error* may easily give rise to an unknown error when an observation is made.

In addition to this error, there is the limitation of the sextant itself, depending on the accuracy aimed at in the observation. The ordinary sextant reads to the nearest 0′·2. The possible error from this source is therefore 0′·1.

Personal Error. Personal error, as the name suggests, is peculiar to the observer himself, and affects all his observations. Unless it is abnormal, it is of no practical importance when bearings are measured, but it may be when altitudes are measured because the precision required is then considerable.

ERRORS IN TERRESTRIAL POSITION LINES

Errors in Taking and Laying Off Bearings. In practical chart work, involving observations of terrestrial objects, there is the possibility that the lines of bearing are plotted inaccurately or to a degree of accuracy less than that with which the observations were made.

Bearings taken with a magnetic compass are particularly liable to error because the deviation is not constant, and although it may be practically eliminated when the compass is corrected, it may not be negligible or even accurately known several days later. The effect of sub-permanent magnetism, the heating of funnels, and change in latitude all combine to vary the deviation.

The diameter of the card in the standard magnetic compass is 6½ inches, and a degree is represented by two lines on the circumference approximately six-hundredths of an inch apart. It is not

easy, therefore, to take a bearing with certainty, and if the ship is unsteady, the difficulty is further increased.

In the gyro compass, the 'hunt' of the card makes accurate observation difficult, and there is also the possibility of an unknown residual error in the compass itself.

When these sources of error are borne in mind, it is seen that the resulting error in the line of bearing drawn on the chart may easily reach $\frac{1}{2}°$, and this may lead to an appreciable displacement of the fix.

Displacement of Fix when the Same Error Occurs in Two Lines of Bearing. The fix by two lines of bearing from terrestrial

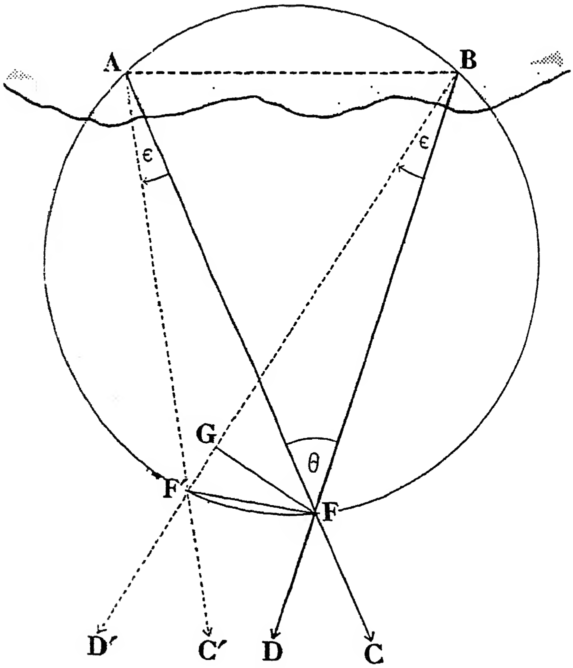


FIGURE 74.

objects is one of the simplest methods of finding a ship's position. In figure 74, A and B are these objects and AC and BD the two *accurate* lines of bearing intersecting at F, the ship's true position.

If the errors in the position lines drawn on the chart are the same in magnitude and sign—that is, the angle CAC' is equal to the angle DBD'—these position lines may be represented by AC' and BD', and F', their point of intersection, is the fix obtained. The displacement is FF'.

Let the error in the bearing be denoted by ϵ and the true angle

of cut, AFB, by θ. Then the angle of cut actually obtained, $AF'B$, is given by:

$$\angle AF'B + \angle BAF' + \angle F'BA = 180° = \angle AFB + \angle BAF + \angle FBA$$

i.e.

$$\angle AF'B + (\epsilon + \angle BAF) + (\angle FBA - \epsilon) = \angle AFB + \angle BAF + \angle FBA$$

i.e. $$\angle AF'B = \angle AFB = \theta$$

Hence, if errors in the two position lines are the same in magnitude and sign, F' will lie on a circle passing through A, B and F.

To find FF', draw FG perpendicular to BF'. Then:

$$BF \sin GBF = GF = FF' \sin BF'F$$

i.e. $$FF' = \frac{BF \sin GBF}{\sin BF'F}$$

But the angles $BF'F$ and BAF are equal, and from the rule of sines:

$$\frac{BF}{\sin BAF} = \frac{AB}{\sin AFB}$$

Therefore: $$FF' = \frac{AB \sin GBF}{\sin \theta}$$

If ϵ, which is a small angle, is now expressed in circular measure, the displacement is given in the form:

$$FF' = \frac{\epsilon\, AB}{\sin \theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This formula shows that the error in the fix resulting from a constant error ϵ in the observation is least when θ is 90°, and increases as θ decreases, this increase becoming rapid after θ has reached about 30°. When θ is 30°, the error is $\epsilon\, AB$ cosec 30°, or twice the error when the angle of cut is 90°.

It is required to find the errors in the fix obtained when the true bearings of two points, A *and* B, 14 *miles apart, are* (a) 060° *and* 030°, (b) 010° *and* 100°, *and the errors in the observed bearings are each* 1°.

(*a*) The angle of cut, being the difference of the true bearings, is 30°, and the error in circular measure is:

$$\frac{60}{3{,}438} \text{ or } \frac{1}{57} \qquad \text{(approximately)}$$

Hence the displacement of the fix is given by:

$$FF' = 14' \times \frac{1}{57} \times \text{cosec } 30°$$
$$= 0'{\cdot}5 \qquad \text{(approximately)}$$

(*b*) The angle of cut is 90°, and the displacement is given by:

$$FF' = 14' \times \frac{1}{57} \times \text{cosec } 90°$$
$$= 0'{\cdot}25 \qquad \text{(approximately)}$$

The Cocked Hat. When the lines of bearing from three objects, observed simultaneously, are drawn on the chart, it is usually found that they do not meet in a point but form what is known as a *cocked hat.* This is the shaded triangle in figure 75.

This cocked hat results from :

(1) the unknown and therefore uncorrected error of the compass, which may be as much as $\frac{1}{2}°$.

(2) the error in observation resulting from the limitations of the compass, which may be $\frac{1}{4}°$.

(3) the error in the actual plotting of the lines of bearing, which may also be $\frac{1}{4}°$.

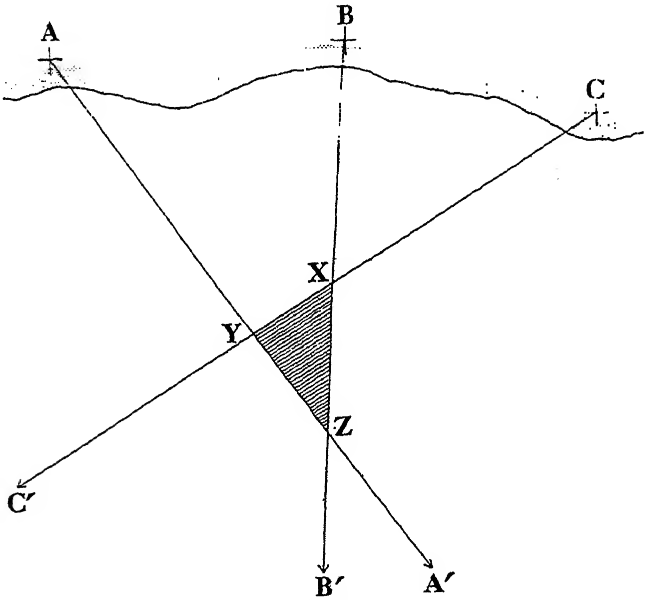


FIGURE 75.

Of these errors, (2) and (3) are fortuitous and may have either sign. That is, the plotted results may be either $\frac{1}{2}°$ high or $\frac{1}{2}°$ low on what the bearings should be. The remaining error (1), however, has a definite sign, and although it may be high or low, it is the same in each plotted bearing. It is thus convenient to investigate this error first.

The Cocked Hat Arising from the Same Error in Three Lines of Bearing. In figure 76, F is the true position, and A, B and C are three objects, the true bearings of which are observed. Suppose these bearings are 221°, 276° and 313°.

If the bearings are taken and laid off correctly, the three position lines intersect in F, but if there is a compass error of 1° low, say—it

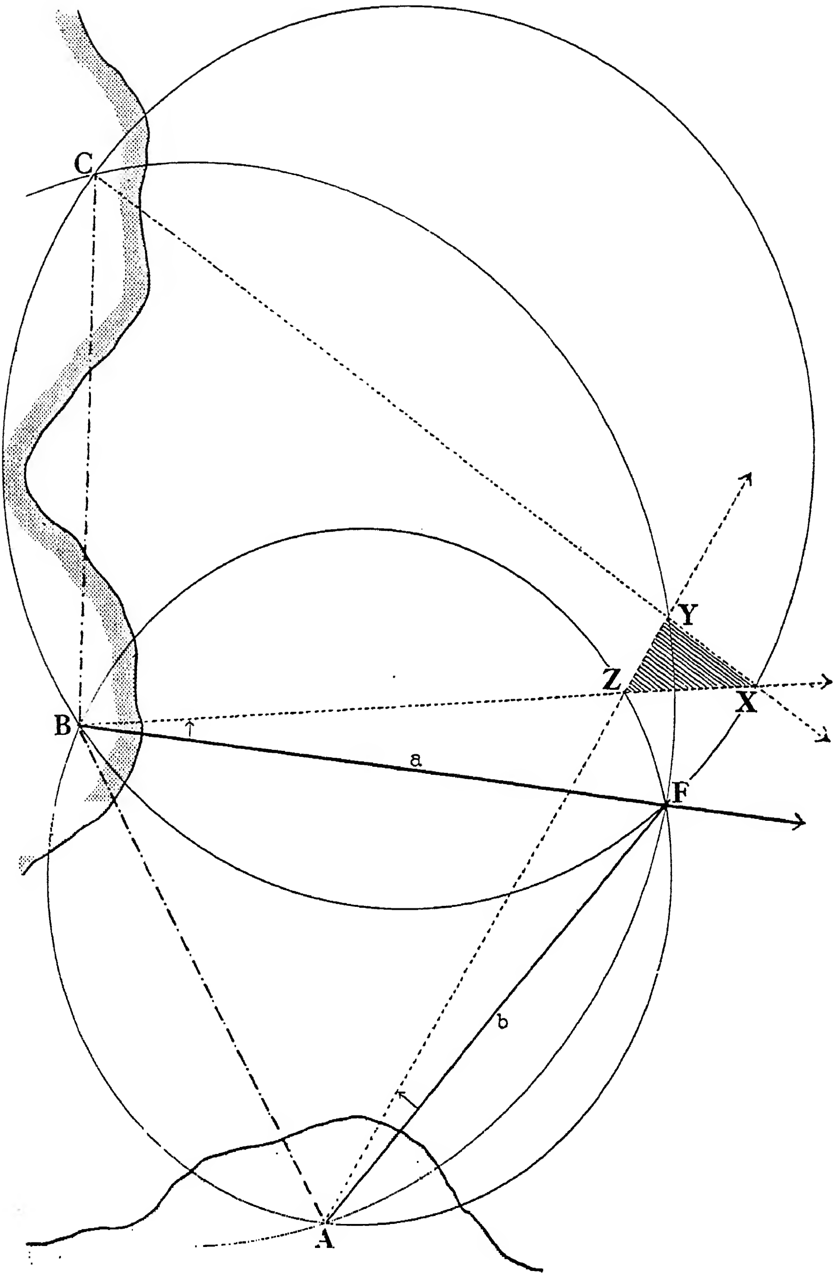


FIGURE 76.

is shown in the figure as 10° for the sake of clarity—the bearings plotted on the chart are AZ (220°), BX (275°) and CY (312°), and they form the cocked hat XYZ.

Since the difference between the bearings of A and B will be the same whether the compass error is correct or incorrect, the angle AZB must be equal to the angle AFB, and Z, the point of intersection of the two bearings, AZ and BZ, must lie on the circle through A, B and F. Similarly X, the point of intersection of the two bearings, BX and CX, must lie on the circle through B, C and F; and Y, the point of intersection of the two bearings, AY and CY must lie on the circle through A, C and F.

The true position F, it is seen, lies *outside* the triangle XYZ.

The distances of F from the vertices of the triangle are given by formula (1) on page 161. Thus, if ϵ is the constant error of the bearings in circular measure and θ_1, θ_2 and θ_3 are the angles between the bearings of A and B, B and C, and C and A:

$$FZ=\frac{\epsilon\, AB}{\sin\theta_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$FX=\frac{\epsilon\, BC}{\sin\theta_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$FY=\frac{\epsilon\, CA}{\sin\theta_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The distances AB, BC and CA can be taken from the chart.

If, for example, they are respectively 2′, 5′ and 7′, and the values of θ_1, θ_2 and θ_3 are 30°, 90° and 120°, the constant error being 1°, the separate displacements of F are given by formulæ (2), (3) and (4) as:

$FZ=0{\cdot}7$ cables
$FX=0{\cdot}9$ cables
$FY=1{\cdot}4$ cables

When the cocked hat results from an inaccuracy in the assumed value of the compass error and from no other cause, the amount and sign of the inaccuracy may be obtained approximately from figure 76. In this figure Z is the intersection of the position lines through A and B, and F lies on the circle through A, B and Z. But, for a similar reason, F lies on the circle through B, C and X. F is therefore the point of intersection of the two circles, and if the plotting is accurate and there are no errors apart from the constant error in the bearings, the circle through C, A and Y should also pass through F.

The position of F, obtained by this construction, is the true position of the ship.

If it is found that the circle drawn through A, C and Y does not pass through the point of intersection of the circles ABZ and BCX, the errors in the bearings are not all the same and must arise from other causes, such as errors in observation and plotting.

It is required to find the compass error when the bearings of three objects, carefully taken by gyro compass, are such that the difference between the bearings of A and B is 45°. AB is 6′, and FZ, as measured on the chart, is 0′·4.

From formula (2):

$$FZ = \frac{\epsilon\, AB}{\sin \theta_1}$$

$$\epsilon = \frac{0{\cdot}4 \times \sin 45^\circ}{6} \quad \text{(in circular measure)}$$

$$= \frac{57 \times 0{\cdot}4 \times 0{\cdot}7}{6} \quad \text{(in degrees)}$$

$$= 2{\cdot}7$$

The sign of ϵ can be obtained from the chart.

The Cocked Hat in General. When errors of observation and plotting are included with the compass error, the resulting errors in the position lines may all differ, and a cocked hat is formed as shown in figure 77.

The constant error ϵ is now replaced by separate and unequal

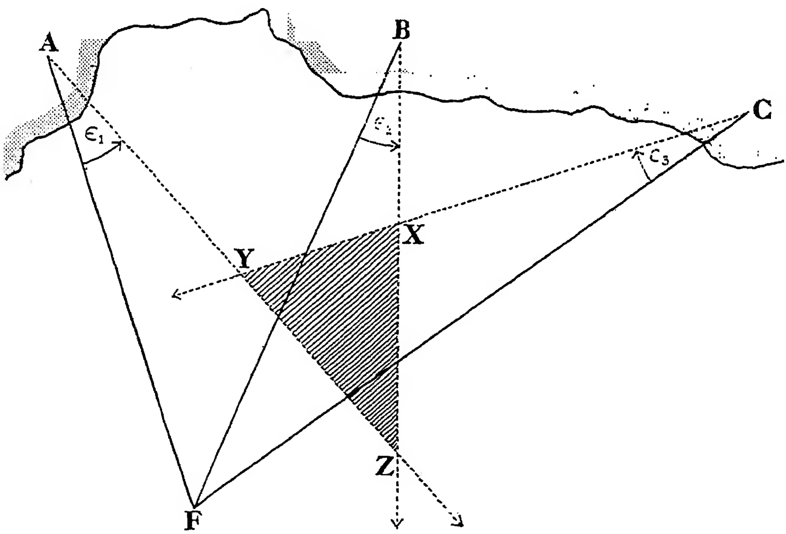


FIGURE 77.

errors ϵ_1, ϵ_2 and ϵ_3. In this example, ϵ_1 and ϵ_2 have the same sign, which is opposite to that of ϵ_3.

As before, F is the true position of the ship, and AZ, BX and CY are the lines of bearing actually plotted and forming the cocked hat XYZ.

Unless the errors ϵ_1, ϵ_2 and ϵ_3 are definitely known, it is impossible to locate the position of F from this cocked hat.

By taking particular values and signs of ϵ_1, ϵ_2 and ϵ_3, it can be seen that the chance of F's falling inside the cocked hat is only 1 in 4. Also, the value and sign of ϵ_3 may cause the plotted line of bearing CY to pass through Z. Z is then the fix by observation, but it is still a distance FZ in error. It is thus clear that even when all three lines of bearing intersect in a single point, the resultant fix may be considerably in error.

In the practice of navigation, when a cocked hat is obtained, it is customary to place the ship's position on the chart in the most dangerous position that can be derived from the observations because the existence of the cocked hat is evidence that the observations are inaccurate, and by interpreting them to his apparent disadvantage, the navigator gives himself a margin of safety which he might not otherwise have.

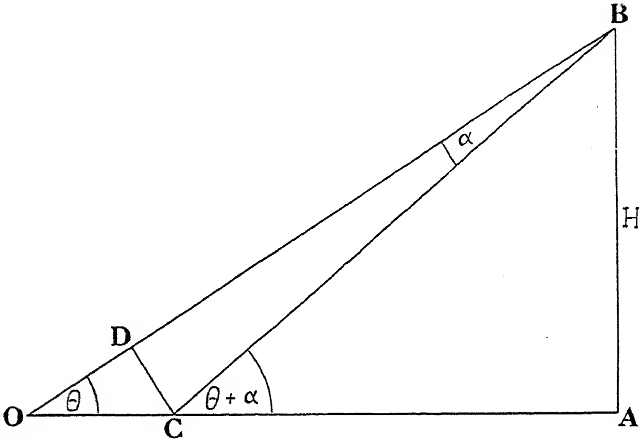


FIGURE 78.

Distance by Vertical Sextant Angle. If an object B is at a known height AB above sea-level, and this height, when measured with a sextant, is found to subtend an angle θ, the position of the observer O lies on a circle of centre A and radius $AB \cot \theta$.

Figure 78, from which the complications introduced by the observer's height of eye, refraction and the Earth's curvature have been omitted for the sake of clarity, shows this situation.

The accuracy of the value of OA derived from $AB \cot \theta$ depends on two factors :

(1) the accuracy of the measured angle.

(2) the accuracy with which the height of B above sea-level is known.

(1) *The Error in* θ.

If α is the actual error in the measured altitude, which is θ, the true altitude of B is $(\theta+\alpha)$. Let C be a point on OA such that the

angle BCA is $(\theta+\alpha)$. Then the angle OBC is α, and if CD is drawn perpendicular to OB:

$$OC \sin \theta = CD = CB \sin \alpha$$

Also, if AB is denoted by H:

$$OB \sin \theta = H$$

$$\therefore \qquad OC \sin^2 \theta = H\left(\frac{CB}{OB}\right)\sin \alpha$$

Since α is small, the ratio $\frac{CB}{OB}$ may be taken as unity.

Hence:
$$OC \quad \frac{H \sin \alpha}{\sin^2 \theta}$$

For an error in observation of 1′, the error in the radius is thus given by:

$$OC = \frac{H \sin 1'}{\sin^2 \theta}$$
$$= \frac{H}{3{,}438 \sin^2 \theta}$$

If H is given in feet, OC is also given in feet.

Since in practice θ is also a small angle, the formula can be further simplified. Thus:

$$OC = \frac{H}{3{,}438}\left(\frac{3{,}438}{\theta}\right)^2$$
$$= \frac{3{,}438\ H}{\theta^2} \quad \ldots\ldots\ldots \quad (5)$$

The altitude is now expressed in minutes of arc, and the formula gives the error in the radius of the position circle in feet, corresponding to an error in observation of 1′.

It is required to find the error in the radius of the position circle, when the error in the observed altitude of a lighthouse 100 *feet above sea-level is* 1′, *the altitude being* 30′.

From formula (5) the error in the radius is given by:

$$OC = \frac{3{,}438 \times 100}{(30)^2}$$
$$= 382 \text{ feet}$$
$$= 0{\cdot}6 \text{ cables} \qquad \text{(approximately)}$$

The actual radius is given by:

$$OA = H \cot \theta$$
$$= \frac{100 \times \cos 30'}{\sin 30'}$$
$$= \frac{100 \times 3{,}438}{30} \qquad \text{(approximately)}$$
$$= 11{,}460 \text{ feet}$$
$$= 19{\cdot}1 \text{ cables}$$
$$AC = 18{\cdot}5 \text{ cables}$$

If the true altitude is $(\theta-\alpha)$, the radius of the position circle is $(19{\cdot}1+0{\cdot}6)$ or 19·7 cables. The observation of the altitude therefore shows that the true radius of the position circle lies between 18·5 cables and 19·7 cables.

If the navigator had to clear a danger at 16 cables from A, he would assume that his observation placed him at the lesser distance of 18·5 cables. His margin of safety would then not be less than 2·5 cables.

(2) *The Error in* H.

In finding the height of B above sea-level, it is necessary to take into account the state of the tide, and this introduces the possibility of error in H. Suppose this error is h in feet, so that the true height of B above sea-level is $(H+h)$ feet.

The radius OA of the position circle is now given by:

$$OA=(H+h)\cot\theta$$

—and the error in this is $h \cot \theta$.

If the altitude of the previous example is used, and h is taken as 1 foot, the error in OA arising from this inaccuracy in H is:

$$1 \cot 30' \text{ feet}$$

$$=\frac{1}{\sin 30'} \text{ feet} \qquad \text{(approximately)}$$

$$=\frac{3{,}438}{30} \text{ feet}$$

The error in the radius is thus about 0·2 cables for each foot of inaccuracy in H.

If the error in H is believed not to exceed 4 feet, the error in the radius will not exceed 0·8 cables, and in the previous example the true radius will not be less than:

$$(19{\cdot}1-0{\cdot}6-0{\cdot}8) \quad \text{or} \quad 17{\cdot}7 \text{ cables}$$

The margin of safety when the danger is 16 cables from A is therefore 1·7 cables.

Station-Pointer Fixes by Horizontal Sextant Angles. When the horizontal angle subtended by two objects at an observer is measured, it tells him that his position lies on the arc of a circle passing through the points. If the angle subtended by one of these objects and a third object is now measured, a second position circle is obtained, intersecting the first position circle at the common object. The other point of intersection is the observer's position. These position circles can be plotted directly, or the angles can be set on a station pointer. But, whatever method is used for finding the observer's position, there will be a possibility of error in the position found owing to:

(1) error in the actual measurement of the angles.

(2) plotting error, or the instrumental error inherent in the station pointer.

(3) error arising from the fact that, in general, the three objects and the observer will not lie in a horizontal plane.

In figure 79, AFB and BFC are the accurate position circles, and $AF'B$ and $BF'C$ are those actually obtained by observation and plotting. F is the true position of the observer, and F' is that obtained.

X is the point in which the circles BFC and $AF'B$ intersect, and BX produced cuts the circle AFB in Y.

For the purpose of this investigation it is assumed that each horizontal angle has the same error α. That is, if the angle AYB (which is equal to the angle AFB) is the true horizontal angle, the angle AXB, which is plotted, is equal to $(AYB+\alpha)$. Similarly the angle $BF'C$ is equal to $(BFC+\alpha)$.

From the triangle AXY:

$$\angle AXB=\angle AYB+\angle XAY$$

i.e. $$\angle XAY=\alpha$$

Also, by the rule of sines:

$$XY=\frac{AX \sin \alpha}{\sin AYB}$$

Or, since α is a small angle expressed in circular measure:

$$XY=\frac{\alpha\, AX}{\sin AYB}$$

From the triangle XYF, by the rule of sines:

$$XF=\frac{XY \sin XYF}{\sin XFY}$$

Therefore, by substitution:

$$XF=\frac{\alpha\, AX}{\sin XFY}\times\frac{\sin XYF}{\sin AYB}$$

If ϕ denotes the angle at which the two circles AFB and BFC cut, then, since F is close to F', the arcs XF and YF are so small that ϕ may be taken as the angle between the chords XF and YF. The expression giving XF may therefore be written:

$$XF=\frac{\alpha\, AX}{\sin \phi}\times\frac{\sin XYF}{\sin AYB}$$

Also, since Y is a point on the circle AFB close to F, YF is approximately the tangent at Y to the circle AFB. Hence the angles YAB and BYF (or XYF) are equal, and:

$$\frac{\sin XYF}{\sin AYB}=\frac{\sin YAB}{\sin AYB}=\frac{BY}{AB}$$

$$\therefore \qquad XF=\frac{\alpha\, AX}{\sin \phi}\times\frac{BY}{AB}$$

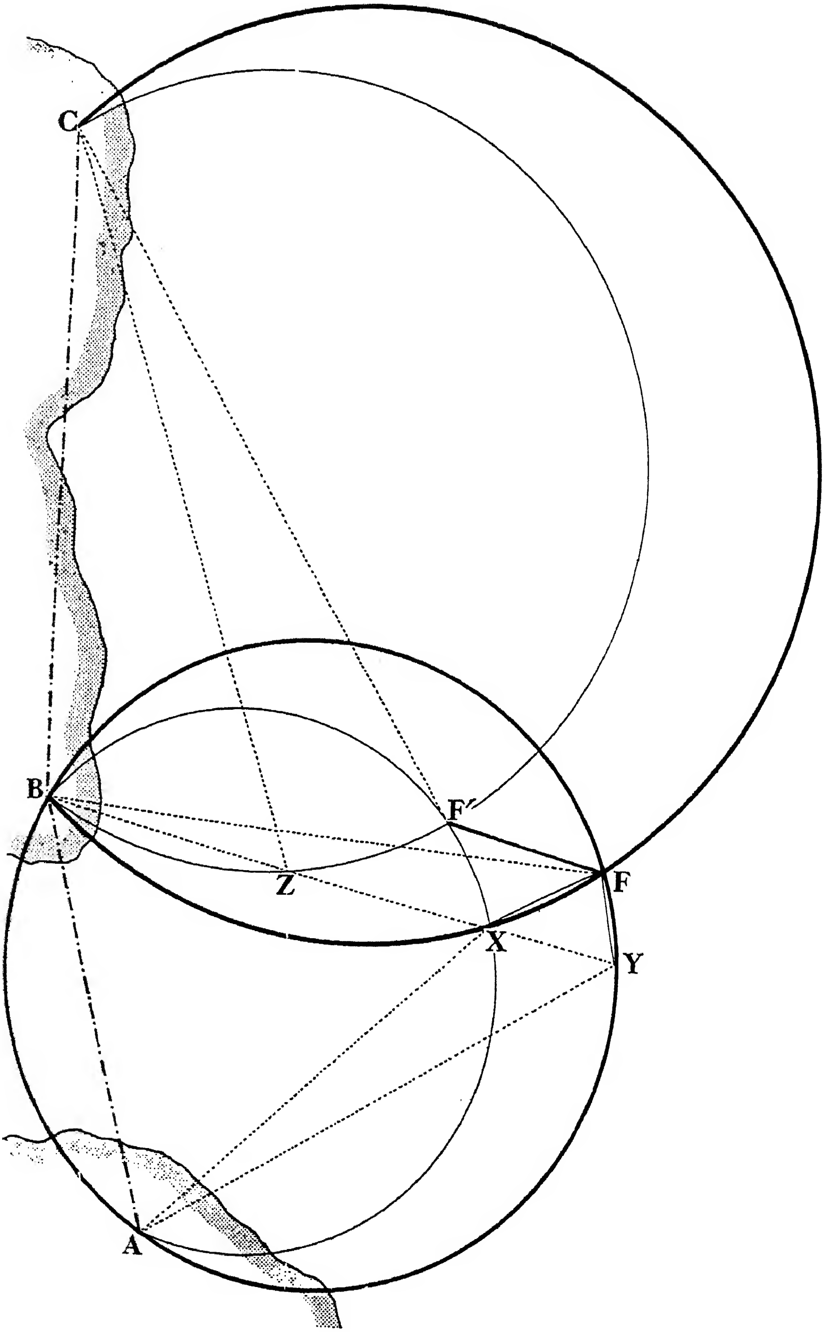


FIGURE 79

But AX is approximately equal to AF, and BY to FB. Hence:

$$XF=\frac{\alpha\ FB}{\sin\phi}\times\frac{AF}{AB}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In the same way it can be proved that:

$$XF'=\frac{\alpha\ FB}{\sin\phi}\times\frac{CF}{BC}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Since the chords XF and XF' are small in comparison with the distances of F' and X from A and B, it can be assumed with sufficient accuracy that the angle FXF' is equal to ϕ. Also, if r_1 is the ratio of the observer's distance from the first object A to the distance of A from B, the middle object, and if r_2 is the ratio of the observer's distance from the third object C to the distance of C from B; that is, if:

$$\frac{AF}{AB}=r_1 \qquad \text{and} \qquad \frac{CF}{BC}=r_2$$

—the displacement from the true position, FF', can be expressed in terms of r_1 and r_2 thus:

$$(FF')^2=XF^2+(XF')^2-2\ XF\ XF'\cos FXF'$$

Therefore, from (7) and (8):

$$(FF')^2=\frac{\alpha^2FB^2}{\sin^2\phi}[r_1{}^2+r_2{}^2-2r_1r_2\cos\phi]$$

i.e.

$$FF'=\frac{\alpha\ FB}{\sin\phi}\sqrt{r_1{}^2+r_2{}^2-2r_1r_2\cos\phi}$$

If ϕ is not an acute angle, it is equal to $(180°-\theta)$ where θ is acute, and the expression for FF' becomes:

$$FF'=\frac{\alpha\ FB}{\sin\theta}\sqrt{r_1{}^2+r_2{}^2+2r_1r_2\cos\theta}\quad . \quad . \quad . \quad (9)$$

—where θ is called the *acute angle of cut.*

Maximum Errors in the Fix by Station Pointer. These follow at once from formula (9), which shows that FF' varies directly as α and the distance of the fix from the object common to both observations, and inversely as $\sin\theta$; and also depends on the values of the ratios r_1 and r_2.

When θ is small, cosec θ and $\cos\theta$ are both large, and FF' is large.

When θ is equal to 90°, cosec θ is unity, and $\cos\theta$ is zero, and FF' is given by:

$$\alpha\ FB\sqrt{r_1{}^2+r_2{}^2}$$

For values of r_1 and r_2 not exceeding 5, and for values of $\frac{1}{2}(r_1+r_2)$ less than 3, the mean ratio $\frac{1}{2}(r_1+r_2)$ can be used instead of each separate ratio without appreciably altering the maximum error, and when this is done, and the value of α is taken as $\frac{1}{2}°$, formula

(9) can be adjusted to give a still more approximate value of the maximum error. Thus:

$$FF'_{max}=\frac{a\,FB}{\sin}\sqrt{\tfrac{1}{2}(r_1+r_2)^2(1+\cos\theta)}$$

$$=\frac{a\,FB}{\sin\frac{1}{2}\theta}\times\left(\frac{r_1+r_2}{2}\right)$$

$$=\frac{30\,FB}{3{,}438\sin\frac{1}{2}\theta}\times\left(\frac{r_1+r_2}{2}\right)$$

$$=\frac{FB}{\theta^\circ}\left(\frac{r_1+r_2}{2}\right)\text{ in miles}\qquad\text{(approximately)}$$

$$=\frac{\text{distance of middle object}}{\text{acute angle of cut in degrees}}\times\text{mean ratio}\quad.\quad(10)$$

If the distance FB is taken as unity, it is clear that a table can be constructed in terms of θ and the mean ratio, giving the values of FF', and from this table the error for any other value of FB can be found by multiplying the tabulated error by that value. The error itself is tabulated in cables instead of decimals of a mile.

TABLE GIVING THE MAXIMUM ERROR IN CABLES OF A STATION-POINTER FIX, FOR EACH MILE OF DISTANCE FROM THE MIDDLE OBJECT, AND FOR VALUES OF θ AND THE MEAN RATIO, WHEN THE ERROR IN EACH ANGLE OBSERVED IS $\frac{1}{2}$°.

Acute Angle of Cut	Mean Ratio $\frac{1}{2}$	Mean Ratio 1	Mean Ratio $1\frac{1}{2}$	Mean Ratio 2	Mean Ratio $2\frac{1}{2}$
10°	0·5	1·0	1·5	2·0	2·5
20°	0·3	0·5	0·8	1·0	1·3
30°	0·2	0·3	0·5	0·7	0·8
40°	0·1	0·3	0·4	0·5	0·6
50°	0·1	0·2	0·3	0·4	0·5
60°	0·1	0·2	0·3	0·3	0·4
70°	0·1	0·1	0·2	0·3	0·4
80°	0·1	0·1	0·2	0·2	0·3
90°	0·1	0·1	0·2	0·2	0·3

In figure 80, A, B and C are three objects. The circles of position corresponding to the horizontal sextant angles have been drawn, and it is seen that they intersect at an angle of about 30°. It is also seen that the ratios r_1 and r_2, being FA/AB and FC/BC, are about 1 and 2. The mean ratio is thus about $1\frac{1}{2}$.

The table shows that when FB is 1′, the maximum error in these circumstances is 0·5 cables.

If, for example, FB is 3′, the maximum error in the fix corresponding to an error of $\frac{1}{2}$° in observation and plotting, is 1·5 cables.

Reliability of Station-Pointer Fixes. Formula (10) makes it possible to lay down rules for deciding whether a station-pointer fix is reliable or not, and indicating the extent of its reliability.

There are three factors to consider: the angle of cut θ; the distance of the fix from the middle object or object common to both observations; and the mean of the ratios r_1 and r_2. The formula shows that the error in the fix will be least when three conditions are fulfilled:

(1) The distance of the fix from the middle object is as small as possible. That is, the nearest of the three objects should be chosen as the middle object when practicable.
(2) The angle of cut should be as near 90° as possible.
(3) The mean ratio should be as small as possible.

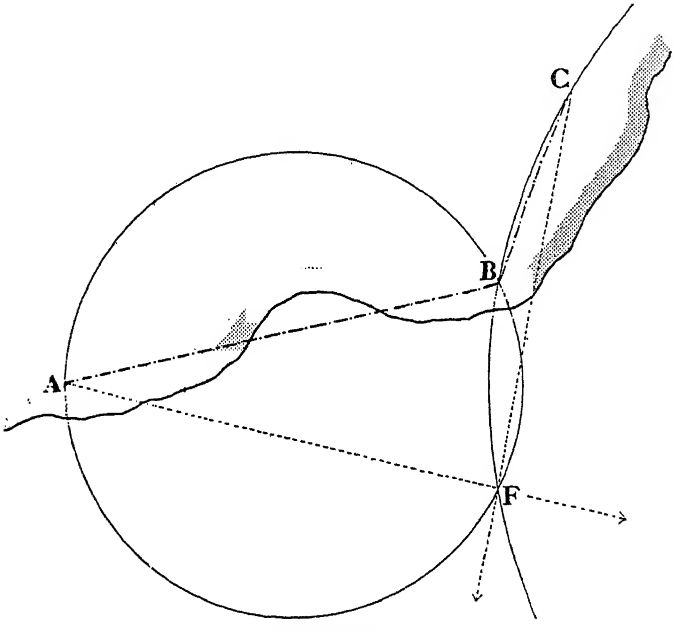


FIGURE 80.

As a rule it is unlikely that all three conditions can be fulfilled at any one time, but it does happen that a reliable fix results even when only two are fulfilled. The fulfilment of a single condition is not sufficient to determine the reliability of a fix. The angle of cut, for example, may be 90°, but the other two factors can easily outweigh this advantage. On the other hand, if in addition to an angle of cut equal to 90° the distance from the middle object is small, the resulting error will be small.

The Angle of Cut. In order to ensure that the angle of cut shall not be too small, it is desirable to have an approximate idea of what it will be before the observations are actually made. This can be obtained by considering the angle subtended at the middle object by the other two objects.

In figure 81, FL and FM are tangents at F to the position circles. Then θ is the angle LFM.

Since FL is a tangent to the circle BFC at F, the angle LFB is equal to the angle BCF. Similarly the angle MFB is equal to the angle BAF. Hence:

$$\begin{aligned}\theta &= \angle BAF + \angle BCF \\ &= 180° - (\angle AFB + \angle ABF) + 180° - (\angle BFC + \angle CBF) \\ &= 360° - (\angle AFC + \angle ABC)\end{aligned}$$

Of these angles, ABC can be measured or estimated from the chart. Also, if E is the estimated position of the ship at the time

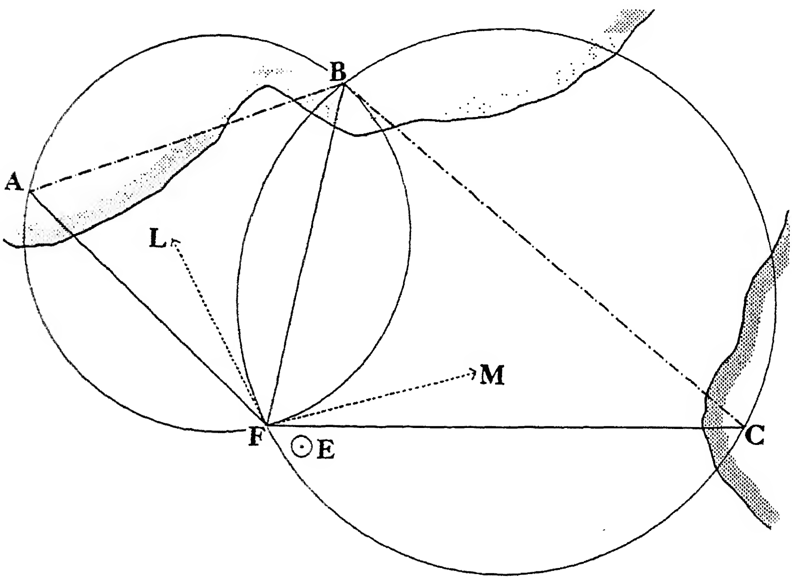


FIGURE 81.

the observations will be made, the angles AEC and AFC will be approximately equal, so that, approximately:

$$\theta = 360° - (\angle AEC + \angle ABC) \quad . \quad . \quad . \quad . \quad (11)$$

The angle AEC can also be estimated from the chart, and a value of θ obtained before the actual observations are taken. A glance at the table on page 172 will then give some idea of the reliability that can be attached to the fix when it is obtained.

As it stands, formula (11) is not general because θ must be less than 90°, and the sum of the angles AEC and ABC will not always be greater than 270°. When they are not, it can be shown, by adjusting the positions of A, B, C and F, that:

$$\theta = (\angle AEC + \angle ABC) - 180°$$

or

$$\theta = 180° - (\angle AEC + \angle ABC)$$

The rule giving θ is therefore: add the angles AEC and ABC, and subtract the sum from 360° or 180°, or subtract 180° from the sum, so as to obtain a value of θ less than 90°.

Examples of Satisfactory Station-Pointer Fixes. In the two examples that follow, E denotes the estimated position of the ship at the time the fix is obtained.

In figure 82a, the angle ABC is estimated to be 170°, and the angle AEC 160°. The angle of cut is therefore given by:

$$[360° - (170° + 160°)]$$
$$= 30°$$

The ratio r_1, being FA/AB or EA/AB approximately, is about $\frac{3}{4}$; and r_2 is about $\frac{5}{4}$. The mean ratio is therefore about unity.

Suppose FB is 2′, and that the error in each angle observed is $\frac{1}{2}$°.

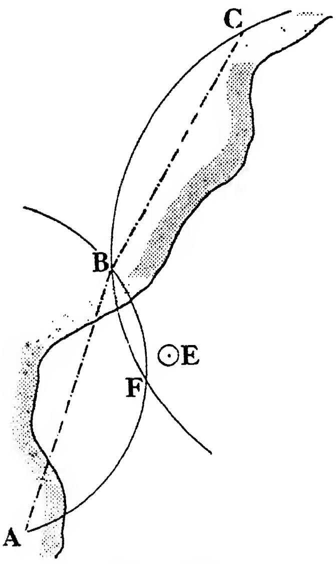


Figure 82a.

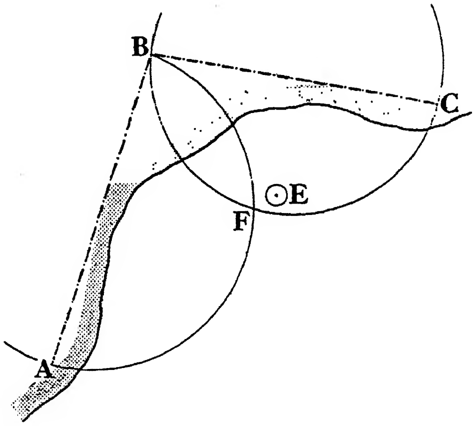


Figure 82b.

The table on page 172 shows that the possible error in the fix is $2 \times 0{\cdot}3$ or 0·6 cables.

In figure 82b, the angle ABC is estimated to be 100°, and the angle AEC 190°. The angle of cut is therefore 70°. Also r_1 and r_2 are both about $\frac{3}{4}$, and the mean ratio is therefore about $\frac{3}{4}$. If the distance FB in this example is 7′, the possible error in the position when the error in each angle observed is $\frac{1}{2}$°, is seen from the tables to be $7 \times 0{\cdot}1$ or 0·7 cables.

The small and apparently unfavourable angle of cut in the first example is counterbalanced by the small value of FB. In the second example FB is large, but θ is 70°. The two fixes are thus widely separated in their angles of cut, yet their reliability is practically the same, and both would be regarded as satisfactory.

Example of an Unsatisfactory Station-Pointer Fix. When the middle object lies near the circle passing through the other two objects and the fix, the fix cannot fail to be unreliable because, in the limiting position when the middle object lies on that circle, it is impossible to obtain a fix.

Figure 83 shows the middle object badly placed. The angle ABC is about 100° and the angle AEC 60°. The angle of cut is therefore [180°−(100°+60°)] or 20°.

Also r_1 and r_2 are each about $1\frac{3}{4}$. The mean ratio is therefore about $1\frac{3}{4}$. If FB is 6′, the error in the fix resulting from an error of $\frac{1}{2}°$ in each angle observed is $6 \times 0{\cdot}9$ or 5·4 cables, an error sufficiently large to render the fix unreliable.

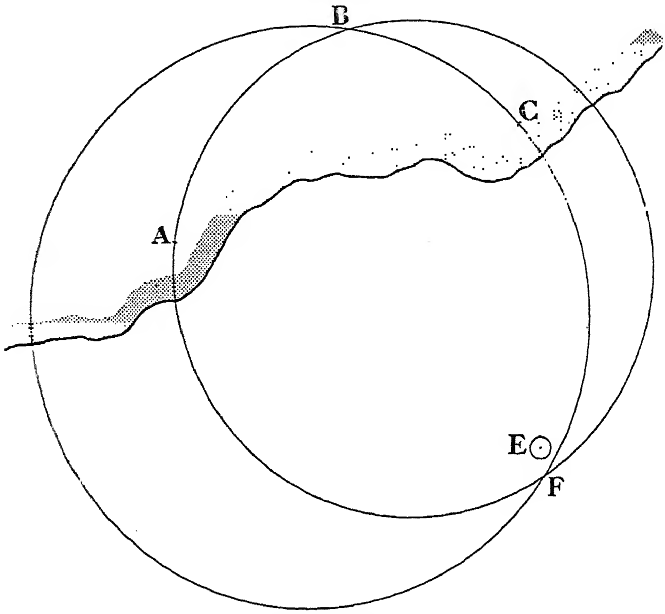


FIGURE 83.

Doubling the Angle on the Bow and the Effect of Current. It is apparent that if a ship holds a steady course until the bearing of an object on her bow is doubled, the position at which this occurs forms an isosceles triangle with the first position and the object, and it is distant from the object by an amount equal to the run between the observations. If the ship experiences a current or tidal stream in the meantime, it must be allowed for in order to avoid an error in her final position.

In practice it will usually be more convenient to solve a problem of this type by plotting it on the chart and transferring the position lines as necessary. The following theory, however, may be regarded as general.

In figure 84, AB is the direction of the fore-and-aft line of the ship, and BC is the direction of the current or tidal stream. AB and BC combine to give the course made good. Thus, if AB is the distance moved through the water in a given interval, and BC the set experienced in the same interval, then AC gives the course and distance made good in that time.

Suppose X is some object observed from the ship.

When the ship is at A, the angle on the bow is XAB, denoted by α. When the ship is at C, it is assumed for the purpose of this problem that the angle on the bow has been doubled. At this point the fore-and-aft line is in the direction CE, parallel to AD, and the angle XCE is thus 2α.

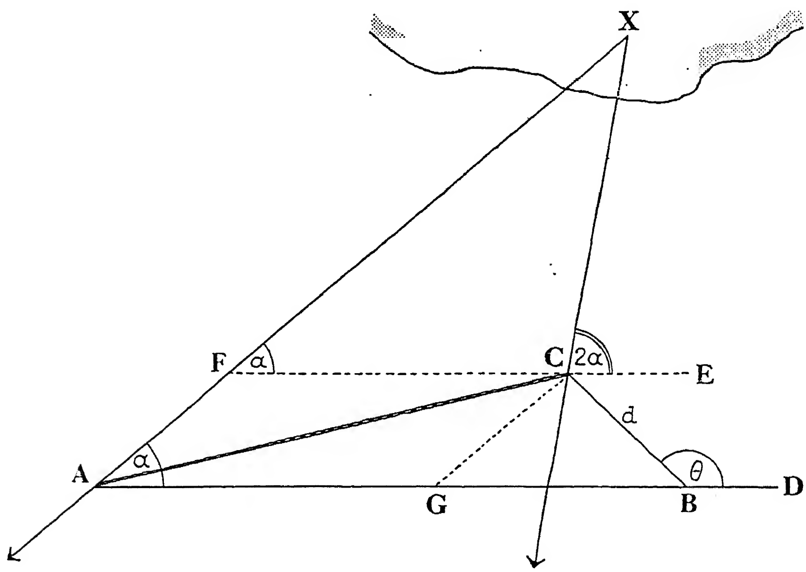


FIGURE 84.

EC produced meets AX in F. The angle CFX is therefore equal to the angle CXF, and FC is equal to CX.

CG is drawn parallel to XA. The angle CGB is therefore equal to α, and, since $FAGC$ is a parallelogram :

$$CX = AG = AB - GB$$

AB, the distance resulting from the ship's known speed and the duration of the run, can be found at once, but GB must be calculated from the triangle GCB. Thus :

$$\frac{GB}{BC} = \frac{\sin GCB}{\sin BGC}$$

i.e.
$$GB = BC\,\frac{\sin GCB}{\sin \alpha}$$

If BC is denoted by d—the amount of drift during the run—and the angle CBD by θ, the angle GCB is $(\theta-\alpha)$ and :

$$CX = AB - \frac{d \sin(\theta-\alpha)}{\sin\alpha}$$

If θ is less than α, CX is given by :

$$AB + \frac{d \sin(\alpha-\theta)}{\sin\alpha}$$

These formulæ suffice when the current or tidal stream carries the ship to port, and the object is also to port. They also suffice when the ship is carried to starboard and the object is to starboard. When, however, the ship moves to starboard and the object is to port, or vice versa, it can easily be shown that CX is given by :

$$AB + \frac{d \sin(\theta+\alpha)}{\sin\alpha}$$

When the direction of the current or tidal stream is known, θ can be found, and GB can be calculated, either by logarithms or the traverse table. The distance of the ship from the object at the instant of the second observation can therefore be found.

At 1000 *an object is seen to bear* 040° *to an observer on board a ship steaming* 075° *at* 16 *knots in a tidal stream setting* 300° *at* 3 *knots. At* 1030 *the same object bears* 005°. *How far is the ship from the object at* 1030 ?

At 1000 the angle on the port bow is (075°—040°) or 35°. At 1030 the angle is (075°—005°) or 70°. Also the angle θ is (75° —300°+360°) or 135°.

The ship's run in 30^m is 8′, and d is 1′·5.

Both the set of the tidal stream and the object are to port. The distance of the ship from the object at 1030 is therefore :

$$8' - \frac{1'{\cdot}5 \sin(135°-35°)}{\sin 35°}$$

$$= 8' - \frac{1'{\cdot}5 \sin 100°}{\sin 35°}$$

$$= 8' - 2'{\cdot}6$$

$$= 5'{\cdot}4$$

The position of the ship at 1030 is thus fixed by a bearing and distance of 005° and 5′·4, and it is necessary to plot only the true bearing.

If the set had been in the opposite direction, 120°, θ would have been equal to (120°—75°) or 45°, and the distance would have been :

$$8 + \frac{1'{\cdot}5 \sin(45°+35°)}{\sin 35°}$$

$$= 8' + 2'{\cdot}6$$

$$= 10'{\cdot}6$$

Effect of the Current when θ has Particular Values. The general formula is simplified considerably when θ has certain values. These values and adjustments are :

(1) *When θ is equal to zero.* This means that the direction of the current or tidal stream is the same as the course steered. Then, by substitution :

$$CX=AB+d$$

(2) *When θ is equal to* 180°. The set is now in a direction opposite to the course steered, and :

$$CX=AB-d$$

(3) *When θ is equal to α.* This means that the direction of the current or tidal stream is that of the first true bearing, and :

$$CX=AB$$

(4) *When θ is equal to* $(180°-\alpha)$. The set is now in a direction opposite to the first true bearing, and again :

$$CX=AB$$

(5) *When θ is equal to* 2α. This means that the direction of the current or tidal stream is that of the second true bearing, and :

$$CX=AB-d$$

(6) *When θ is equal to* $(180°-2\alpha)$. The set is now in a direction opposite to the second true bearing and :

$$CX=AB+d$$

All these circumstances can be investigated with simple diagrams, and when this is done, it will be clear that in (5) and (6) the distances $(AB-d)$ and $(AB+d)$ are *not* distances ' over the ground '.

ERRORS IN ASTRONOMICAL POSITION LINES

Error may be introduced into the astronomical position line by contributory errors which come under four headings :

(1) Error in the observed altitude.
(2) Uncertainty in deck-watch error.
(3) Error inherent in the method by which the sight is worked.
(4) Error in the reckoning between observations.

These contributory errors may be considered separately.

Error in the Observed Altitude. When the sextant altitude of a heavenly body has been corrected for index error, dip, refraction, semi-diameter and parallax, the resulting altitude, and therefore the true zenith distance, may be incorrect owing to a combination of the actual errors of observation and incorrect values of the dip and refraction. This resultant error, whatever its value, is reflected

in the intercept, which may be either too large or too small, and in consequence the position line itself is plotted incorrectly.

In figure 85, D is the D.R. position or a special position from which the sight is worked; DJ is the true intercept and the line HJK is the true position line.

If, for example, the error in the intercept does not exceed 2′, all position lines obtained from observations of the same heavenly body at the particular instant in question will lie between the limiting position lines $H'J'K'$ and $H''J''K''$, where DJ' and DJ'' are respectively $(DJ-2')$ and $(DJ+2')$. Therefore, when a ship's position is decided by two astronomical position lines in which errors are suspected on account of uncertainty in the observed

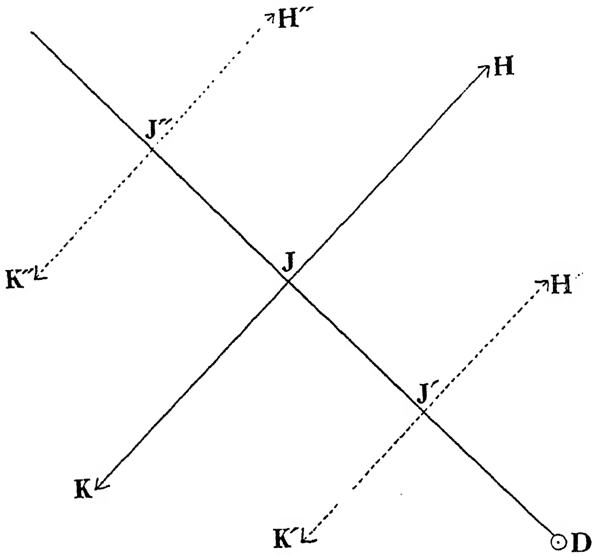


FIGURE 85.

altitudes, it must lie within a parallelogram formed by the limiting position lines, and, if a course had to be shaped to clear a known danger, the navigator would take for his position that point in the parallelogram which would place him at the greatest disadvantage.

The error to which an altitude is most liable is that arising from the choice of incorrect values for dip and refraction.

In normal circumstances, when the altitude is above 15°, a pronounced error in the refraction is not likely to occur. The dip, however, being affected by refraction, is a more uncertain quantity, and when atmospherical conditions are abnormal, the actual value of the dip may differ from the tabulated value by anything up to 10′. For this reason no reliance should be placed

on a single position line obtained when abnormal refraction is suspected.

Errors arising from uncertainty in the value of the dip can be reduced considerably by observing, if possible, stars that bear north, south, east and west. The first pair will give a mean latitude, and the second a mean longitude.

Uncertainty in the Deck-Watch Error. If the deck-watch error is incorrect, the G.M.T. will be incorrect and the hour angle also. An error in the hour angle will lead to an error in the calculated zenith distance, and the position line will again be displaced.

In figure 86, H denotes the true hour angle in which the error is h. The zenith distances corresponding to the hour angles H and

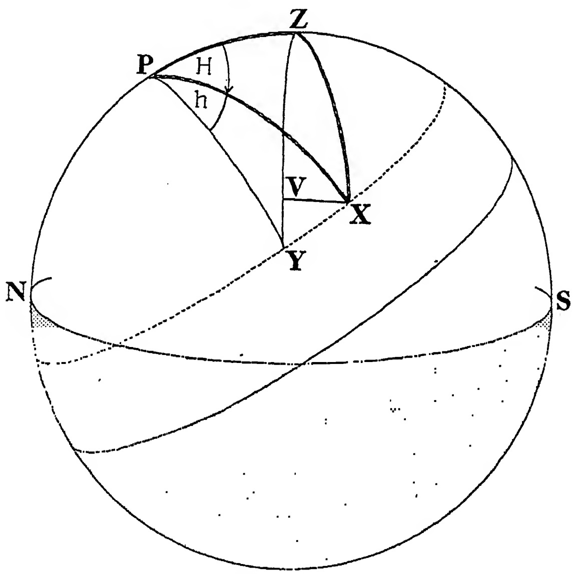


FIGURE 86.

$(H+h)$ are ZX and ZY, and the length of the small-circle arc XY is $h \cos d$, where d is the declination.

If V is taken on ZY so that ZV is equal to ZX, VY is the error in the zenith distance corresponding to the error h in the hour angle, and XV is a small-circle arc perpendicular to ZY.

Since h is small, the triangle VXY may be considered a plane triangle, so that:

$$\begin{aligned} VY &= XY \cos VYX \\ &= XY \sin PYZ \\ &= h \cos d \sin PYZ \end{aligned}$$

But the angles PYZ and PXZ are approximately equal. Hence:

$$VY = h \cos d \sin PXZ$$

From the spherical triangle PZX, in which the azimuth is denoted by α:

$$\frac{\sin PXZ}{\sin PZ}=\frac{\sin PZX}{\sin PX}$$

i.e.

$$\sin PXZ=\frac{\sin \alpha \cos l}{\cos d}$$

Therefore, by substitution:

$$VY=h \sin \alpha \cos l \quad . \quad . \quad . \quad . \quad . \quad (12)$$

If h is expressed in minutes of arc, VY will be expressed in the same units.

In figure 87, LMN is the position line derived from the correct hour angle H, and $L'M'N'$ the position line derived from the

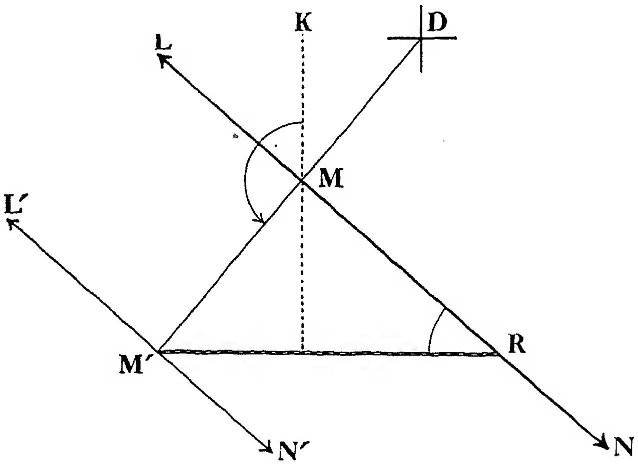


FIGURE 87.

incorrect hour angle $(H+h)$. MM' is thus the error in the calculated zenith distance, and this distance in nautical miles is equal to VY (figure 86) in minutes of arc. The displacement of the position line in longitude is RM′.

The azimuth of the heavenly body is KMM', denoted as before by α, and the angle MRM' is therefore $(180°-\alpha)$. Then:

$$MM'=RM' \sin (180°-\alpha)$$

i.e.

$$RM'=MM' \operatorname{cosec} \alpha$$

If MM' is expressed in nautical miles, RM' is the departure between R and M', and the d'long between R and M' is given by:

$$\begin{aligned} \text{d'long} &= RM' \sec l \\ &= MM' \operatorname{cosec} \alpha \sec l \end{aligned}$$

i.e.

$$MM'=\text{d'long} \sin \alpha \cos l \quad . \quad . \quad . \quad . \quad . \quad (13)$$

This relation, combined with (12), in which VY is equal to MM', shows that the d'long between R and M' is equal to h. The position line is thus displaced in longitude by an amount equal to the error in the hour angle expressed in minutes of arc, and the effect of the error is analogous to that shown in figure 85 where the true position line lies between two parallel lines separated by a distance depending on the magnitude of the error involved.

When the heavenly body is on the meridian or below the pole, α is 180° or 0°, so that the error VY given by (12) vanishes. The latitude found from a meridian altitude will thus be correct even though there is a small error in the hour angle. When the heavenly body is near the meridian, α is small, and the error in the calculated zenith distance is negligible.

The greatest error in a position line for any given latitude, arising from an error in hour angle, occurs when the heavenly body is on the prime vertical; α is then 90°, and the error in the calculated zenith distance is $h \cos l$. Near the equator, where $\cos l$ may be taken as unity, this error becomes the initial error in the deck-watch error expressed in minutes of arc. Thus, if the deck-watch error is incorrect by 16^s, at the equator the position line is displaced by 4′ in longitude in a direction 090° or 270° if the heavenly body is observed on the prime vertical.

In general the direction of the displacement can be found from the formula giving the geographical position of the heavenly body. Thus, the westerly longitude of this position is given by:

Long. W. = G.M.T. + E (for the Sun)

or Long. W. = G.M.T. + R − R.A.✱ (for a star)

In these relations the quantities E and R and the right ascension are constant for the particular observations, and it is thus evident that if the G.M.T. is too large, the sight is worked for a geographical position that is too far west. The position line is thus displaced to the west, and it must be shifted east parallel to itself through an amount equal to the error in G.M.T. expressed in minutes of longitude, in order that the error may be corrected.

If the G.M.T. is too small, the position line must be shifted to the west.

In working a sight, the error of the deck watch on G.M.T. was taken as 24^s *slow instead of* 24^s *fast, and the index error was applied as* −2′·0 *instead of* +2′·0. *The D.R. position was* 30°*N.,* 60°*W.; the true bearing of the heavenly body was* 150°; *and the intercept found in these circumstances was* 1′·7 *away. What was the correct intercept?*

The G.M.T. was 48^s too large, and the position line must be moved 12′ in longitude or 10′·5 in departure farther east. This affects the intercept by 10′·5 cos 60° or 5′·25 towards.

The incorrect application of the index error makes the true zenith distance 4′ too large, so that the intercept must be extended a further 4′ towards the geographical position.

The correct intercept is therefore :

$$4'+5'{\cdot}2-1'{\cdot}7$$
$$=7'{\cdot}5 \text{ towards}$$

Figure 88 shows these adjustments.

Error inherent in the Method by which the Sight is worked. This error reveals itself as an error in the calculated zenith distance, and therefore affects the intercept in the way that an error in the observed altitude affects it.

The reason for this error is two-fold. There is the accumulative

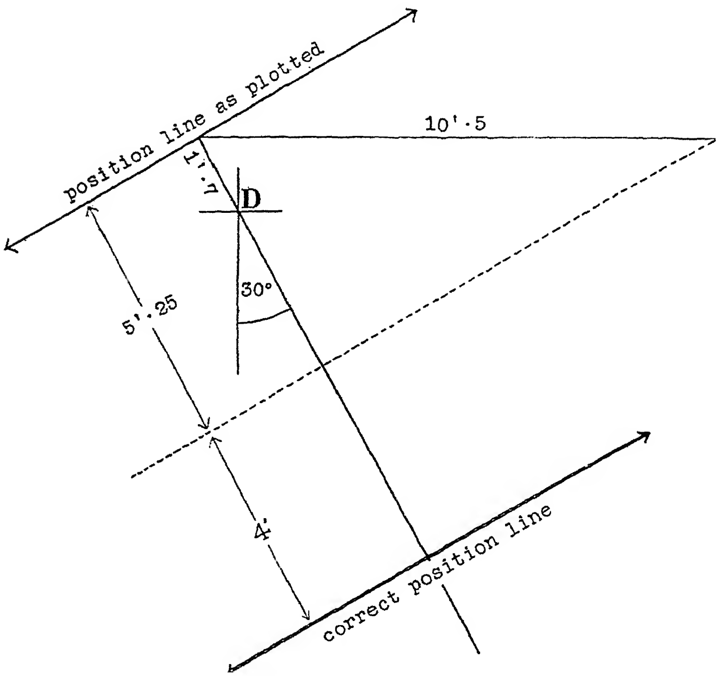


FIGURE 88.

and unavoidable error caused by the addition and rounding-off in the formation of quantities taken from the almanac, and there is the error, also unavoidable, in the method by which the astronomical triangle is solved. This second error arises because the calculations must be simplified by working to only a few figures, and it varies according to the method adopted. If an almanac giving quantities to 0'·1 is used and the triangle is solved by the cosine-haversine method with five-figure logarithms, the resulting error will be negligible, although the zenith distance thus found will seldom be correct to 0'·1. Tabular methods of solution will not increase this error appreciably, but if an almanac or a tabular method working to

an accuracy of 1′ is used, there will be the possibility of an error in the intercept that may, at the worst, rise to 2′ or 3′.

The effect of using quantities that are tabulated to the nearest 1′ is discussed in detail on page 122 of Volume II of this Manual, and the accuracy of altitude-azimuth tables on page 164 of the same volume. General questions of accuracy are also dealt with on pages 110 and 128 of this volume.

Error in the Reckoning Between Observations. When some time elapses between two observations of heavenly bodies, there

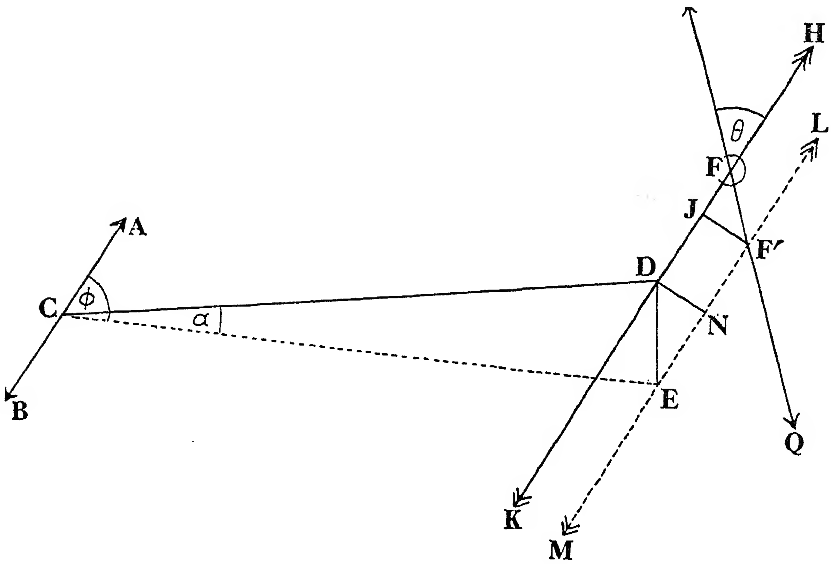


FIGURE 89.

is a likelihood that the first position line will be incorrectly transferred for two reasons :

(1) the course laid down on the chart may differ from the course actually made good during the run between sights.

(2) the distance estimated to have been made good may differ from the correct distance made good.

(1) *Error arising from incorrect course.* Errors in the course may result from any combination of inaccurate plotting, the effects of wind on the ship, the existence of an unsuspected current, and the faulty correction of the compass.

In figure 89, *ACB* is the position line obtained from the first observation, and *CE* gives the course and distance plotted on the chart. The true course is shown by *CD*, so that the error in plotting the course is the angle *DCE*. The correct transferred position line is thus *HDK*, and the incorrect one is *LEM*.

The position line obtained from the second observation is PQ. It is independent of the position from which it is calculated; that is, PQ will be obtained whether the intercept is calculated from D or E.

The true observed position is F, and the incorrect position F'. The error is thus FF'.

Let ϕ denote the angle ACE, and α the error in the course. Then, since α is small and CD is equal to CE, the angle DEC is approximately a right-angle.

Draw DN and $F'J$ perpendicular to LM and HK. Then, since the angle CEM is equal to ϕ and the angle DEC differs little from 90°, the angle EDN is approximately equal to the angle CEM, and therefore to ϕ. Also:

$$DN = ED \cos \phi$$

If θ is the angle of cut between the position lines HK and PQ:

$$JF' = FF' \sin \theta$$

Also JF' is equal to DN. Therefore:

$$FF' = \frac{ED \cos \phi}{\sin \theta}$$

ED can be expressed in terms of the run CD and the error α, and if α is expressed in degrees:

$$ED = CD \sin \alpha$$
$$= \frac{60\ \alpha}{3{,}438}\ CD$$
$$= \frac{\alpha\ CD}{57} \qquad \text{(approximately)}$$

Therefore, by substitution:

$$FF' = \frac{\alpha\ CD \cos \phi}{57 \sin \theta} \quad \ldots \ldots \quad (14)$$

This equation shows that if ϕ is equal to 90°, as it is when the first observation is taken of a heavenly body that is either ahead or astern, the error in the observed position vanishes.

The navigator of a ship, steaming on a course 075° *at* 15 *knots, obtained his position from two observations taken at an interval of* 3ʰ. *The angle of cut of the position lines was* 30°; *the true bearing of the first heavenly body was* 135°, *and the estimated error in laying down the course on the chart was* 3°. *What was the error in the position obtained?*

The angle between the course and the true bearing of the first heavenly body is 60°, so that ϕ is 30°. The run between sights is 45′, and the error, by formula (14), is:

$$\frac{3}{57} \times 45 \times \frac{\cos 30°}{\sin 30°}$$
$$= 4'{\cdot}1$$

The error in the position obtained is thus considerable.

(2) *Error arising from incorrect distance.* The error is now in the length of CD, the distance run between sights, not in ϕ, and it results from an inaccurate estimation of the ship's speed over the ground.

In figure 90, CD represents the correct distance made good, and CE the distance actually plotted on the chart. HK is the correct transferred position line, and LM the incorrect one. The position line obtained from the second observation is PQ, and is independent of D or E. F is the true observed position, and F' the incorrect position. FJ and EN are drawn perpendicular to LM and HK,

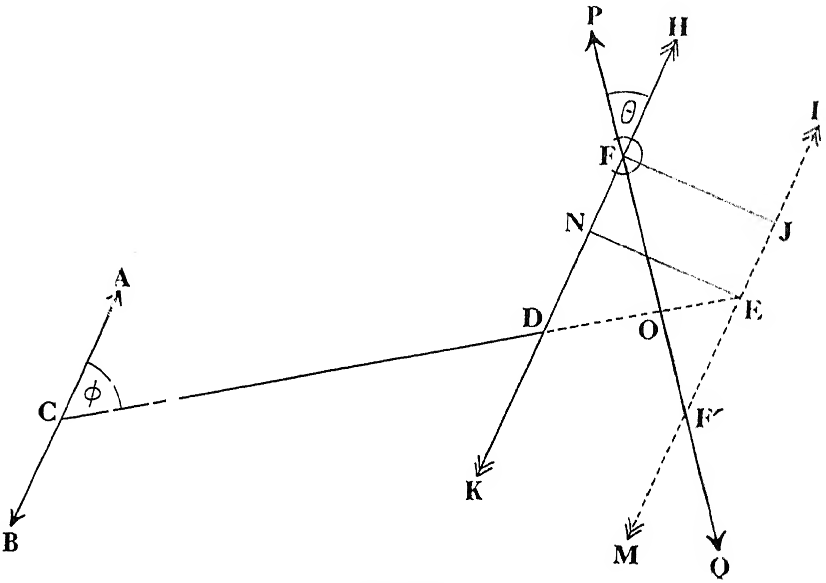


FIGURE 90.

and x denotes the error DE in the estimated distance between observations. Then the angle NDE is ϕ, and :

$$NE = DE \sin\phi = FJ$$

Also, since the angle $FF'J$ is equal to θ, the angle of cut :

$$FJ = FF' \sin\theta$$

i.e.

$$FF' = \frac{x \sin\phi}{\sin\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

This formula shows that if ϕ is zero, as it is when the position line obtained from the first observation is parallel to the course, the error in the observed position vanishes, because CD then coincides with the first position line which is thus transferred along itself. The observed position is therefore at O, the intersection of CD and PQ.

Formula (15) also shows that FF' depends on the error in the distance and not the distance itself, as in formula (14).

For given values of x and θ, FF' is a maximum when ϕ is 90°; that is when the first heavenly body is ahead or astern.

If, for example, the error in estimating the distance made good is $5'$, the angle of cut is 30° and ϕ is 60°, the resulting error in the observed position is:

$$5 \times \frac{\sin 60^\circ}{\sin 30^\circ}$$
$$= 8' \cdot 7$$

The Cocked Hat Formed by Astronomical Position Lines. In general, the position lines obtained from three astronomical observations (which, for simplicity, are considered as being taken simultaneously) are no more likely to pass through a common point than three terrestrial position lines are likely to pass through one, though for different reasons. By far the most important reason for their failure to do so lies in the fact that the observed zenith distances are seldom correct. Each position line is thus displaced parallel to itself as in figure 91, and a cocked hat is formed.

Unless the separate errors are known, the true observed position cannot be found. If, however, all the errors are assumed to be equal in magnitude and sign, as they would be if, for example, the only source of error lay in an inaccurate value of the index error, then simple constructions to find the fix can be obtained.

(1) *All intercepts 'towards' or 'away'*. In figure 91, D is the position from which the intercepts are calculated, and DS, DT and DR give the directions of the heavenly bodies observed. AA', BB' and CC' are the position lines obtained.

If a constant error of $1'$ in the index error is assumed, all intercepts must change by this amount, and the position lines AA' and BB' must be displaced so as to intersect in K. Since EE' is equal to FF', it follows that KL is equal to KM, and that K lies on the bisector of the angle ZXY. Hence, so long as the error in the observation of the heavenly body S is the same as that in the observation of T, the observer's position will lie on the line OX. By taking the observations of R and S, it follows that the true position lies on OZ, the bisector of the angle XZY. These bisectors intersect in O, which is the observer's position when all the intercepts are drawn the same way and the error is the same in magnitude and sign.

(2) *Intercepts 'towards' and 'away'*. Figure 92 shows the construction when the intercepts derived from the observations of R and S are drawn towards, and the intercept derived from the observation of T is drawn away.

The true position lies on the bisector of XZY.

The assumption of a constant error in the intercepts derived from S and T moves X to K where K is so placed that KX is the bisector of the exterior angle YXA'. This bisector meets the bisector of the angle XZY in O, which is the observer's position in the given circumstances.

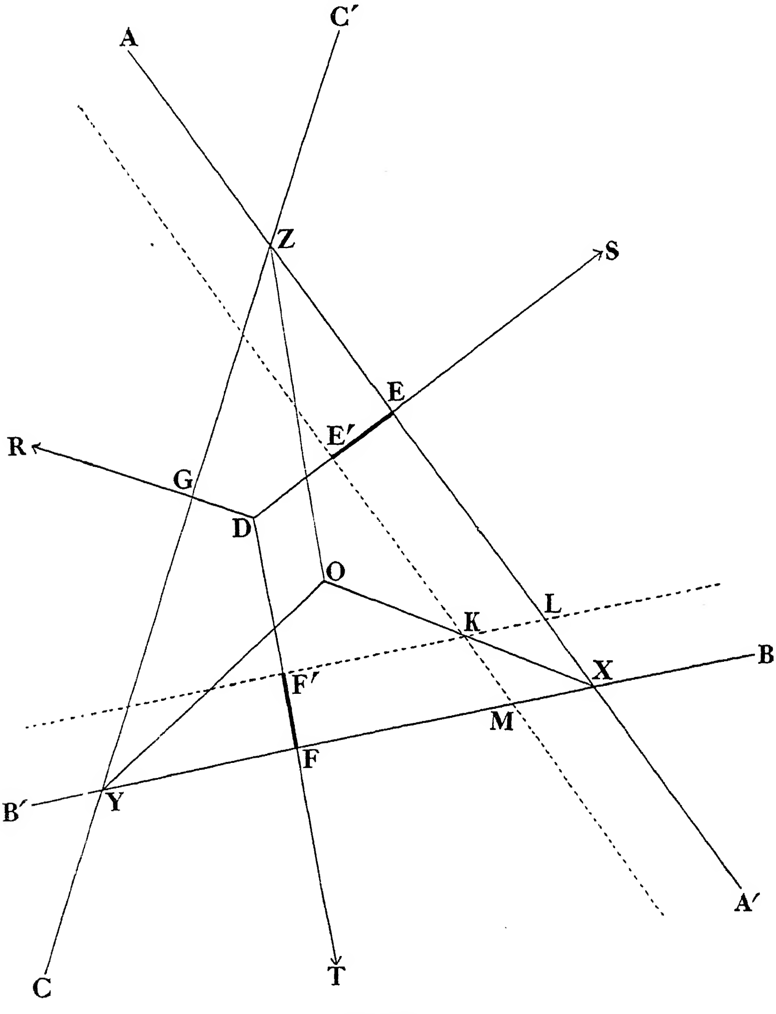


FIGURE 91.

It should be borne in mind that these constructions are valid only when the errors in the zenith distances obtained from sextant observations are equal in magnitude and sign, as they are when the index error is inaccurate; and it should be noticed that the constructions can be made whether or not other errors are taken into consideration. For these reasons, no reliance should be placed on

such constructions unless it is suspected that the total errors in each intercept are equal in magnitude and sign.

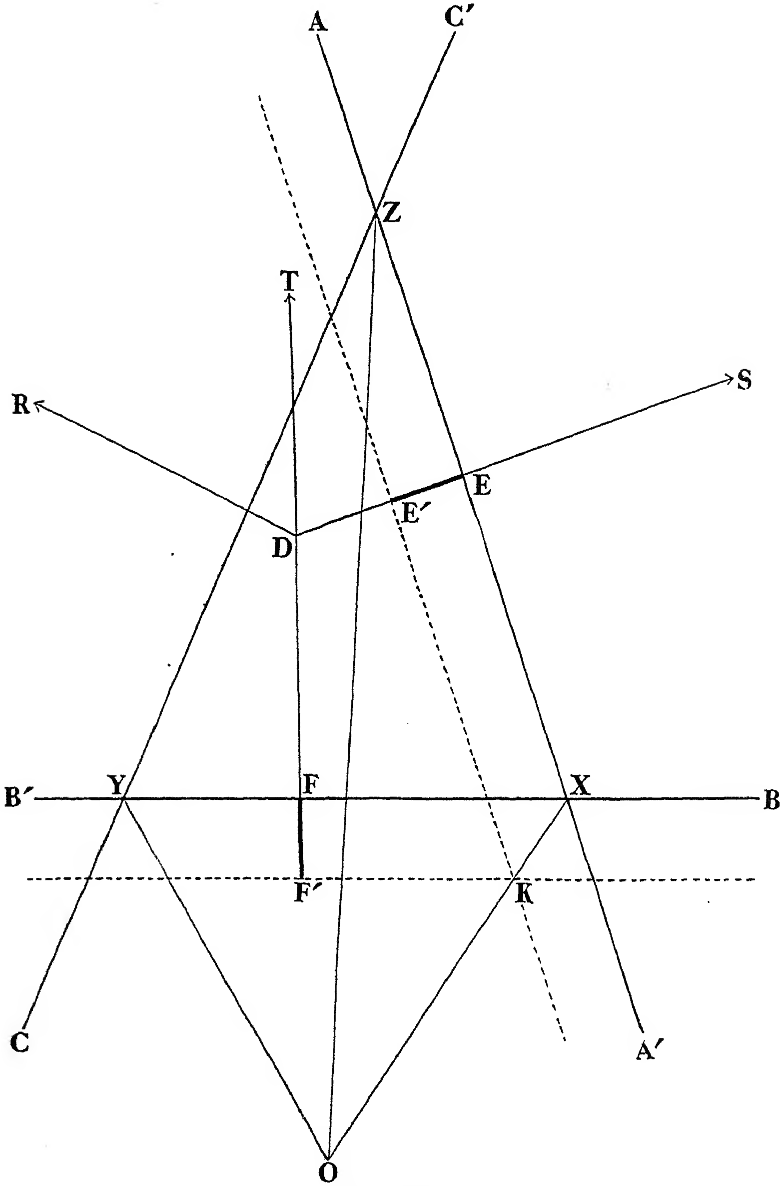


FIGURE 92.

It should also be noticed that the observer's position lies outside the cocked hat only if 180° will enclose all three bearings.

CHAPTER XIV

LINES OF BEARING AND POSITION LINES FROM DIRECTIONAL WIRELESS TELEGRAPHY

The path of a W/T signal between two places on the Earth's surface is the arc of the great circle joining them. The geographical position of the shore station that makes or receives the signal is always known, and this fact, together with the great-circle bearing recorded, provides the navigator with the data for obtaining a position line from the signal.

The problem calls for different methods of approach according as:

(1) the station takes the bearing of the ship.
(2) the ship takes the bearing of the station.

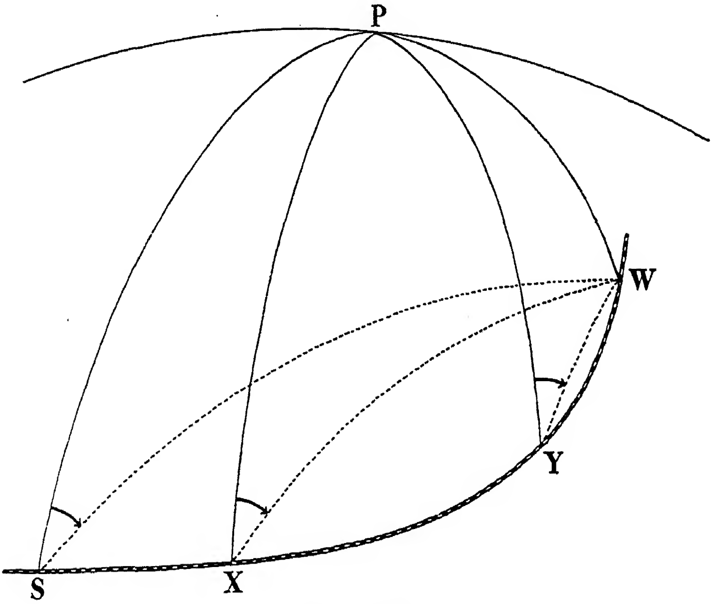


FIGURE 93.

When the station takes the bearing of the ship, the problem is relatively straightforward because the position line is part of the great circle defined by the position of the station and the bearing at the station; but when the ship takes the bearing of the station, the problem is complicated by the fact that the bearing of a known position from one that is unknown does not define a great circle but a *curve of constant bearing*.

The Curve of Constant Bearing. This, as the name suggests, is the curve joining all points on the Earth's surface from which the great-circle bearings of a given point—a particular station, for example—are the same.

Figure 93 shows a part of the curve when the constant bearing is about 35°, W being the station of which the bearing is taken. The dotted line SW is a great circle passing through W and making an angle of 35° with the meridian SP, but the ship can equally well be at X or Y, points at which the great circles joining them to W also make angles of 35° with the meridians through them.

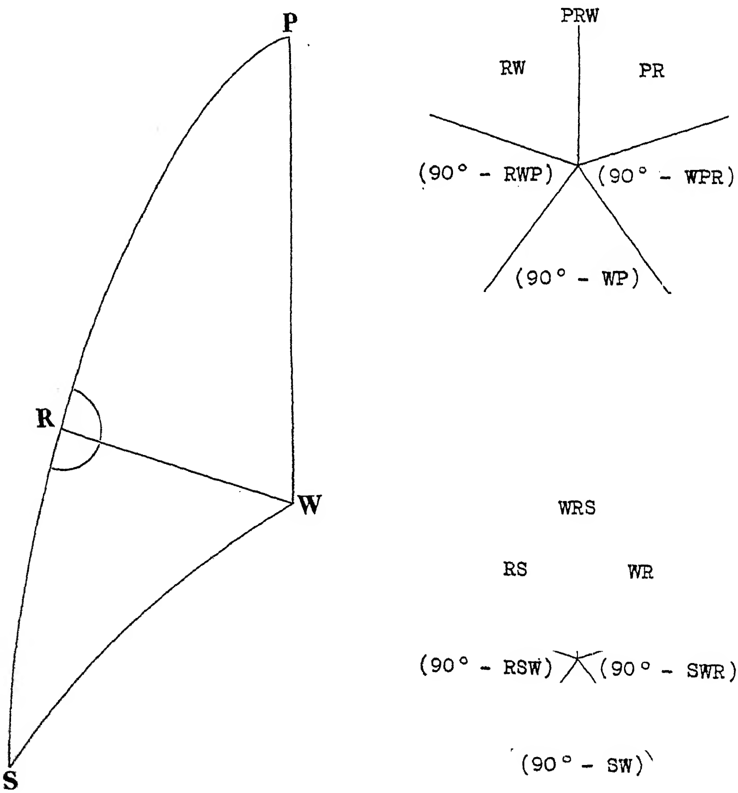


FIGURE 94.

Trigonometrical Solution when the Station Takes the Bearing. Since the position of the station and the bearing of the ship from this station are known, it is possible to calculate either the longitude in which the great circle thus defined cuts any particular parallel of latitude, or the latitude in which the great circle cuts any particular meridian, and the direction of the great circle at the position obtained. The most convenient latitude or longitude to assume is clearly that of the D.R. position because the arc of the great circle,

which is the position line, can then be drawn as a straight line with the least loss of accuracy.

Either the four-part formula or Napier's rules will solve the spherical triangle, but if the four-part formula is used, the D.R. longitude must be assumed. Napier's rules suffice for an assumed longitude or an assumed latitude.

A ship in D.R. position 48°20′*N.*, 10°00′*W.*, *is told that she bears* 240° *from Land's End D/F station, the position of which is* 50°07′·1*N.*, 5°40′·1*W. What is the latitude of the point on the D.R. meridian through which the position line must be drawn, and what is the direction of the position line?*

In figure 94, P is the pole, W the station and S the point on the D.R. meridian through which the great circle passes. The angle at P is then the d'long between the station and the ship's D.R. position, and is given by:

D.R. long.	10°00′·0W.
Station	5°40′·1W.
∠SPW	4°19′·9

WR is the perpendicular from W on PS, and the circular parts for the two right-angled spherical triangles thus formed are as shown.

Napier's rules applied to the triangle PRW give:

$$\sin RW = \sin WP \sin WPR \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$= \sin 39°52'{\cdot}9 \sin 4°19'{\cdot}9$$
$$\therefore \quad RW = 2°46'{\cdot}5$$

$$\cos WPR = \tan PR \cot WP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$
$$\text{i.e.} \quad \tan PR = \cos WPR \tan WP$$
$$= \cos 4°19'{\cdot}9 \tan 39°52'{\cdot}9$$
$$\therefore \quad PR = 39°48'{\cdot}1$$

$$\cos WP = \cot RWP \cot WPR \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$
$$\text{i.e.} \quad \cot RWP = \cos WP \tan WPR$$
$$= \cos 39°52'{\cdot}9 \tan 4°19'{\cdot}9$$
$$\therefore \quad RWP = 86°40'{\cdot}4$$

Napier's rules applied to the triangle WRS, in which the angle SWR is equal to ($PWS-PWR$) or (360°—240°—86°40′·4), that is 33°19′·6, give:

$$\cos RSW = \sin SWR \cos WR \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$
$$= \sin 33°19'{\cdot}6 \cos 2°46'{\cdot}5$$
$$\therefore \quad RSW = 56°43'{\cdot}1$$

$$\sin WR = \tan RS \cot SWR \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$
$$\text{i.e.} \quad \tan RS = \sin WR \tan SWR$$
$$= \sin 2°46'{\cdot}5 \tan 33°19'{\cdot}6$$
$$\therefore \quad RS = 1°49'{\cdot}4$$

The latitude of S, being equal to $[90° - (PR+RS)]$ is therefore :

$$90° - 39°48'{\cdot}1 - 1°49'{\cdot}4$$
$$= 48°22'{\cdot}5$$

The required position line is thus a straight line drawn through the point 48°22′·5N., 10°00′W., in a direction N.56°43′·1E.

If the four-part formula is used, both PS and the angle PSW are found directly. Thus :

$$\cos 39°52'{\cdot}9 \cos 4°19'{\cdot}9$$
$$= \sin 39°52'{\cdot}9 \cot PS - \sin 4°19'{\cdot}9 \cot 120°$$

and:

$$\cos PS \cos 4°19'{\cdot}9$$
$$= \sin PS \cot 39°52'{\cdot}9 - \sin 4°19'{\cdot}9 \cot PSW$$

The first equation gives PS—41°37′·5, as before—and the substitution of this value in the second equation enables the angle PSW to be found.

Trigonometrical Solution when the Ship Takes the Bearing. When the ship takes the bearing of a shore station, the latitude can be assumed and the longitude of a position in that latitude can be found, the position being such that the great circle passing through it in the direction recorded also passes through the station ; or the longitude can be assumed and the latitude of a position in that longitude found, the position again fulfilling those requirements. If several assumptions are made and a number of positions obtained, these positions must lie on the curve of constant bearing, and the position line is the part of that curve in the neighbourhood of the D.R. position.

If, in the last example, the ship had taken a bearing of the Land's End station, she would have said that the station bore 056¾°, and the position line would be the part of the curve of constant bearing in the neighbourhood of 48°20′N., 10°00′W. Actually the bearing of the station has been found to be N.56°43′·1E., and this figure will be taken in the calculation of the necessary points on the curve of equal bearing in order to ensure that the two examples agree.

As before, a longitude is assumed. Let it be the D.R. longitude, 10°00′W. Equations (1) and (2) then give RW (2°46′·5) and PR (39°48′·1), and since in the spherical triangle WRS the observed bearing is now the angle RSW (56°43′·1), RS can be calculated from the formula :

$$\sin RS = \tan WR \cot RSW \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$
$$= \tan 2°46'{\cdot}5 \cot 56°43'{\cdot}1$$
$$\therefore \quad RS = 1°49'{\cdot}4$$

The latitude of S is thus $(90° - 39°48'{\cdot}1 - 1°49'{\cdot}4)$ or 48°22′·5 and the position 48°22′·5N., 10°00′W., is one point on the curve from which the great-circle bearing of Land's End is N.56°43′·1E.

To find other points on this curve, choose longitudes 30′ each side of 10°00′W. The working is then:

Assumed long.	10°30′·0W.	9°30′·0W.
Station	5°40′·1W.	5°40′·1W.
d'long	4°49′·9	3°49′·9

By formula (1):

sin RW=sin 39°52′·9 sin 4°49′·9 =sin 39°52′·9 sin 3°49′·9

∴ RW=3°05′·7 =2°27′·4

By formula (2):

tan PR=cos 4°49′·9 tan 39°52′·9 =cos 3°49′·9 tan 39°52′·9

∴ PR=39°46′·9 =39°49′·1

By formula (6):

sin RS=tan 3°05′·7 cot 56°43′·1 =tan 2°27′·4 cot 56°43′·1

∴ RS=2°02′·0 =1°36′·8

PR=39°46′·9 =39°49′·1

PS=41°48′·9 =41°25′·9

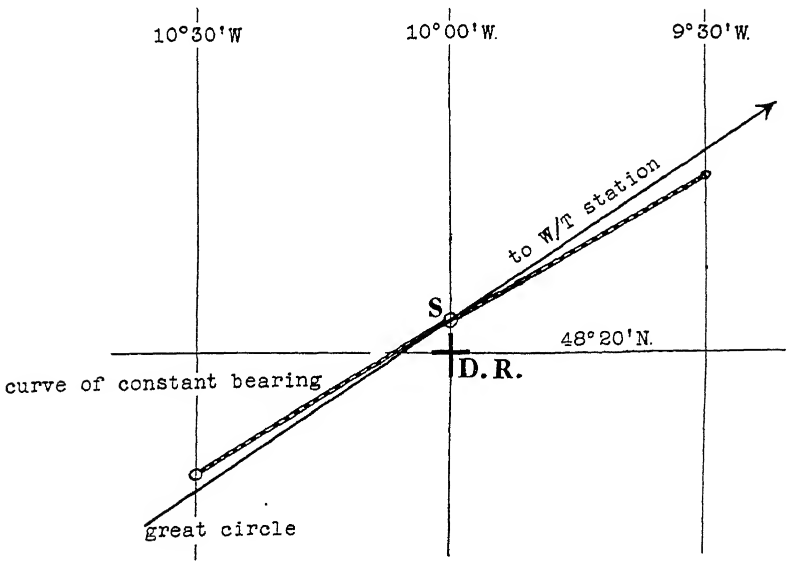


FIGURE 95.

Three points on the curve of equal bearing have thus been obtained:

$\begin{cases} 48°11'·1N. \\ 10°30'·0W. \end{cases}$ $\begin{cases} 48°22'·5N. \\ 10°00'·0W. \end{cases}$ $\begin{cases} 48°34'·1N. \\ 9°30'·0W. \end{cases}$

—and these points, joined by a smooth curve, give the position line in the neighbourhood of the D.R. when the ship takes the bearing of the shore station.

Figure 95 shows this position line and also the part of the great circle that is the position line when the station takes the bearing of the ship, and it is seen that if bearings are taken simultaneously by the ship and the station, the ship lies at the intersection of the great circle defined by the bearing of the ship from the station, and the curve of constant bearing defined by the bearing of the station from the ship. It must be remembered, however, that in this example the two bearings have been made to agree exactly. In practice there would be an error in each bearing, and the small angle of cut between the great circle and the curve of constant bearing would lead to a considerable error in the position found. For this reason the method of finding a ship's position by simultaneous W/T bearings between the ship and a shore station, though theoretically sound, is open to practical objections.

The Convergency of the Meridians. The trigonometrical methods just described are accurate and general in that they enable a position line to be derived from a W/T bearing whatever the distance of the ship from the station. When that distance is small, certain assumptions can be made and the position line drawn by the convergency method, the practical working of which is described in Chapter V of Volume I.

A great circle, other than the equator, cuts all meridians at different angles since the meridians converge towards the pole, and the difference between the initial and final directions of the great circle is known as *the convergency of the meridians* or simply *the convergency*.

In figure 96, the initial bearing is α and the final bearing β, so that c, the convergency of the meridians, is $(\beta-\alpha)$.

In terms of the angles A and B in the spherical triangle PAB, the convergency is given by:

$$c=180^\circ-B-A$$

i.e.

$$\frac{c}{2}=90^\circ-\frac{A+B}{2}$$

Napier's Analogies (pages 253 and 254, Volume II) can now be employed to express the convergency in terms of the latitude and longitude of A and B. Thus, by Napier:

$$\tan\frac{A+B}{2}=\frac{\cos\frac{1}{2}(PA-PB)}{\cos\frac{1}{2}(PA+PB)}\cot\frac{APB}{2}$$

i.e.

$$\cot\tfrac{1}{2}c=\frac{\cos\frac{1}{2}(\text{d'lat})}{\cos\,(\text{mean co-lat.})}\cot\tfrac{1}{2}(\text{d'long})$$

or

$$\tan\tfrac{1}{2}c=\frac{\sin\,(\text{mean lat.})}{\cos\frac{1}{2}\,(\text{d'lat})}\tan\tfrac{1}{2}(\text{d'long})$$

This is an accurate formula, and it is true for all values of c, d'lat and d'long; but, when these factors are small, as they usually are when W/T bearings are taken, it can be modified considerably since:

$$\tan \tfrac{1}{2}c = \tfrac{1}{2}c$$

$$\cos \tfrac{1}{2}\,(\text{d'lat}) = 1$$

$$\tan \tfrac{1}{2}\,(\text{d'long}) = \tfrac{1}{2}\,\text{d'long}$$

The approximate formula is thus:

$$\tfrac{1}{2}c = \tfrac{1}{2}\,\text{d'long} \times \sin\,(\text{mean lat.})$$

or $$c = \text{d'long} \times \sin(\text{mean lat.}) \quad . \quad . \quad . \quad . \quad (7)$$

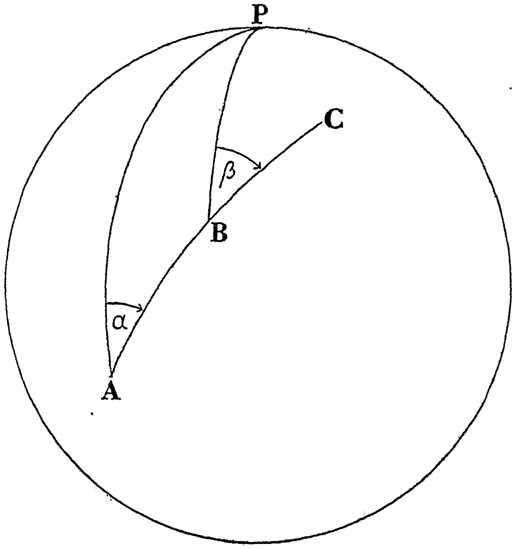


FIGURE 96.

This is the form in which the convergency is most conveniently expressed for use with W/T bearings, but for the requirements of surveying it is further adjusted by the substitutions:

$$\text{d'long} = \text{departure} \times \sec\,(\text{mean lat.})$$

and $$\text{dep.} = \text{distance} \times \sin\,(\text{course or mercatorial bearing})$$

It then becomes:

$$c = \text{departure} \times \tan\,(\text{mean lat.}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and $$c = \text{distance} \times \sin\,(\text{mercatorial bearing}) \times \tan\,(\text{mean lat.}) \quad (9)$$

If the d'long, departure or distance in any of these formulæ is expressed in minutes of arc, the convergency is given in minutes of arc.

Each formula shows that the convergency is zero when the mean latitude is zero. This occurs when the latitudes are equal and of contrary names, or when the two places are on the equator. The convergency is also zero when the d'long, departure or mercatorial bearing is zero; that is when the two places are on the same meridian.

Alternative Proof of the Convergency Formulæ. The three convergency formulæ can be established directly by considering the polar relations between the various great circles, the pole of a great circle being that point which is 90° from every point on the great circle.

In figure 97, A and B are two places on the Earth a short distance

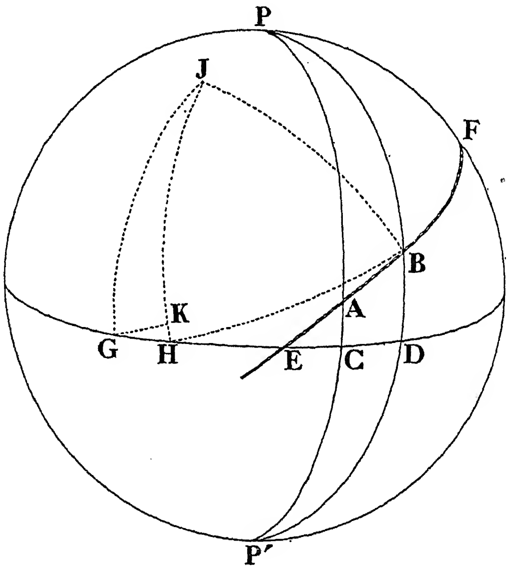


FIGURE 97.

apart, and PAP' and PBP', the meridians through them, cut the equator in C and D. The great circle through AB cuts the equator in E.

Since any meridian cuts the equator at right-angles, its pole must lie on the equator. The poles of the meridians through A and B are therefore G and H, where GC and HD are each equal to 90° and GH is equal to CD, the d'long between A and B.

The pole of the great circle $EABF$ is J, and JG, the great-circle arc joining the poles of the two great circles, $EABF$ and $PACP'$, measures the angle PAF since the angle between two great circles is equal to the angle between their poles. Similarly the great-circle arc JH measures the angle PBF.

The convergency between A and B, being the difference between the angles PBF and PAF, is thus $(JH-JG)$.

If a small circle is now drawn with centre J and radius JG, it cuts JH in K, and, since GH is small, the triangle GHK may be considered a plane right-angled triangle. Hence :

$$\begin{aligned} c &= JH-JG \\ &= KH \\ &= GH \cos JHG \\ &= \text{d'long} \cos JHG \end{aligned}$$

But JB and HB are both 90°. B is therefore the pole of JH, and since P is the pole of the equator, PB measures the angle JHG, which is thus the co-latitude of B.

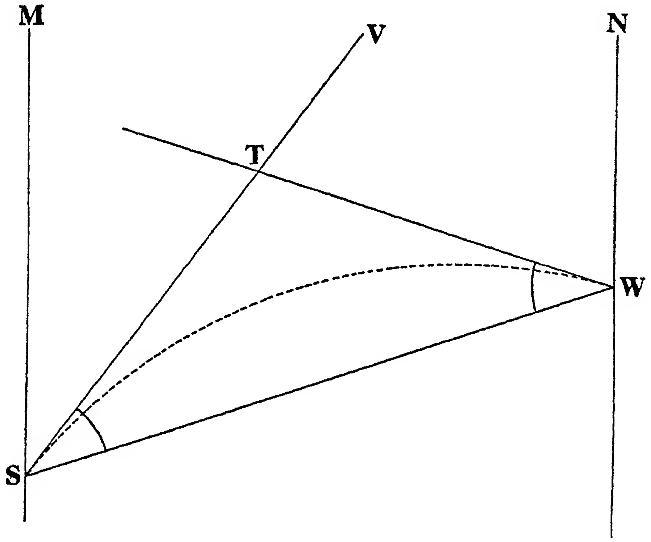


FIGURE 98.

The convergency between A and B is therefore given approximately by d'long sin (lat. B). It can be shown that the same convergency is also given approximately by d'long sin (lat. A). The mean latitude of A and B may thus be taken as the latitude of either without appreciable error when the distance between A and B is small, and the resulting formula, as before, is :

$$c = \text{d'long} \times \sin \text{ (mean lat.)}$$

Difference Between Great-Circle and Mercatorial Bearings. The straight line joining the positions of the ship and the station on a Mercator chart is a rhumb line which cuts all meridians at the same angle. This angle is the mercatorial bearing.

In figure 98, S and W are the positions of the ship and the station on a Mercator chart. The straight line SW is the rhumb line joining them, and the dotted line is the great circle.

The tangents to this great circle at S and W cut at T, and since over short distances (distances, that is, less than 500′) SW approximates to the arc of a circle on the plane of the chart, T is so placed that the angles TSW and TWS are equal.

The angle VTW between the tangents, being the difference between the great-circle bearings at W and S, is the convergency. Therefore:

$$\begin{aligned} c &= \angle TSW + \angle TWS \\ &= 2\angle TSW \end{aligned}$$

Hence the angle between the great-circle bearing ST and the mercatorial bearing SW is equal to half the convergency, and the mercatorial bearing can be obtained by applying half the convergency to the great-circle bearing.

Half-Convergency Solution when the Station Takes the Bearing. Since the great circle always lies on the polar side of the rhumb line joining the same two places, there is no difficulty in deciding the way to apply the half-convergency which, in practice, is obtained from the table in Volume II of the *Admiralty List of Wireless Signals*, or from the ordinary traverse table. (See Chapter IV, Volume I, of this Manual.)

When the half-convergency is applied to the bearing noted by the station, the mercatorial bearing is obtained, and this can be laid off on the chart from the position of the station.

Fig. 99 shows this mercatorial bearing WS cutting the meridian through the D.R. position in S, and if S and W are on the same chart, the position of S is obtained at once. If they are not on the same chart, and a chart that does cover them is not available, the position of S can be found from the formula for rhumb-line sailing:

$$\text{d.m.p.} = \text{d'long} \times \cot \text{(mercatorial bearing)}$$

The ship herself lies on that part of the great circle in the neighbourhood of her D.R. position, and, from what was said in the last section, the great circle makes the same angle with the rhumb line WS at S as it makes at W. The half-convergency must therefore be applied again to give the direction of the great circle at S. The short length on which the ship must lie is then drawn as a straight line.

The following example is the same as that worked by the accurate trigonometrical method on page 193, and a comparison of the results obtained will show that the assumptions made in the approximate formula are justified.

A ship in D.R. position 48°20′*N.*, 10°00′*W.*, *is told that she bears* 240° *from Land's End D/F station, the position of which is* 50°07′·1*N.*,

5°40′·1W. *What is the latitude of the point on the D.R. meridian through which the position line must be drawn, and what is the direction of the position line?*

D.R. long.	10°00′·0W.	D.R. lat.	48°20′·0N.
Station	5°40′·1W.	Station	50°07′·1N.
d'long	4°19′·9	mean lat.	49°13′·5N.

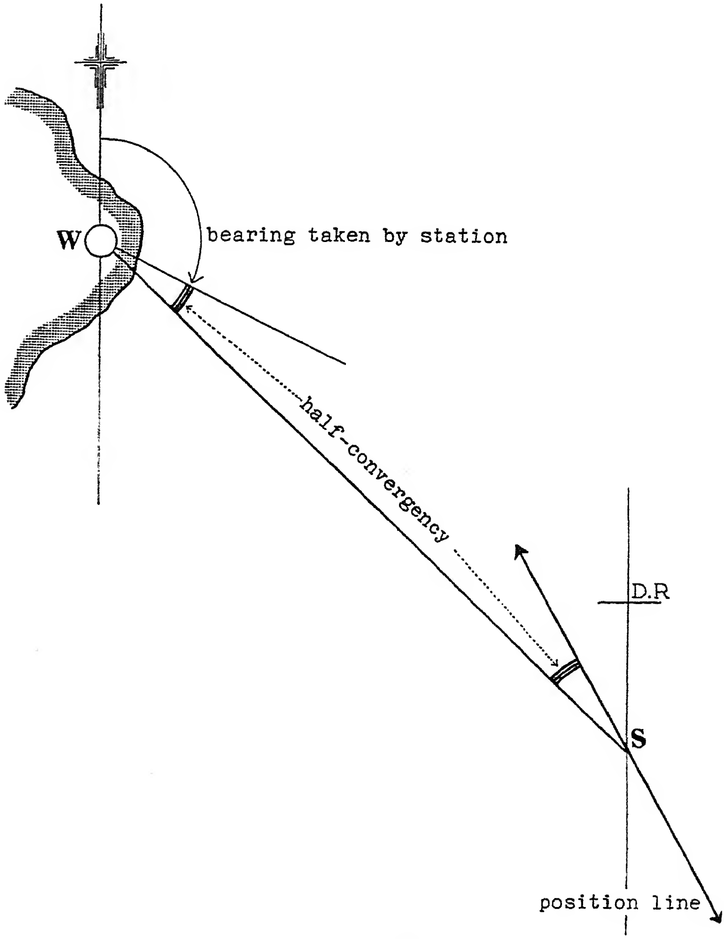


FIGURE 99.

From the table in the *Admiralty List of Wireless Signals*, Volume II, the half-convergency is seen to be 1°·6.

The mercatorial bearing of the ship is therefore (240°—1°·6) or S.58°·4W.

The latitude of the point in which this bearing cuts the meridian of 10°00′W, is given by:

$$\begin{aligned}\text{d.m.p.} &= \text{d'long} \times \cot\ (\text{mercatorial bearing}) \\ &= 259'{\cdot}9 \cot\ (58^\circ{\cdot}4) \\ &= 159'{\cdot}9\end{aligned}$$

The meridional parts of the station latitude are	3485·5
d.m.p.	159·9
∴ the meridional parts of the required position are	3325·6

The latitude in 10°00′W. through which the position line must be drawn is thus 48°22′·7N. By the accurate method, the latitude is 48°22′·5N.

The direction in which the position line must be drawn is given by:

mercatorial bearing of station from ship	N.58°·4E.
half-convergency	1°·6
direction of position line	N.56°·8E.

By the accurate method, this direction is N.56°43′·1E.

In both methods of working the above example, it will be noticed that a point was found on the meridian of the D.R. longitude, and this may be accepted as the procedure to be followed although it is sometimes just as satisfactory to find a point on the parallel through the D.R. latitude.

The assumption that the position line is a straight line means that the position line drawn on the chart is actually a tangent to the great circle. Nevertheless, when the D.R. position is itself close to the ship's true position, this tangent will pass sufficiently close to the true position to avoid practical error, whatever factor is taken—latitude or longitude. If, however, the D.R. position is considerably removed from the true, the tangent will be nearer the true position if the longitude is assumed for bearings near east or west, and there is little point in assuming the latitude when the bearings are near north and south because, when a bearing is near 000° or 180°, the great circle approximates to a straight line on the chart, and the tangent itself will therefore approximate to the great circle over a long distance so that no appreciable error will be introduced even if the longitude is assumed.

When the half-convergency is used, the longitude must be assumed. Hence the assumption of the longitude may be taken as the general procedure.

Difference between the Curve of Constant Bearing and the Mercatorial Bearing. Before a convergency solution of the problem can be found when the ship takes a bearing of the station, it is necessary to establish a connexion between the curve of constant

bearing and the mercatorial bearing, exactly as a connexion was established between the great-circle bearing and the mercatorial bearing.

In figure 100, ϕ and ϕ_0 are the latitudes of the ship's position S, and the station W, and λ is the difference of longitude between S

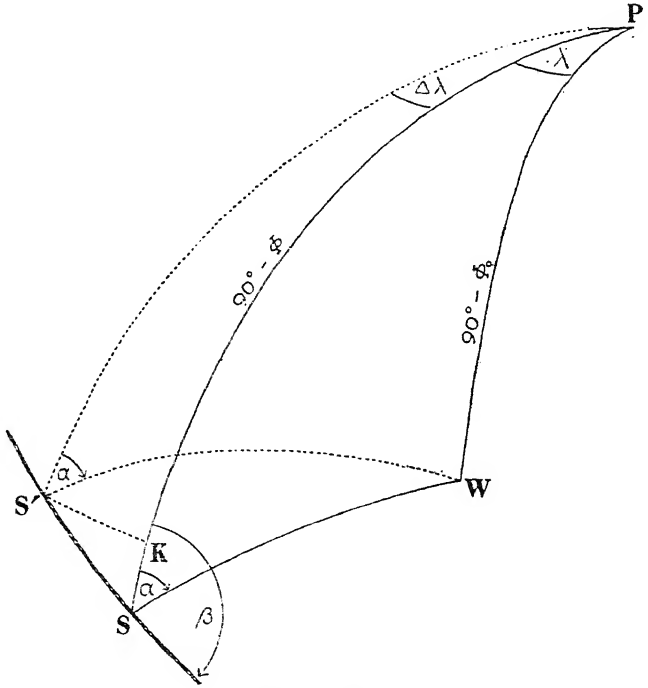


FIGURE 100.

and W. Then, for a bearing α taken of the station by the ship, the position line is given by the four-part formula. Thus:

$$\cos\lambda \sin\phi = \cos\phi \tan\phi_0 - \sin\lambda \cot\alpha$$

i.e.
$$\tan\phi_0 = \cos\lambda \tan\phi + \sin\lambda \cot\alpha \sec\phi \quad . \quad . \quad (10)$$

In this equation α and ϕ_0 are constants, and ϕ and λ are variables.

Let β be the angle that the curve of constant bearing makes with the meridian at the point S. Then, if S' is a close point on the curve corresponding to an increment $\Delta\lambda$ in the d'long, and K is placed so that SK is the increment in latitude equal to $\Delta\phi$, it follows from the geometry of the figure that:

$$\tan(180° - \beta) = \frac{KS'}{KS}$$

$$= \frac{\Delta\lambda \cos\phi}{\Delta\phi}$$

i.e.
$$\tan\beta = -\cos\phi \frac{d\lambda}{d\phi}$$

Equation (10) differentiated with respect to ϕ gives:

$$\sec^2\phi\cos\lambda+\sec\phi\tan\phi\sin\lambda\cot\alpha$$
$$=\frac{d\lambda}{d\phi}(\tan\phi\sin\lambda-\sec\phi\cos\lambda\cot\alpha)$$

Hence it follows that:

$$\tan\beta=-\frac{\sec\phi\cos\lambda+\tan\phi\sin\lambda\cot\alpha}{\tan\phi\sin\lambda-\sec\phi\cos\lambda\cot\alpha}$$
$$=\frac{\tan\alpha+\sin\phi\tan\lambda}{1-\sin\phi\tan\lambda\tan\alpha}$$

or $$\tan(\beta-\alpha)=\sin\phi\tan\lambda$$

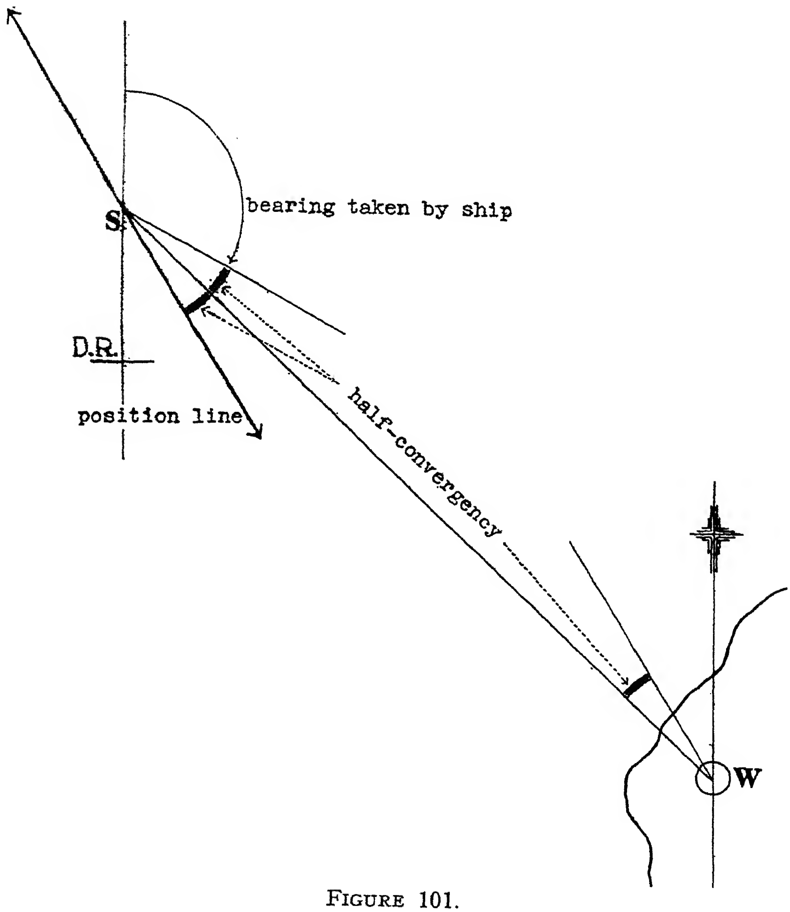


FIGURE 101.

When the distances involved are small, as in the example given on page 194, this expression reduces to:

$$\beta-\alpha=\lambda\sin\phi$$

i.e. $$\beta-\alpha=\text{d'long}\times\sin(\text{lat. } S)$$

This difference is of the same order as the convergency, and is equal to the convergency with the exception that the latitude term involves the latitude of S instead of the mean latitude of S and W, and, as already stated, the error involved in taking the mean latitude of two close points for the latitude of one of them is of no practical importance. The position line that is part of the curve of constant bearing is thus obtained by applying the full convergency to the great-circle bearing taken by the ship, or half the convergency to the mercatorial bearing between the ship and the station, on the equatorial side.

Figure 101 shows the position line thus obtained.

In order to afford a comparison between this method and the trigonometrical, the example on page 193 is repeated.

A ship in D.R. position 48°20′*N.*, 10°00′*W. notes that the D/F bearing of the Land's End station in* 50°07′·1*N.*, 5°40′·1*W. is* 056¾°. *What is the latitude of the point on the D.R. meridian through which the position line must be drawn, and what is the direction of the position line?*

D.R. long.	10°00′·0W.	D.R. lat.	48°20′·0N.
Station	5°40′·1W.	Station	50°07′·1N.
d'long	4°19′·9	mean lat.	49°13′·5N.

From the table in the *Admiralty List of Wireless Signals*, Volume II, the half-convergency is seen to be 1°·6.

The mercatorial bearing of the station is therefore (056°·8+1°·6) or N.58°·4E.

The latitude of the point in which this bearing cuts the meridian of 10°00′W. is given by:

$$\begin{aligned}\text{d.m.p.} &= \text{d'long} \times \cot\text{ (mercatorial bearing)}\\ &= 259'{\cdot}9 \cot (58^\circ{\cdot}4)\\ &= 159'{\cdot}9\end{aligned}$$

The meridional parts of the station latitude are	3485·5
d.m.p.	159·9
∴ the meridional parts of the required position are	3325·6

The latitude in 10°00′W. through which the position line must be drawn is thus 48°22′·7N.

The direction in which the position line must be drawn is given by:

mercatorial bearing of station from ship	N.58°·4E.
half-convergency (on equatorial side)	1°·6
direction of position line	N.60°E.

This agrees with the direction shown in figure 95.

Half-Convergency in Practice. In practice, the half-convergency is usually small over the range of accuracy in the actual D/F bearings, and—as figure 102 shows—it is thus possible to omit the second application of the half-convergency and to take the mercatorial bearing as the position line without introducing an error more serious than that already in the D/F bearing.

In figure 102, the correct position lines obtained from simultaneous D/F bearings of the ship from *A* and *B* are shown in red,

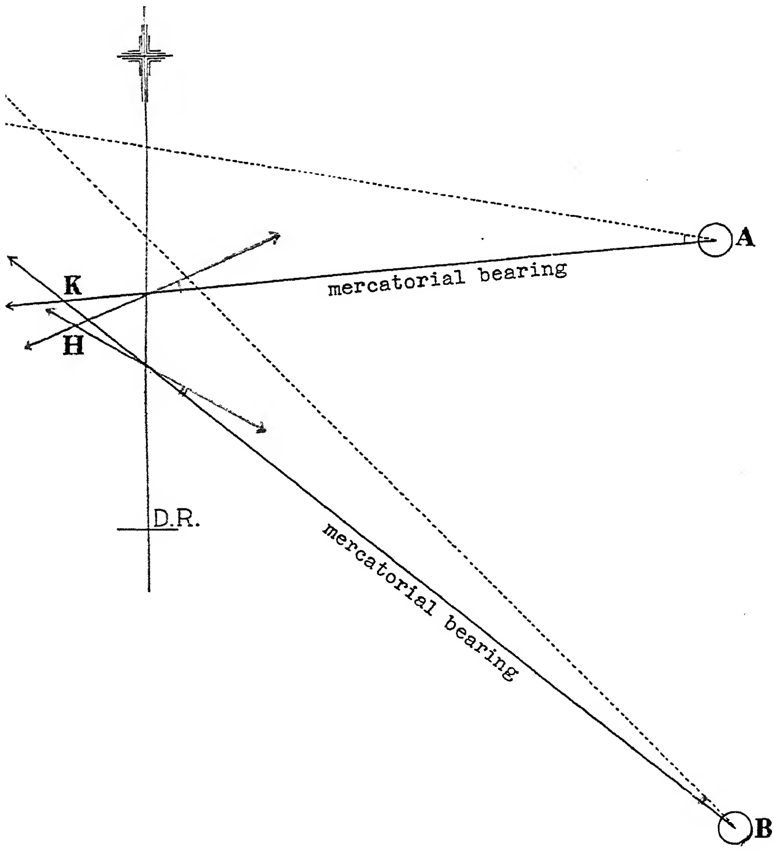


FIGURE 102.

and their intersection at *H* determines the ship's position. The mercatorial bearings, obtained by applying the half-convergency to the bearings taken, cut in *K*, and it is seen that, in spite of the large convergency used for the sake of clarity in drawing, the displacement *HK* is small.

When a bearing is close to 000° or 180°, the position line can always be taken as the mercatorial bearing because, as already stated, the great circle itself approximates to a straight line.

The same omission can be made when the ship takes the bearing, if the convergency is small.

Accuracy of Methods Dealing with D/F Bearings. In both the preceding theory and the examples illustrating it, the distance between the ship and the station has been assumed to be small because the errors in the reception of signals over long distances are at present too great to allow the ship's position to be obtained with the accuracy necessary for safe navigation.

When a method of plotting a D/F bearing is arrived at by making certain approximations that are admissible over short distances only, by which is meant distances less than 100′, that method cannot be used over long distances without introducing an additional error which is independent of the error resulting from inaccurate reception. The method that involves a single application of the half-convergency is such a method. It is entirely adequate over short distances, but it breaks down if used over long distances, as figure 102 shows. On the other hand, the method based on the direct trigonometrical solution of the spherical triangle *PSW* (discussed at the beginning of this chapter) holds for any distance. The working of that method, however, is tedious, and the following 'intercept' method is therefore recommended if the occasion should arise when it is necessary to plot a wireless bearing over distances up to about 1,000′ without loss of accuracy other than that resulting from the unavoidable error in reception.

The Intercept Method of using a D/F Bearing.* This method has the advantage of using the same procedure for plotting a position line from a D/F bearing as that used when a position line is plotted from an astronomical observation; that is, a straight line, corresponding to the line of bearing of the heavenly body, is laid off from the D.R. position, and at right-angles to this line, through a point on it arrived at by calculation, the actual position line is drawn.

Like the position line obtained from an astronomical observation, the position line obtained by this method is independent of the D.R. position within certain limits of error in that position, and it can be transferred for the run of the ship exactly as the ordinary position line can be transferred.

The Intercept Method when the Station takes the Bearing of the Ship. In figure 103, *W* and *S* are the positions of the station and the ship's D.R. The observed bearing of the ship is α_0, and the position line is therefore part of the great-circle arc *WL* which lies in the neighbourhood of *J*. The great-circle bearing of *S* from *W* is α, and the great-circle distance is d.

Then, since the spherical triangle *SJW* is right-angled at *J*:

$$\sin SJ = \sin WS \sin JWS$$

i.e.
$$\sin p = \pm \sin d \sin (\alpha_0 - \alpha)$$

* This method has been devised by Instr. Commander A. J. Low, R.N.

This formula is exact. The distance p, however, is small, and $\sin p$ can therefore be taken as p without loss of accuracy. Also, if the distance of the ship from the station lies within limits to be discussed, $\sin d$ can be taken as d.

The error involved in this second approximation for any given distance and mean latitude is greatest when the D.R. position and the station are on the same parallel, the bearing then being east or west, and least when they are on the same meridian, the bearing then being north or south.

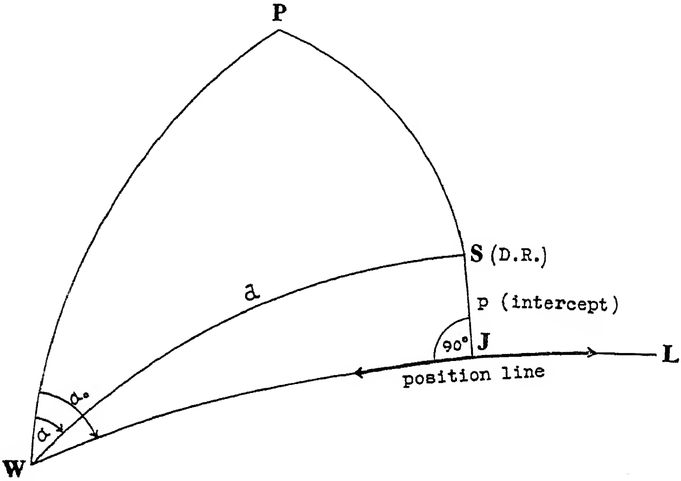


FIGURE 103.

If, for example, the station is 1,000′ from the D.R. position, and the mean latitude is 60°, the greatest percentage error in the intercept is 2·47. If the mean latitude is 30°, the error is 1·53. This percentage error also falls rapidly with the distance. Over 400′ for a mean latitude of 60° it is only 0·39. If the intercept is now taken as 30′, a percentage error of 2·47 in this corresponds to an error of 0°02′·5 in the bearing. In the worst circumstances considered (which are not likely to be encountered in practice) the error in the method arising from the assumption that $\sin d$ is equal to d is negligible in comparison with the probable error in reception, and over distances up to 1,000′ in these circumstances the approximate formula :

$$p = \pm d \sin (a_0 - a)$$

—may be used without appreciable loss of accuracy.

If β is the mercatorial or rhumb-line bearing of the D.R. position from the station, and c is the convergency of the meridians through W and S, then, approximately :

$$\beta = a + \tfrac{1}{2}c$$

—and, by substitution :

$$p = \pm d \sin (\beta_0 - \beta)$$

—where β_0 may, for convenience, be called the mercatorial bearing of the ship from the station, although this is not strictly true because the convergency is calculated for the D.R. position and not for the ship's actual position. The difference, however, is negligible.

To find the length of the intercept, p, it is therefore necessary to know the values of d, β_0 and β. These values can be found by the following steps :

(1) Find the convergency of the meridians from the tables in the *Admiralty List of Wireless Signals*, Volume II, or calculate it from the formula :

$$c = \text{d'long} \times \sin \text{(mean latitude)}$$

(2) Apply half the convergency to the observed bearing, adding it in north latitudes if the bearing is less than 180° and subtracting it if the bearing is greater, and reversing the rule in south latitudes. This gives β_0.

(3) Find from the chart or by calculation the rhumb-line distance d (which approximates to the great-circle distance d over the short distance involved) between the station and the D.R. position, and also the rhumb-line bearing β.

It remains now to find the direction of the intercept, and this can be done by applying the full convergency to the observed bearing, a rule that holds whether the bearing is taken by the station or by the ship, and then drawing the intercept at right-angles to this direction. A comparison between the observed bearing and the D.R. bearing will tell whether 90° should be added or subtracted from the direction of the position line, the rule being : $\beta_0 > \beta$, add 90° ; $\beta_0 < \beta$, subtract 90°.

The error in applying a convergency that is found for the D.R. position and not the ship's actual position can be shown to be negligible in comparison with the probable error in reception. When, for example, the D.R. position is 30′ from the ship's actual position, and the intercept runs east and west in latitude 60°, the error in the position obtained is only 0′·45.

Intercept Method when the Ship takes the Bearing of the Station. When the ship takes the bearing of the station, the ship's position lies on an arc of the curve of constant bearing in the neighbourhood of J, as shown in figure 104.

In this figure the angle PJW, α_0, is the great-circle bearing of the station actually taken from the ship, and the angle PSW, α, is the great-circle bearing of the station from the D.R. position. Then, if ϵ is the small angle that S and J subtend at W, and JW makes an angle θ with the curve of constant bearing at J, from the spherical triangle SWJ :

$$\frac{\sin SJ}{\sin SW} = \frac{\sin SWJ}{\sin SJW}$$

i.e.
$$\sin p = \pm \sin d \sec \theta \sin \epsilon$$

Also, if the convergency between S and W is taken as being equal to that between J and W:

$$\epsilon = a - a_0$$

—and if, as before, β is the mercatorial bearing of the station from the D.R. position and β_0 is the mercatorial bearing of the station

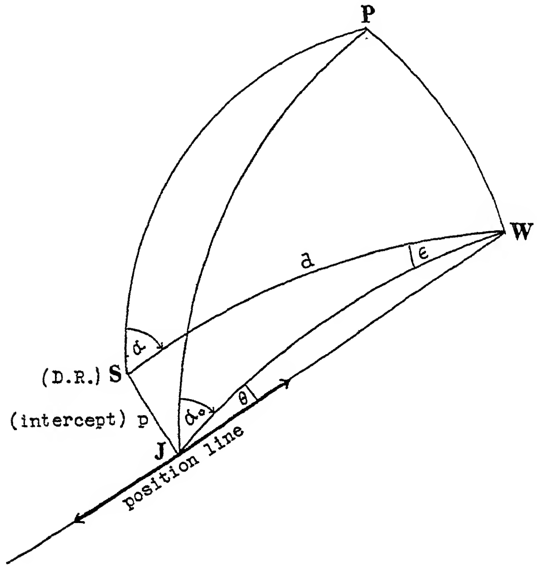


FIGURE 104.

from the ship, obtained by applying half the convergency (calculated for the D.R. position) to the observed bearing:

$$a - a_0 = \beta - \beta_0$$

The formula therefore reduces to:

$$p = \pm d \sec \theta \sin (\beta - \beta_0)$$

—and if sec θ is taken as unity, the formula is the same as that obtained when the station takes the bearing of the ship. This assumption is justified because the error introduced in the intercept is negligible in comparison with the probable error in the reception. When, for example, the distance is 1,000′ and the mean latitude 60°, the greatest percentage error in the intercept is 10·22, corresponding to an error of 0°10′·5 in the bearing, and this falls to 0·12 when the mean latitude is 30°. Also, when the mean latitude is 60° and the distance 400′, the greatest percentage error is only 1·64, corresponding to an error of 0°04′·2 in the bearing.

The same formula thus holds whether the ship takes the bearing of the station or the station takes the bearing of the ship, and the

only difference in procedure lies in the rule that tells whether 90° should be added or subtracted from the direction of the position line. When the ship takes the bearing of the station that rule becomes: $\beta_0 > \beta$, subtract 90°; $\beta_0 < \beta$, add 90°.

A ship in D.R. position 39°50′*N.*, 62°00′*W. takes the bearing,* 044°, *of a W/T station in* 46°40′*N.*, 53°05′*W., and at the same time a D/F station in* 43°35′*N.*, 70°12′*W. takes a bearing,* 121°, *of the ship. Required the ship's position.*

D/F station	43°35′N.	70°12′W.
D.R. position	39°50′N.	62°00′W.
d'lat	3°45′S.	d'long 8°12′E.

The mean latitude is therefore $41\frac{3}{4}$°, and the departure is 367′E. From the traverse table with this data, the distance and bearing of the D.R. position from the D/F station is seen to be 431′, $121\frac{1}{2}$°. Also, by calculation, the convergency is 328′ or $5\frac{1}{2}$°.

To Find the Mercatorial Bearing of the Ship from the D/F station.

Great-circle bearing	121°
Half-convergency	2°·7
(sum)	123°·7

To Find the Length of the Intercept.

Mercatorial bearing of ship	123°·7
Mercatorial bearing of D.R.	121°·5
(difference)	2°·2

$$\text{intercept} = 431' \sin 2°\cdot2$$
$$= 16'\cdot5$$

To Find the Direction of the Intercept.

Great-circle bearing	121°
Whole convergency	5°·5
Direction of pos^{n} line	126°·5
	+90°
Direction of intercept	216°·5

The intercept obtained from the D/F 'sight' is therefore 16′·5 in a direction $216\frac{1}{2}$°.

The same steps applied to the 'sight' of the W/T station give:

W/T station	46°40′N.	53°05′W.
D.R. position	39°50′N.	62°00′W.
d'lat	6°50′N.	d'long 8°55′E.

The mean latitude is therefore $43\frac{1}{4}°$, and the departure is 389′·7E. From the traverse table the distance and bearing of the W/T station from the D.R. position is seen to be 566′, $043\frac{1}{2}°$. The convergency is 6°·1.

Length of Intercept		*Direction of Intercept*	
Great-circle bearing	044°	Great-circle bearing	044°
Half-convergency	3°	Whole convergency	6°
Mercatorial bearing	047°	Direction of pos^n line	050°
D.R. bearing	$043\frac{1}{2}°$		−90°
(difference)	$3\frac{1}{2}°$	Direction of intercept	320°

Distance = 566′
Length = 566′ sin $3\frac{1}{2}°$
= 34′·5

By plotting on the chart or by calculation the ship's position is seen to be:

$$\begin{cases} 40°00'\cdot5\text{N.} \\ 62°54'\cdot5\text{W.} \end{cases}$$

Although the distances in this example are considerable and the D.R. position is 43′ from the position obtained, the method itself introduces no appreciable error. The accurate bearings, calculated from the position obtained, are:

Bearing of position from the D/F station	120°·8
Bearing of the W/T station from position	043°·7

These bearings, it is seen, are well within half a degree of the observed bearings, so that the errors introduced are negligible in comparison with those introduced by the errors in reception.

D/F Bearings on a Gnomonic Chart. On a gnomonic chart, all straight lines represent great circles and therefore the paths of W/T signals, but angles do not preserve their true values except at the tangent point. Bearings from any other point cannot therefore be laid off directly.

In figure 105, W is the station from which the true bearing of a ship S is α. The ship then lies on the great circle WS which appears on the chart as the straight line ws, and the angle PWS is α. Unless W is the tangent point, this angle PWS will not be equal to the angle pws.

Denote the angle Kwp by λ, the angle Kws by μ, and the angle pKw by θ. Then, since K is the tangent point:

$$\angle PKW = \angle pKw = \theta$$

and

$$\angle Kpw = 180° - (\lambda + \theta)$$

Also, since CK is perpendicular to all lines in the tangent plane and is taken as unity:

$$Kw = CK \tan KCw$$

i.e.

$$Kw = \tan KW$$

Similarly

$$Kp = \tan KP$$

From the triangle pKw:

$$\frac{\sin Kpw}{\sin Kwp} = \frac{Kw}{Kp}$$

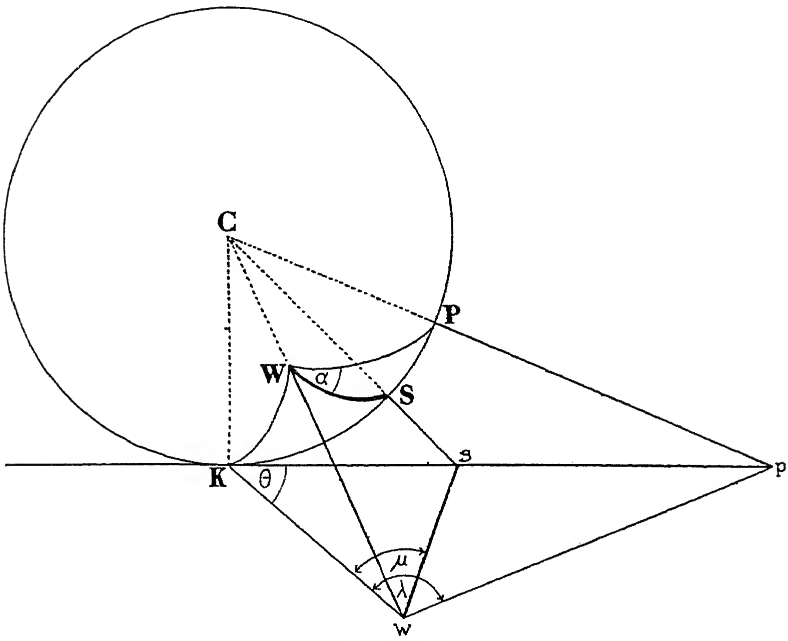


FIGURE 105.

Hence, by substitution:

$$\frac{\sin(\lambda+\theta)}{\sin \lambda} = \frac{\tan KW}{\tan KP}$$

i.e.

$$\sin\theta \cot\lambda = \frac{\tan KW}{\tan KP} - \cos$$

$$= \frac{\sin KW \cos KP - \cos KW \sin KP \cos PKW}{\cos KW \sin KP}$$

$$= \frac{\sin PW \cos PWK}{\cos KW \sin KP}$$

(For the explanation of this step, see the proof of the four-part formula on page 252 of Volume II.)

$$\therefore \qquad \tan\lambda = \frac{\cos KW \sin KP \sin PKW}{\sin PW \cos PWK}$$

i.e.
$$\tan\lambda = \cos KW \tan PWK \quad . \quad . \quad . \quad . \quad (11)$$

Similarly it can be shown that:

$$\tan\mu = \cos KW \tan SWK \quad . \quad . \quad . \quad . \quad . \quad (12)$$

If the true bearing of K from W is denoted by β, it is apparent that the angle SWK is $(\beta-\alpha)$. Also β is a fixed angle for the chart, just as λ is a fixed angle since Kw and pw are fixed lines.

In equation (11), KW and PWK can be calculated from the known latitudes of W and K and the known difference of longitude between W and K, and λ can therefore be found.

On the chart the true bearing α is represented by the angle pws which is $(\lambda-\mu)$ or α'. Hence μ is equal to $(\lambda-\alpha')$, and equation (12) becomes:

$$\tan(\lambda-\alpha') = \cos KW \tan(\beta-\alpha)$$

This relation enables the values of α' to be found corresponding to given values of α.

If a gnomonic chart were specially designed for plotting D/F bearings, a circle would have to be drawn about each station on the chart with the station as centre, and the direction of lines such as ws, corresponding to every degree of α, would have to be shown on the circumference. Thus, if the true bearing of the ship from the station is 50°, the straight line joining the station to the point marked 50° on the circumference of the circle described about the station would give the straight line that represents on the chart the great circle WS, the angle PWS being 50°. This line is the position line, and the part in the neighbourhood of the D.R. position can be transferred to a Mercator chart.

Compass Roses on Gnomonic Charts of Large Areas. From the above considerations, it is seen that:

(1) in order to use a gnomonic chart for plotting D/F bearings, a specially constructed compass rose in which the gradutions are not equally spaced, must be engraved round each station shown on the chart.

(2) compass roses for plotting the D/F bearings of shore stations taken from the ship cannot be engraved since a special rose would be required at the ship's position each time bearings are plotted.

Compass Roses on Gnomonic Charts of Small Areas. Although Admiralty plans and charts with natural scales over 1/50,000 are published as gnomonic charts, the compass roses engraved on the meridians are of the ordinary type with graduations equally spaced. But, since these charts cover only small areas of the Earth's surface, the maximum error in bearing that arises from using these compass roses is inappreciable.

CHAPTER XV

REFRACTION, DIP AND MIRAGE

Optical refraction is the bending of light rays that results from the passage of those rays through media of different densities. Its effect is twofold: a displacement of any heavenly body that is observed, and a 'lifting' of the horizon above which the altitude of the heavenly body is measured. For this reason it is customary to speak of *astronomical refraction* and *terrestrial refraction*.

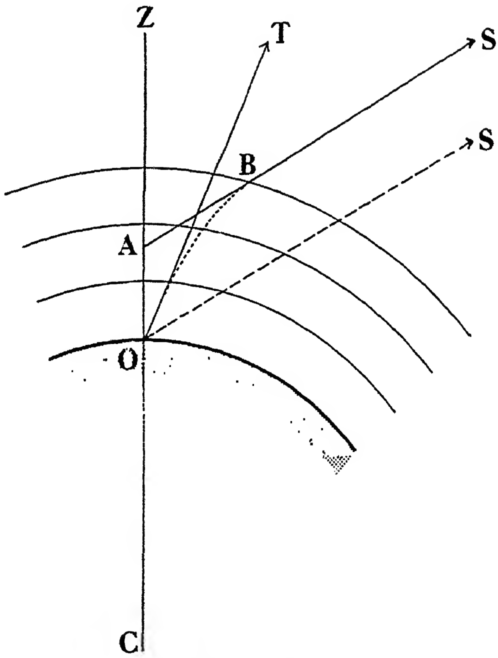


FIGURE 106.

Astronomical Refraction. This is the refraction that results from the passage of the rays of light from a heavenly body through the Earth's atmosphere.

In normal circumstances, which are defined as the state of the atmosphere when the barometer shows 1016 millibars or 30 inches and the temperature is 50°F., the refraction when the observed zenith distance of a heavenly body is ζ is accurately given by the formula:

$$r_0 = 58''{\cdot}29 \tan \zeta - 0''{\cdot}067 \tan^3 \zeta \quad . \quad . \quad . \quad . \quad (1)$$

—and r_0 is called the *mean refraction*.

For example, if the observed zenith distance is 80°, the mean refraction is:

$$58''\cdot29 \tan 80° - 0''\cdot067 \tan^3 80°$$
$$= 5'30''\cdot6 - 12''\cdot2$$
$$= 5'18''$$

Clearly the second term in this formula is negligible for values of ζ less than 70°. Hence, for ordinary altitudes, the mean refraction is given by:

$$r_0 = 58''\cdot3 \tan \zeta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In figure 106, the atmosphere is shown as a series of layers surrounding the Earth, the air in each layer being considered

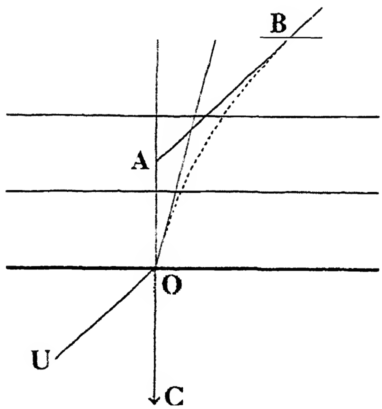


FIGURE 107.

uniform. Since the density of the air in the layers decreases with the distance of the layers from the Earth, the path of a ray from the star S is shown by the line SBO, where the part BO in the Earth's atmosphere curves as shown. The tangent OT to this curve gives the direction in which an observer at O sees the star, but the true direction of the star is a line OS parallel to ABS. The angle ZOS therefore measures the true zenith distance, and the angle ZOT the apparent zenith distance. The angle of refraction r_0, given by formula (1) in normal circumstances, is thus TOS.

Derivation of the Refraction Formulæ. When the zenith distance is small, the layers of air are approximately horizontal, and formula (2) is easily established. Figure 107 shows the layers thus arranged.

The apparent zenith distance of T is the angle ZOT. At B, the ray passes from the vacuum of space into the Earth's atmosphere and is refracted, and if it could emerge at O into a similar vacuum, then, whatever the variation in the density and temperature of the air in the intervening layers, SB and OU would be parallel. That is, the angle ZAS would be equal to the angle COU, and each would be equal to $(\zeta+r)$ where ζ is the observed zenith distance and r the angle of refraction.

If μ is the refractive index of the air at O on the Earth's surface :

$$\sin COU=\mu \sin ZOT$$

i.e. $$\sin (\zeta+r)=\mu \sin \zeta$$

Since r is small, this equation may be written :

$$\sin \zeta+r \cos \zeta=\mu \sin \zeta$$

i.e. $$r=(\mu-1) \tan \zeta$$

or $$r=A \tan \zeta$$

—where A is a constant depending on μ. When r is expressed in seconds, A is found by observation to be $58''\cdot 3$.

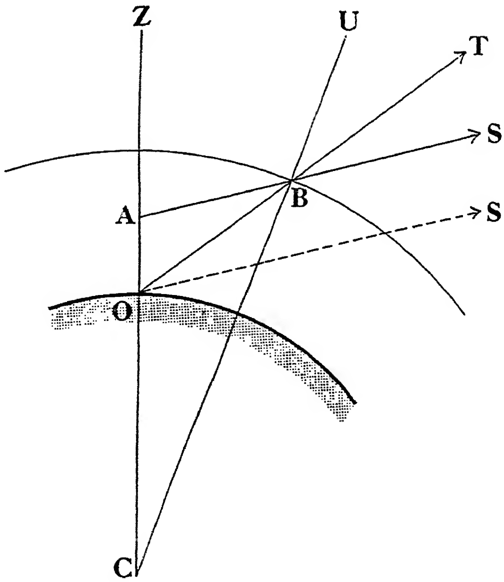


FIGURE 108.

When the zenith distance is large, the spherical shape of the atmosphere must be considered, and Cassini's assumption adopted. This assumption replaces the Earth's varying atmosphere by an equivalent shell of uniform density and temperature, chosen so that a ray passing through it is refracted by the same amount as it is when passing through the actual atmosphere. Figure 108 shows this equivalent shell.

SB is the ray that is refracted at the surface of the shell so as to pass along BO. The apparent zenith distance ζ is the angle ZOT. OS is the true direction of the star. Then the angles TOS and TBS are each equal to r, and if the angle OBC (or UBT) is denoted by ϕ:

$$UBS=\phi+r$$

If μ is the refractive index for the shell:

$$\sin UBS=\mu \sin OBC$$

i.e.

$$\sin (\phi+r)=\mu \sin \phi$$

Since r is small, this may be written:

$$r=(\mu-1) \tan \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From the triangle OBC:

$$\frac{\sin BOC}{\sin OBC}=\frac{BC}{OC}=\frac{R+H}{R}$$

—where H is the thickness of the shell and R is the radius of the Earth. But:

$$\sin BOC=\sin ZOB=\sin \zeta$$

$$\therefore \qquad \sin \zeta=\left(1+\frac{H}{R}\right) \sin \phi$$

i.e.

$$\sin \phi=\frac{\sin \zeta}{1+n}$$

—where n denotes the ratio of H to R and is therefore small.

By squaring and subtracting each side from unity:

$$\cos^2 \phi=\frac{(1+n)^2-\sin^2 \zeta}{(1+n)^2}$$

$$=\frac{\cos^2 \zeta+2n+n^2}{(1+n)^2}$$

i.e.

$$\cos \phi=\frac{\cos \zeta\, (1+2n \sec^2 \zeta)^{\frac{1}{2}}}{(1+n)}$$

—since the term in n^2 may be neglected. Hence, by combining these equations giving $\sin \phi$ and $\cos \phi$:

$$\tan \phi=\tan \zeta\, (1+2n \sec^2 \zeta)^{-\frac{1}{2}}$$
$$=\tan \zeta\, (1-n \sec^2 \zeta)$$

—the binomial theorem being used to expand the right-hand side and the squares and higher powers of n being omitted. That is:

$$\tan \phi=\tan \zeta\, [1-n(1+\tan^2 \zeta)]$$
$$=(1-n) \tan \zeta-n \tan^3 \zeta$$

Equation (3) therefore becomes:

$$r=A \tan \zeta-B \tan^3 \zeta$$

—where A and B depend on μ and n. Observations of the Sun and the stars show that their values are, in normal circumstances, $58''{\cdot}29$ and $0''{\cdot}067$ respectively.

Effect of Atmospheric Pressure and Temperature on Refraction. The amount of refraction depends on the density of the air, and that, in turn, depends on the atmospheric pressure and temperature. The exact effect of these two factors is still unknown, but for zenith distances not exceeding 80°, and for atmospheric conditions that are not extreme, r may be found with sufficient accuracy from the formula :

$$r=\frac{17\,b}{460+t}\times r_0$$

—where b is the barometric pressure in inches, t the temperature in degrees Fahrenheit, and r_0 the mean refraction.

If, for example, the observed zenith distance is 80°, the barometer 31 inches, and the temperature 30°F., the mean refraction is 5′18″ or 318″ and :

$$\begin{aligned} r&=\frac{17\times 31}{460+30}\times 318'' \\ &=342'' \\ &=5'42'' \end{aligned}$$

When b is measured in millibars instead of inches, as it must be if the latest type of barometer is used, the formula becomes :

$$r=\frac{17b}{460+t}\times\frac{r_0}{33{\cdot}86}$$

In practice, the refraction corresponding to known barometer and thermometer readings is found by applying a correction to the mean refraction for the particular altitude, the correction being taken out (*Inman's Tables*) in two parts.

Terrestrial Refraction. This is the refraction that results from the passage of the rays of light through pockets of air of different densities adjacent to the Earth's surface. In normal circumstances it will be negligible, but if the temperature of the water above which the rays pass is not uniform, the air in contact with the water is then heated differently ; its density varies, and there is refraction, the usual effect of which is a 'lifting' of the horizon.

In figure 109, O is the observer and B the terrestrial object observed. BO is the path of the ray by which the observer sees the object, and, since it is curved as shown, he sees the object in the direction OT where OT is a tangent to BO. The angle TOB is thus the angle of refraction.

If B and O are close to the Earth's surface, the curve BO approximates to an arc of a circle, and, if BT is the tangent at B, the angles TBO and TOB are equal. In normal circumstances the angle TOB is, by Biot's law, equal to about one-thirteenth of the angle

BCO, and this latter angle, when expressed in minutes of arc, is the number of nautical miles between O' and B'. Therefore :

$$\text{terrestrial refraction} = \frac{\text{distance of object from observer}}{13}$$

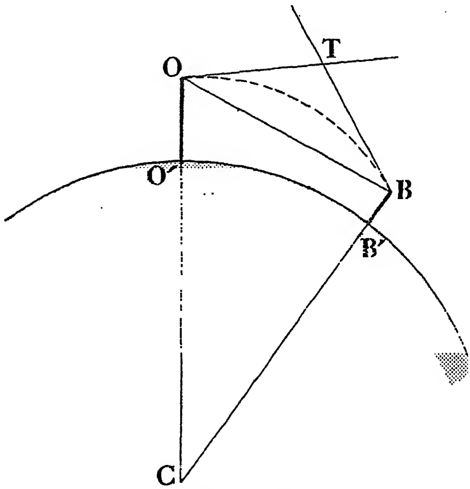


FIGURE 109.

Formulæ for the Dip and Distance of the Sea-Horizon. Dip is defined as the angle between the horizontal plane through the observer's eye and the direction of the horizon.

(1) *When refraction is neglected.*

If there is no refraction, the horizon is fixed by the tangent from the observer to the surface of the sea.

In figure 110, the observer O is at a height h above sea-level. HOC is a vertical plane through the observer, and T marks the horizon. The angle HOT is therefore the dip, and the angle VCT is the distance of the horizon.

Denote the dip by θ, the distance by d, and the Earth's radius by R. Then, since the angles HOC and OTC are each 90° :

$$\angle TOC + \angle TCO = 90° = \angle TOC + \angle HOT$$

i.e.
$$d = \angle TCO = \angle HOT = \theta$$

That is, the distance of the sea-horizon is equal to the true dip. Also, from the triangle TOC :

$$\cos\theta = \frac{R}{R+h}$$

i.e.
$$1 - 2\sin^2\frac{\theta}{2} = \frac{(R+h)-h}{R+h} = 1 - \frac{h}{R+h}$$

∴
$$\sin\frac{\theta}{2} = \sqrt{\frac{h}{2(R+h)}}$$

The radius of the Earth is 3,438 nautical miles, or 3,438×6,080 feet. Also h, in practice, is not likely to exceed 100 feet, and is therefore small compared with R. The formula can thus be written :

$$\sin\frac{\theta}{2}=\sqrt{\frac{h}{2R}}=\sqrt{\frac{h}{2\times 3{,}438\times 6{,}080}}$$

When θ is expressed in minutes of arc :

$$\frac{\theta}{2}\sin 1'=\sqrt{\frac{h}{6{,}876\times 6{,}080}}$$

i.e.
$$\theta=6{,}876\sqrt{\frac{h}{6{,}876\times 6{,}080}}$$
$$=1{\cdot}06\sqrt{h}$$

The true dip in minutes is thus equal to the distance of the sea-horizon in nautical miles, each being equal to a little more than the square root of the observer's height of eye in feet.

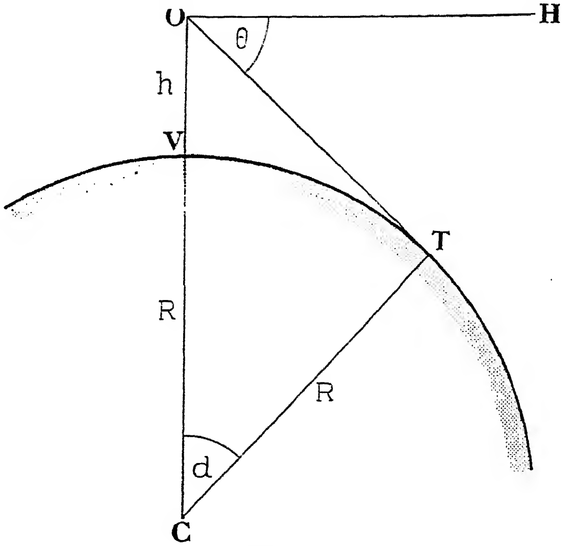


FIGURE 110.

(2) *When allowance is made for refraction.*

The observer now sees points on the surface of the sea that lie beyond the horizon fixed by the tangent from him to the surface.

In figure 111, OT' is the curved path of the ray by which the observer sees T'. It touches the surface at T', and OD and $T'D$ are tangents intersecting in D. The angle HOD is the dip, θ, and the angle VOT' measures the distance, d.

Since $T'D$ is a tangent to a curve which touches the surface of the sea, it is also a tangent to that surface. The angle $DT'C$ is therefore 90°.

The angle of refraction, r, is DOT', and this is assumed to be equal to the angle $DT'O$. By Biot's law, r is equal to one-thirteenth of the angle subtended by O and T' at the Earth's centre. That is :

$$r=\frac{1}{13}d$$

By the rule of sines :

$$\frac{R}{R+h}=\frac{\sin COT'}{\sin OT'C}$$
$$=\frac{\sin(180°-d-90°+r)}{\sin(90°-r)}$$
$$=\frac{\cos(d-r)}{\cos r}$$
$$=\cos d+\sin d\tan r$$

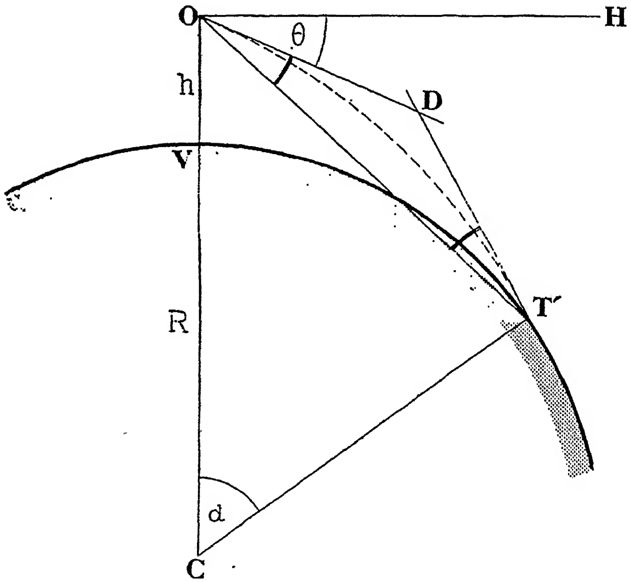


FIGURE 111.

When r and d, which are both small, are expressed in circular measure, this equation becomes :

$$\frac{R}{R+h}=1-\frac{d^2}{2}+rd$$

i.e.

$$\frac{d^2}{2}-\frac{d^2}{13}=1-\frac{R}{R+h}$$

or

$$\frac{11\,d^2}{26}=\frac{h}{R}$$

$\therefore$

$$d=\sqrt{\frac{26\,h}{11\,R}}$$

The distance of the sea-horizon in nautical miles is thus given by:

$$d \sin 1' = \sqrt{\frac{26\,h}{11 \times 3{,}438 \times 6{,}080}}$$

i.e. $$d = 1{\cdot}15\sqrt{h}$$

—where h is the observer's height of eye in feet.

When, for example, h is 100 feet, the distance of the sea-horizon is $11'{\cdot}5$.

The dip, θ, is the angle HOD, and is given in terms of d and r by the relation between the angles of the triangle OCT'. Thus:

$$\angle OCT' + \angle CT'O + \angle T'OC = 180^\circ$$

i.e. $$d + (90^\circ - r) + (90^\circ - \theta - r) = 180^\circ$$

or $$\theta = d - 2r$$

Hence, by substitution:

$$\theta = \frac{11}{13}d = {\cdot}98\sqrt{h}$$

If h is not greater than 100 feet, the formula may be stated thus: the dip in minutes of arc is equal to the square root of the height of eye in feet.

Influence of Temperature-Difference on Dip. When the difference between the air temperature (t_a) and the sea temperature (t_s) is taken into account, the formula for the dip becomes:

$$\theta = A\sqrt{h} - B(t_a - t_s)$$

—where A and B are constants, the exact values of which are still the subject of investigation. The formula does, however, show that the influence of a temperature-difference increases as the height of eye decreases, and that small heights of eye should be avoided in the ordinary practice of navigation.

Abnormal Refraction. Most of the uncertainty in the value of a position line obtained from an observation of a heavenly body, arises from the uncertainty in the value of the dip that results from abnormal refraction.

The amount of refraction at any time depends upon the change of the refractive index along the path of the ray, and when normal atmospheric conditions prevail and the density of the air steadily diminishes with the height above sea-level, a ray of light from an object at sea-level inclines more and more to the horizontal as it proceeds obliquely upward. But if the difference between the sea temperature and the air temperature is such that the density of the air does not decrease uniformly with the height, the ray of light is no longer steadily refracted and errors are at once introduced into the quantities given in the dip tables which are based on a normal refraction.

Abnormal refraction may be experienced anywhere, but it is more prevalent in the Red Sea and the Persian Gulf, off the West

Coast of Africa and near the edges of the Gulf Stream, than it is elsewhere. If it is suspected, the measurement of a heavenly body's altitude above the back horizon, 180° in azimuth from the point on the horizon above which the proper altitude is taken, will afford a check. The difference between the sum of these altitudes and 180° is the sum of the dip and refraction to the horizon in the two directions. If there is reason to believe that the abnormality of the atmosphere is the same in both directions, this sum is twice the dip and refraction to the horizon in either direction.

No definite rules can be laid down for dealing with abnormal refraction, and a sight taken when abnormal refraction is suspected should be used with the utmost caution.

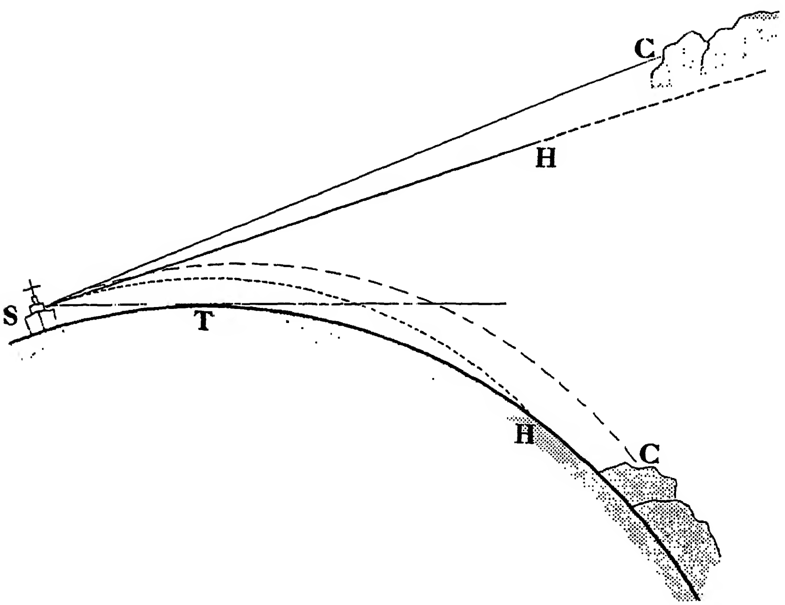


FIGURE 112.

Mirage. This term is used to cover all optical phenomena resulting from abnormal refraction.

As already seen, the passage of warm air over relatively cold water increases the distance of the horizon. It may also produce the phenomenon of *looming*. Outstanding objects, such as buildings and cliffs, that are actually well below the apparent horizon, are then faintly visible.

When the coast is seen in this way, as shown in figure 112, the phenomenon is known as *the loom of the land*.

If cold air passes over relatively warm water, as frequently happens in the region of the Gulf Stream, the density of the air increases with the height for some distance, and rays are bent so that they are convex to the surface of the sea.

The apparent horizon is then lowered, and a distant object is seen inverted in the sky, as shown in figure 113.

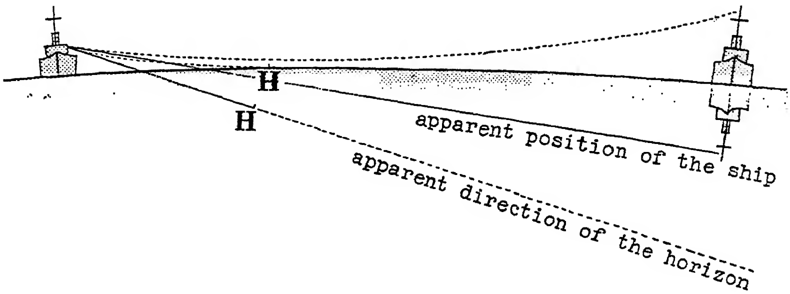


FIGURE 113.

Dip of a Shore-Horizon or a Ship's Waterline. When haze obscures the sea-horizon or the Sun is over the land, a navigator can obtain a true altitude by measuring the altitude above the shore-horizon or the waterline of a conveniently placed ship and applying a special dip.

In figure 114, S is the shore-horizon or ship's waterline, distant d nautical miles from V. Then, in the usual notation, from the triangle COS:

$$\frac{R+h}{R}=\frac{\sin CSO}{\sin COS}$$

$$=\frac{\sin\,[180°-d-(90°-r-\theta)]}{\sin\,(90°-r-\theta)}$$

i.e.
$$1+\frac{h}{R}=\frac{\cos\,(r+\theta-d)}{\cos\,(r+\theta)}$$

or
$$\frac{h}{R}=\frac{\cos\,(r+\theta-d)-\cos\,(r+\theta)}{\cos\,(r+\theta)}$$

$$=\frac{2\sin\frac{d}{2}\sin\,(r+\theta-\frac{d}{2})}{\cos\,(r+\theta)}$$

Since r, θ and d are small angles:

$$\sin\frac{d}{2}=\frac{d}{2}\sin 1'=\frac{d}{2\times 3{,}438}$$

$$\sin\left(r+\theta-\frac{d}{2}\right)=\frac{r+\theta-\frac{d}{2}}{3{,}}$$

$$\cos\,(r+\theta)=1$$

By substitution:

$$d\left(r+\theta-\frac{d}{2}\right)=(3{,}438)^2\frac{h}{R}$$

Also, by Biot's law, r is one-thirteenth of d. Hence :

$$d\left(\theta-\frac{11}{26}d\right)=\frac{(3{,}438)^2\,h}{3{,}438\times6{,}080}$$

i.e.

$$\theta=0{\cdot}565\frac{h}{d}+0{\cdot}423d$$

—where h is in feet and d is in nautical miles.

Inman's Tables give values of θ for distances up to 2,000 yards and heights of eye up to 40 feet, but these values must

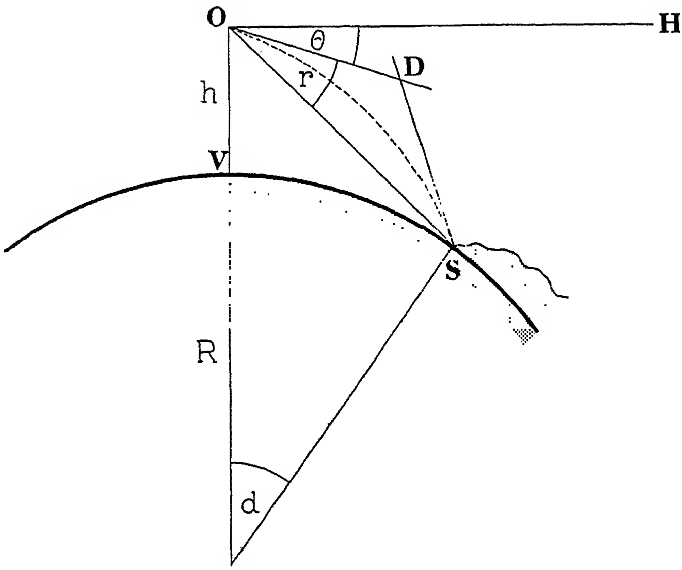


FIGURE 114.

be regarded as approximate if the refraction is known to be abnormal.

It should be clear that, when a ship's waterline is used as the horizon, the ship herself must be on the same bearing as the heavenly body observed and a course at right-angles to this bearing; also that the altitude observed must be corrected separately for the calculated dip, the refraction, semi-diameter and parallax.

The following example shows the procedure :

At Z.T. 0800(+2), *3rd April* 1937, *the D.R. position was* 34°37′*N.*, 28°41′*W.*, *and the deck watch showed* $10^h01^m05^s$ *when the sextant altitude of the Sun's lower limb above the waterline of a ship* 1800 *yards distant was* 28°11′·4. *The deck watch was* 31^s *slow on G.M.T.;*

the index error was —2′·6, and the height of eye 32 feet. Find the position line.

Z.T.	0800 3rd April		Dec.	5°13′·6N.
Zone	+2		E	$11^h56^m34^s$
G.D.	1000 3rd April			
	h m s			
D.W.T.	10 01 05		Sext. Alt.	28°11′·4
Error slow	31		I.E.	—2′·6
G.M.T.	10 01 36	3rd April	Obs. Alt.	28°08′·8
E	11 56 34		Dip (*Inman's*)	—20′·7
G.H.A.	21 58 10			27°48′·1
	=329°32′·5		Refraction	—1′·8
Long. W.	28°32′·5			27°46′·3
H.A.T.S.	301°W. or 59°E.		Semi-diameter	+16′·0
				28°02′·3
			Parallax	+0′·1
			True Alt.	28°02′·4

From the Altitude-Azimuth Tables for latitude 35°N., declination 5°N., and hour angle 59° :

Altitude	28°03′·2	$\triangle d$ 61	Az. N.104°·6 E.
Corrⁿ for 13′·6	+8′·3		
Calc. Alt.	28°11′·5		
True Alt.	28°02′·4		
Intercept	9′·1 away		

The position line can now be laid off in the usual way from 35°00′ N., 28°32′·5 W.

Measurement of Ranges by Shore-Horizon Dip. If the angle between the waterline of a ship and the sea-horizon is measured by a sextant, it is possible to obtain the approximate range of the ship from the observer because the dip of the sea-horizon can be found from tables, and this, added to the sextant angle obtained, gives θ in the formula :

$$\theta = 0{\cdot}565\frac{h}{d} + 0{\cdot}423\,d$$

Either this can be solved for d, or d can be found by interpolation from the tables giving the dip of the shore-horizon.

An observer's height of eye, for example, is 100′, and the sextant angle between the ship's waterline and the horizon is 3′·6.

From the tables, the dip of the sea-horizon is 9′·8. The dip of the ship's waterline is therefore 13′·4. That is :

$$13'\cdot4=\frac{56\cdot5}{l}+0\cdot423d$$

or

$$l=5'$$

This method should give the range to within quarter of a mile, and the greater the height of eye, the greater should be the accuracy, since the base line is the vertical line from the sea to the observer.

The method can also be used for finding the distance of a rock or flat island if the sea-horizon is visible beyond.

Distance of a Mountain. If two points are taken equal in height h above sea-level—O and O' in figure 115—the angle between

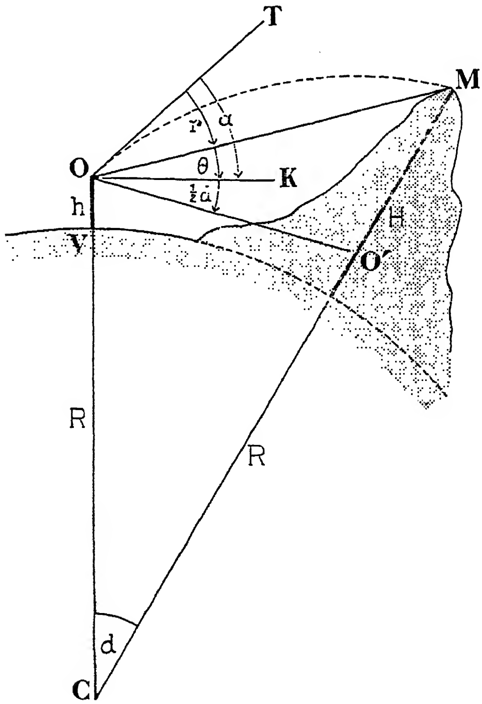


FIGURE 115.

the line joining them and the horizontal through O is equal to half the angle subtended at the Earth's centre.

The angle KOO' in minutes is thus half the distance between the two points in nautical miles, and if θ is the observed altitude

of the summit M diminished by the dip of the sea-horizon (that is, the angle between OM and the horizontal OK), for practical purposes the distance is given by the equation :

$$d=H \cot (\theta+\tfrac{1}{2}d)$$

—where H is the height of the mountain.

This formula, however, is built on the supposition that the triangle MOO' is right-angled at O', and it does not allow for the effect of refraction on the ray that passes from M to O. Normally this ray is convex to OM and the apparent direction of the summit is OT, so that the angle of refraction r is TOM. Hence, by Biot's law :

$$\begin{aligned}\angle MOO' &= \angle TOK+\angle KOO'-\angle TOM \\ &=a+\frac{1}{2}d-\frac{1}{13}d \\ &=a-\frac{11}{26}d\end{aligned}$$

—where a is the observed altitude less the dip.

In Lecky's *Off-Shore Distance Tables* the allowance for refraction is taken as one-twelfth of the approximate distance. The angle MOO' is then given by :

$$a+\frac{1}{2}d-\frac{1}{12}d$$

—so that the correction, in minutes of arc, to the observed altitude above the horizontal is given by :

$$\frac{5}{12}\times(\text{distance in miles})$$

In seconds of arc, this correction is :

$$\frac{5}{12}\times 60\times(\text{distance in miles})$$

or

$$\frac{100}{4}\times(\text{distance in miles})$$

—which is the formula used for correcting a theodolite angle of elevation when the height of a hill is measured.

From a ship at anchor in position 16°02′·4*S*., 51°27′·3*E*., *the sextant altitude of a mountain top above the shore-horizon is* 1°47′20″. *The mountain is marked on the chart, but no height is given. The distance of the mountain top from the ship, measured on the chart in minutes of latitude, is* 17·83, *and the distance of the shore-horizon is* 1,440 *yards. The observer's height of eye is* 35 *feet ; the index error*

is 0′30″ (minus), and the centering error is nil. What is the approximate height of the mountain?

Sextant Altitude	1°47′20″
Index Error	—0′30″
Observed Altitude	1°46′50″
Dip of Shore Horizon	—28′08″
	1°18′42″
$\frac{100}{4}\times 17''{\cdot}83$	7′26″
True Altitude	1°26′08″

In latitude 16°S. the length of 1′ along a meridian is 6,054·4 feet. The height of the mountain in feet is therefore given approximately by:

$$\begin{aligned}&35+17{\cdot}83\times 6{,}054{\cdot}4\times\tan 1°26'08''\\ =&35+2{,}703{\cdot}5\\ =&2{,}738{\cdot}5\end{aligned}$$

The height of the mountain is thus about 2,750 feet.

A more accurate formula giving the height of a mountain can be found by considering the geometry of the triangle MOC. Thus:

$$\begin{aligned}\frac{R+h}{R+H}&=\frac{\sin CMO}{\sin COM}\\ &=\frac{\sin\,[180°-d-(90°+\alpha-r)]}{\sin\,(90°+\alpha-r)}\\ &=\frac{\cos\,(d+\alpha-r)}{\cos\,(\alpha-r)}\end{aligned}$$

i.e.
$$\cos\,(d+\alpha-r)=\frac{R+h}{R+H}\cos\,(\alpha-r)$$

In logarithmic form this may be written:

$$\begin{aligned}\log_{10}\cos\,(d+\alpha-r)&=\log_{10}\cos\,(\alpha-r)+\log_{10}\frac{1+\frac{h}{R}}{1+\frac{H}{R}}\\ &=\log_{10}\cos\,(\alpha-r)-\frac{\log_e\left(1+\frac{H}{R}\right)-\log_e\left(1-\frac{h}{R}\right)}{\log_e 10}\\ &=\log_{10}\cos\,(\alpha-r)-\frac{H-h}{R\log_e 10}\ \text{(approximately)}\end{aligned}$$

If x denotes the last term:

$$\begin{aligned}\log_{10}x&=\log_{10}(H-h)-\log_{10}(R\log_e 10)\\ &=\log_{10}(H-h)-\log_{10}(20{,}890{,}550\times 2{\cdot}30285)\\ &=\log_{10}(H-h)-7{\cdot}682215\end{aligned}$$

The formula therefore reduces to :

$$\log \cos_{10} (d+a-r) = \log \cos_{10} (a-r) - x$$

—where :

$$\log_{10} x = \log_{10} (H-h) - 7{\cdot}682215$$

The sextant altitude of a mountain peak, for example, is 1°01′·5. The peak is 10,000 feet, and the observer's height of eye is 50 feet. The index error is −1′·5. Hence :

Sext. Altitude	1°01′·5
I.E.	−1′·5
Obs. Altitude	1°00′·0
Dip	−7′·0
Corrected Alt.	0°53′·0

The substitution of the heights in the formula for x gives :

$$\begin{aligned} \log x_{10} &= \log_{10} (H-h) - 7{\cdot}682215 \\ &= \log_{10} 9{,}950 \quad - 7{\cdot}682215 \\ &= 3{\cdot}997823 \quad - 7{\cdot}682215 \\ &= \bar{4}{\cdot}315608 \end{aligned}$$

$$\therefore \qquad x = {\cdot}000207$$

Before the refraction can be found by Biot's law, an approximate distance must be calculated, and this can be done with sufficient accuracy by neglecting r in the formula :

$$\log_{10} \cos (d+a-r) = \log_{10} \cos (a-r) - x$$

i.e.

$$\begin{aligned} \log_{10} \cos (d+0°53') &= \log_{10} \cos 0°53' - x \\ &= \bar{1}{\cdot}999948 - {\cdot}000207 \\ &= \bar{1}{\cdot}999741 \end{aligned}$$

$$\therefore \qquad d + 0°53' = 1°58'{\cdot}75$$

or

$$d = 65'{\cdot}75$$

When this approximate distance is divided by 13, the refraction is seen to be 5′. The true altitude, $(a-r)$, is therefore 48′, and :

$$\begin{aligned} \log_{10} \cos (d+48') &= \log_{10} \cos 48' - x \\ &= \bar{1}{\cdot}999958 - {\cdot}000207 \\ &= \bar{1}{\cdot}999751 \end{aligned}$$

i.e.

$$d + 48' = 1°56'{\cdot}5$$

or

$$d = 68'{\cdot}5$$

This is the required distance.

When the distance of a mountain has been accurately found, the position of the ship can be fixed by means of a bearing of the mountain and this distance, but a small complication is introduced if the mountain is a considerable distance from the ship because the bearing observed is a true bearing, and before this can be laid off from the position of the mountain on a Mercator chart, it must be converted into a mercatorial bearing by the addition or subtraction of half the convergency.

CHAPTER XVI

RISING AND SETTING PROBLEMS

Times of visible sunrise and sunset are obtained in practice from the tables given in the abridged edition of the *Nautical Almanac* by the methods described in Chapter XXII of Volume II, but they can also be obtained by what are known as amplitude tables and a method involving the rate of change in altitude of the heavenly body.

Amplitudes. At theoretical rising and setting the difference between 90° and the azimuth is called the *bearing amplitude*, and that between 6^h and the hour angle is called the *time amplitude*.

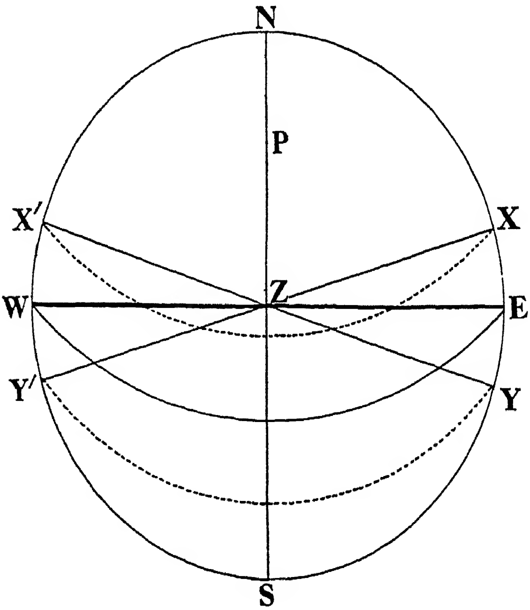


FIGURE 116.

In figures 116 and 117, X and Y are heavenly bodies of equal declinations but opposite names.

Bearing Amplitudes. The angle XZE—figure 116—is the bearing amplitude of the body X at rising, and YZE that of Y at rising. But, since X is just as far north of the equator as Y is south, these two angles are equal.

The azimuth of the body is therefore ($90° \mp$ amplitude) according as the latitude and declination have the same or opposite names.

The same rule holds when the body is setting.

Time Amplitudes. The angle *XPE*—figure 117—is the time amplitude of the body *X* at rising, and the angle *YPE* that of *Y* at rising. As before, these two angles are equal.

The hour angle of the body at rising is therefore ($18^h \mp$ amplitude) according as the latitude and declination have the same or opposite names.

At setting, the hour angle is ($6^h \pm$ amplitude) according as the latitude and declination have the same or opposite names.

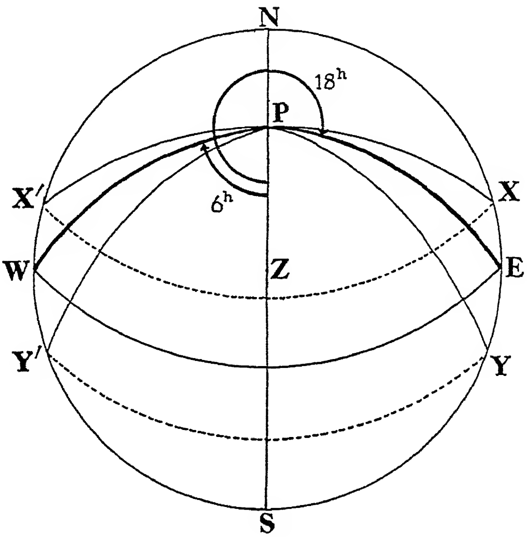


FIGURE 117.

A time amplitude, it will be noted, gives the hour angle of the heavenly body at rising or setting, and before that amplitude can be taken from the table, the latitude of the observer and the declination of the heavenly body at the instant of the phenomenon must be known if the amplitude is to be accurate. But the rapidly changing declination of the Moon, for example, cannot be found until the G.M.T. of the phenomenon has been found. The use of these tables therefore demands the same methods of approximation as are employed in finding the time of the Moon's meridian passage. An approximate declination must be taken and an approximate G.M.T. deduced, and from the declination taken from the almanac for that G.M.T. a more accurate amplitude can be found.

Amplitude Tables. *Inman's Tables* include time and bearing amplitudes for latitudes up to 64° and declinations up to 34°.

Suppose the latitude is 47° and the declination 18°. The tables give a time amplitude of 1^h22^m, and a bearing amplitude of 27°, and these figures apply to any heavenly body.

(1) *Latitude* 47°*N.*, *declination* 18°*S.*

At rising	H.A.	=	18^h	+	1^h22^m	= 19^h22^m
	Azimuth	=	90°	+	27°	= N.117°E.
At setting	H.A.	=	6^h	−	1^h22^m	= 4^h38^m
	Azimuth	=	90°	+	27°	= N.117°W.

(2) *Latitude* 47°*S.*, *declination* 18°*N.*

At rising	H.A.	=	18^h	+	1^h22^m	= 19^h22^m
	Azimuth	=	90°	+	27°	= S.117°E.
At setting	H.A.	=	6^h	−	1^h22^m	= 4^h38^m
	Azimuth	=	90°	+	27°	= S.117°W.

(3) *Latitude* 47°*N.*, *declination* 18°*N.*

At rising	H.A.	=	18^h	−	1^h22^m	= 16^h38^m
	Azimuth	=	90°	−	27°	= N.63°E.
At setting	H.A.	=	6^h	+	1^h22^m	= 7^h22^m
	Azimuth	=	90°	−	27°	= N.63°W.

(4) *Latitude* 47°*S.*, *declination* 18°*S.*

At rising	H.A.	=	18^h	−	1^h22^m	= 16^h38^m
	Azimuth	=	90°	−	27°	= S.63°E.
At setting	H.A.	=	6^h	+	1^h22^m	= 7^h22^m
	Azimuth	=	90°	−	27°	= S.63°W.

Compass Correction by Bearing Amplitude. The bearing of a heavenly body at rising or setting affords an easy check on the compass, but it must always be remembered that the azimuth actually measured is that of the apparent position of the heavenly body, whereas the azimuth calculated is that of the heavenly body when its true zenith distance is 90°.

Inman's Tables include a correction to be added to the apparent azimuth of the Sun's centre in order to give the true azimuth at rising and setting.

The Time of Visible Sunrise or Sunset by Rate of Change in Altitude. Since the Sun's centre is about 1° below the horizon at visible sunrise or sunset, if the time of theoretical sunrise or sunset (when the Sun's centre is on the horizon) is known, the time of visible sunrise or sunset can be found by subtracting or adding the time taken by the Sun to alter its zenith distance by 1°. It is known from Chapter XII of this volume that the zenith distance changes by $15' \cos l \sin (az.)$ per minute. The number of minutes taken to alter by 60′ is therefore :

$$\frac{60}{15 \cos l \sin (az.)}$$
$$= 4 \sec l \operatorname{cosec} (az.)$$

The azimuth of the Sun at theoretical sunrise or sunset may be used with no loss of accuracy and it can be obtained with the H.A.T.S. at theoretical sunrise or sunset from *Davis's Tables* or *Inman's* amplitude tables. Sufficient information is thus available for calculating the time of visible sunrise or sunset.

It is required, for example, to find the local mean time of visible sunset on the 29th March 1937 in 30°N., 94°W. (Zone+6.)

Approximate Z.T. of sunset	1800	29th March
Zone	+6	
Greenwich date	0000	30th March

From the *Nautical Almanac* :

Declination=3°31′N.
E =11^h55^m

From the amplitude tables :

bearing amplitude=4°
time amplitude =8^m

The azimuth at theoretical sunset is therefore N.86°W. and the H.A.T.S. is 6^h8^m. Also the interval between visible and theoretical sunset is 4^m sec 30° cosec 86° or $4\frac{1}{2}^m$. Hence :

	h	m	
H.A.T.S. at theoretical sunset	6	8	
Interval		$4\frac{1}{2}$	
H.A.T.S. at visible sunset	6	$12\frac{1}{2}$	
E	11	55	
L.M.T. of visible sunset	18	$17\frac{1}{2}$	29th March

In the following table the interval in minutes between visible and theoretical sunset is calculated for various latitudes and declinations and a height of eye equal to 20 feet. The error introduced by taking other heights of eye is negligible if those heights do not exceed those customarily found in the practice of navigation.

LATITUDE

Dec.	0°	10°	20°	30°	40°	50°	60°	65°
0°	3·7	3·8	4·0	4·3	4·9	5·8	7·4	8·8
10°	3·8	3·8	4·0	4·4	5·0	6·0	7·9	9·7
20°	4·0	4·0	4·2	4·7	5·4	6·8	10·2	15·0
23°	4·0	4·1	4·4	4·8	5·6	7·3	11·9	23·1

The Sun's Azimuth at Visible Rising and Setting. It was shown in Chapter XII that the rate of change in azimuth is given by 15′ sin (*az.*) cosec h cos β per minute, where β is the angle PXZ in the spherical triangle. The value of β at theoretical sunrise or sunset (which can be taken instead of its value at visible sunrise or sunset without loss of accuracy) is found thus :

$$\cos PZ = \cos PX \cos ZX + \sin PX \sin ZX \cos \beta$$

That is, since ZX is 90° :

$$\cos \beta = \frac{\sin l}{\cos d}$$

If the circumstances are those described in the example worked in the last section :

$$\cos \beta = \frac{\sin 30°}{\cos 3°31'}$$

i.e. $$\beta = 59°56'$$

The rate of change in azimuth is therefore :

$$15' \sin 86° \cos 59°56' \operatorname{cosec} 6^h8^m \text{ per minute}$$
$$= 7\tfrac{1}{2}' \text{ per minute}$$

In the $4\frac{1}{2}^m$ between visible and theoretical sunset the azimuth therefore changes by $4\frac{1}{2} \times 7\frac{1}{2}'$, or 34′.

The azimuth at theoretical sunset is 86°. The azimuth at visible sunset is therefore (86°−0°34′) or N.85°·4W., and the true bearing (360°−85°·4) or 274°·6.

The Time of Moonrise and Moonset. The following example, which is the same as that worked on page 226 of Volume II, illustrates a method of finding the time of moonrise or moonset from the time of meridian passage.

What is the zone time of moonrise at 50°*N.*, 33°*E.*, *on the* 3*rd April* 1937 ?

	h	m	
L.M.T. of ☾'s meridian passage at 50°N., 0°E.	5	27	3rd April
Proportion ($50° \times 33^m \div 360°$)		5	
L.M.T. of ☾'s meridian passage at 50°N., 33°E.	5	22	3rd April
Longitude E.	2	12	
G.M.T. of ☾'s meridian passage at 50°N., 33°E.	3	10	3rd April

The Moon's declination at this G.M.T. is 22°11′·1S.

From the amplitude tables the approximate hour angle of the Moon at rising is $(18^h + 1^h56^m)$ or 19^h56^m. That is, it is 4^h04^m from the meridian, and since meridian passage occurs at 3^h10^m, G.M.T., on the 3rd April, moonrise occurs at $(3^h10^m - 4^h04^m)$ or 23^h06^m, G.M.T. on the 2nd April.

The declination at this G.M.T. is 22°22′·0S.

This new declination gives the time amplitude as 1^h58^m. The Moon's hour angle at rising is $(18^h+1^h58^m)$ or 19^h58^m, and the uncorrected interval between moonrise and transit is therefore $(24^h-19^h58^m)$ or 4^h02^m. The correction to be applied is $(4^h \times 50^m \div 24^h)$ or 8^m, and the interval is seen to be 4^h10^m. Hence:

	h m	
Interval between moonrise and transit	4 10	
G.M.T. of ☾'s transit at 50°N., 33°E.	3 10	3rd April
G.M.T. of moonrise at 50°N., 33°E. is	23 00	2nd April
Zone (−2)	+2	
∴ Z.T.	01 00	(−2) 3rd April

This result, it is seen, agrees with that found from the tables giving the times of moonrise.

CHAPTER XVII

WEIR'S AZIMUTH DIAGRAM

It was shown in Chapter XIV of Volume II that the azimuth of a heavenly body can be found from a diagram composed of latitude ellipses and hour-angle hyperbolas. The reason for this may be arrived at by considering the expression for the azimuth obtained by applying the four-part formula to the spherical triangle PZX.

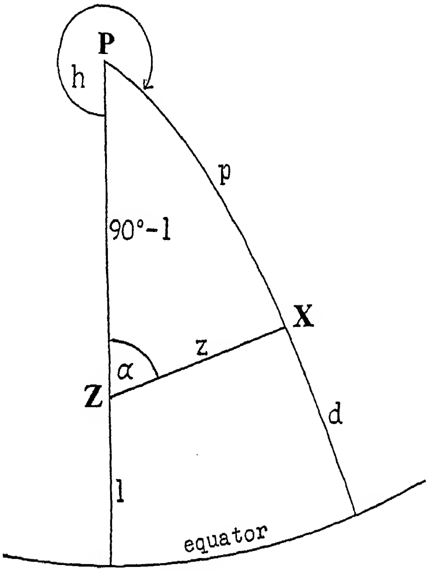


FIGURE 118.

In figure 118, the ordinary notation is adopted, and if α is the azimuth of X, the four-part formula gives:

$$\sin l \cos h = \cos l \cot p - \sin h \cot \alpha$$

i.e.

$$\cot \alpha = \frac{\cos l \cot p - \sin l \cos h}{\sin h}$$

$$= \frac{\tan d - \tan l \cos h}{\sec l \sin h}$$

$$= \frac{y_1 - y_2}{x_2 - x_1}$$

—where:

$$\begin{cases} x_1 = 0 \\ y_1 = \tan d \end{cases} \qquad \begin{cases} x_2 = \sec l \sin h \\ y_2 = \tan l \cos h \end{cases}$$

That is, (x_1, y_1) and (x_2, y_2) represent two points A and B in rectangular co-ordinates, the first of which lies on the y-axis at a distance tan d from the x-axis, and the second at a point determined by l and h. (Figure 119.)

When h is eliminated from the expressions for x_2 and y_2, it is seen that B lies on the ellipse :

$$\frac{x^2}{\sec^2 l}+\frac{y^2}{\tan^2 l}=1$$

When l is eliminated, it is seen that B lies on the hyperbola :

$$\frac{}{\sin^2 h}-\frac{y^2}{\cos^2 h}=1$$

Hence, for any given declination, latitude and hour angle, A is a fixed point and B is the intersection of an ellipse that depends

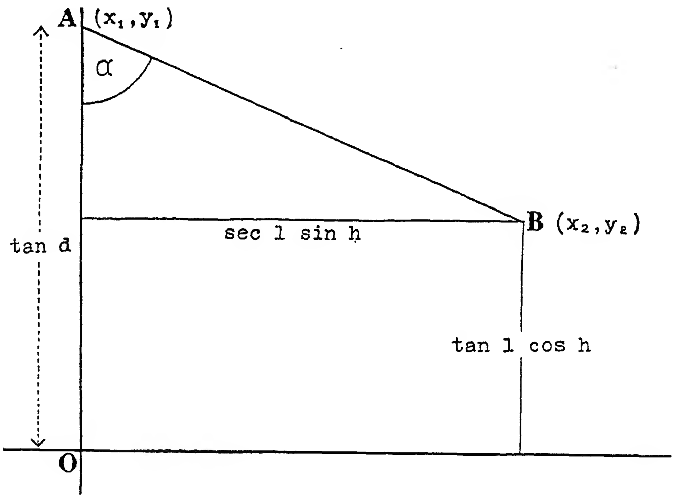


FIGURE 119.

solely on the latitude, and a hyperbola that depends solely on the hour angle. The x-axis of these two conics is the equator, and the y-axis is the observer's meridian. (Figure 120.)

If a number of ellipses and hyperbolas corresponding to different values of the latitude and hour angle are drawn, two families of conics are obtained, and it is seen that each ellipse cuts the hyperbolas at right-angles, and each hyperbola cuts the ellipses at right-angles, a fact which can be established by considering the slope of the tangents at a common point (x, y).

On the ellipse it is given by :

$$\frac{\,}{\sec^2 l}+\frac{y}{\tan^2 l}\times\frac{dy}{dx}=0$$

or

$$\frac{dy}{dx}=-\frac{x\tan^2 l}{y\sec^2 l}$$

On the hyperbola it is given by :

$$\frac{x}{\sin^2 h}\quad\frac{y}{\cos^2 h}\times\frac{dy}{dx}=0$$

or

$$\frac{dy}{dx}=\frac{x\cos^2 h}{y\sin^2 h}$$

The product of these two values is :

$$-\frac{x^2\tan^2 l\cos^2 h}{y^2\sec^2 l\sin^2 h}$$

—and this, since x is equal to sec l sin h, and y to tan l cos h, reduces to -1, thereby proving that the tangents are perpendicular. The two families of conics thus have a common focus F.

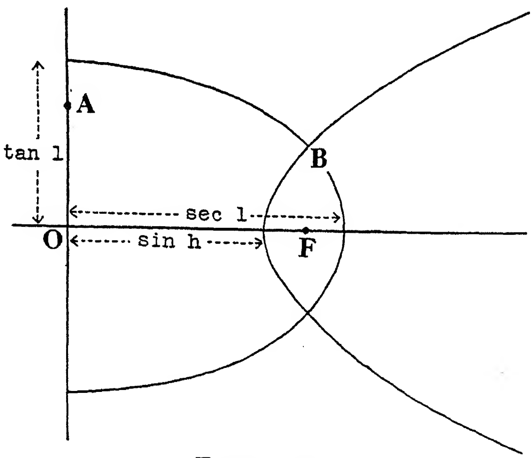


FIGURE 120.

In figure 121, the azimuth α is represented by the angle OAB, which is equal to the angle DOC, OC being parallel to AB. If, then, an azimuth circle is drawn with O as centre, the azimuth can be measured by running a parallel ruler from AB to O and noting the point C where it cuts the azimuth circle. But an adjustment is necessary in order that the convention of having the north point at the top of the azimuth circle may be followed.

In the example taken, the observer's latitude and the heavenly body's declination are both north, and the hour angle is such that α is less than 90° measured from north to east. (See figure 118.)

The north point in figure 121 is thus at the bottom of the azimuth circle, and *A* and *B* are really marked for a south latitude and a south declination equal in magnitude to the actual north latitude

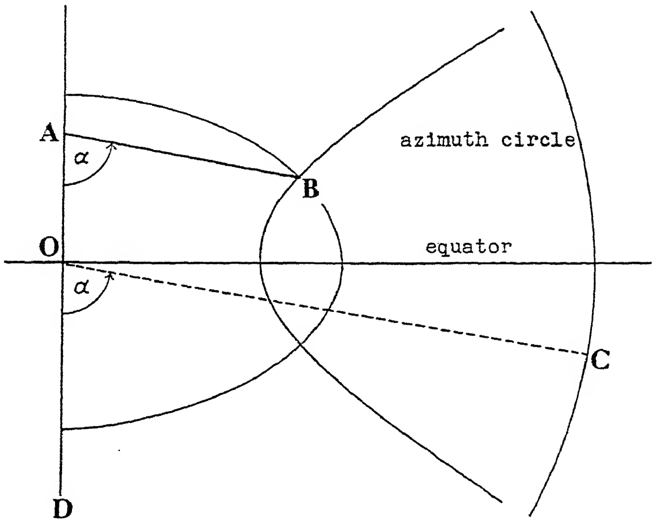


FIGURE 121.

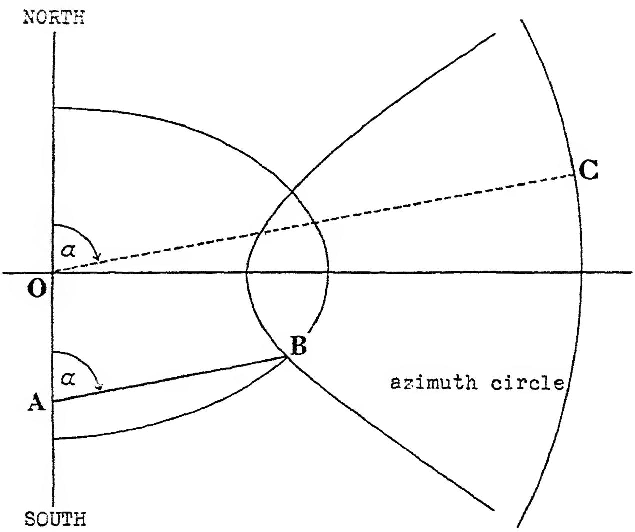


FIGURE 122.

and north declination. The same result is obtained if the convention of having the north point at the top of the azimuth circle is observed, and the names of the latitude and declination are reversed, as shown in figure 122.

CHAPTER XVIII

GRAPHICAL PROBLEMS

Relative Course and Speed. In nearly every problem that involves moving ships, it is essential to find the course and speed of one ship relative to another. If two ships are steaming on straight courses at certain speeds, then the relative course, sometimes called the virtual course, along which they are approaching or diverging can be found by completing a triangle called the *speed triangle.*

For example, figure 123 shows two ships, A and B. A is steaming 160° at y knots; B is steaming 090° at x knots.

Let BX represent x knots on any convenient scale along a course 090°.

Let YX represent y knots on the same scale along a course 160°.

The resultant of these two is found by joining BY and thus

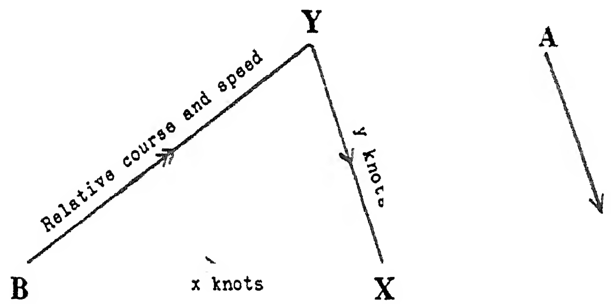


FIGURE 123.

completing the speed triangle BXY. On the chosen scale BY represents the course and speed that B makes good relative to A.

NOTE. The Battenberg course indicator is constructed so as to give a mechanical solution of the speed triangle.

The sides of the speed triangle should be marked with arrow heads to show the directions in which the ships are moving. It is usual to mark the relative course with two arrow heads.

The arrow heads showing the courses of the two ships must either both point towards, or both point away from, the intersection of the courses.

It is seen that when relative courses and speeds are considered, one ship can be plotted at a point and kept there, and the other plotted in relation to that point by moving her along a relative course at a relative speed.

The rules for using the diagram are therefore :

(1) Reverse the name of the declination and mark the point A on the north–south line, south of the equator if the actual declination is north, and north if it is south.
(2) Reverse the observer's latitude and follow the particular ellipse corresponding to that reversed latitude until it cuts the hyperbola corresponding to the given hour angle. This is the point B.
(3) Run a parallel ruler from AB to O, the observer's position, and read the azimuth on the azimuth circle at C.

When the hour angle gives a westerly azimuth, the same diagram can be used since the numerical value of the azimuth is the same for an hour angle of 60° as it is for one of 300°.

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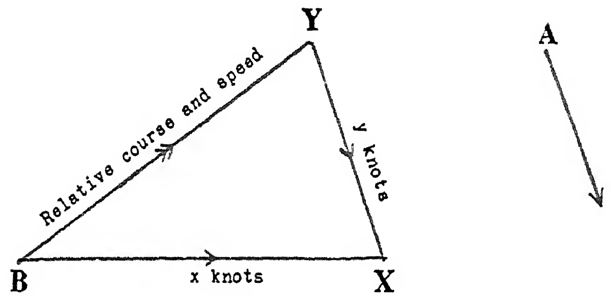

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It is seen that when relative courses and speeds are considered, one ship can be plotted at a point and kept there, and the other plotted in relation to that point by moving her along a relative course at a relative speed.

The lettering in the examples that follow is made consistent, as far as possible, and the colour scheme adopted is :

Course steered by ship A	—blue
Course steered by ship B	—red
Relative courses	—green
All other detail	—black

The scale is necessarily varied.

In all problems of relative motion the following factors are involved :

(1) course and speed of A.
(2) course and speed of B.
(3) relative course and relative speed of A or B.

If any four of the above are known, it is possible, by drawing a speed triangle to scale, to find the other two.

A number of examples illustrating the principle of speed triangles in various circumstances are summarised in the following table and explained in detail on the stated pages.

NOTE. In problems concerned with operations that have to be carried out by a certain time, as for instance, examples 12 and 13, it must be remembered that the first thing to do is to plot the position of the flagship, or the ship being formed on :

(a) at the beginning of the operation.
(b) at the end of the operation.

When this has been done, the problem becomes partly, if not entirely, geographical as opposed to relative.

Example 1. Changing Station on a Moving Ship and Using a Given Speed.

In figure 124, *the flagship* (A) *is steering* 340° *at* 10 *knots. A destroyer* (B) *stationed on the port beam distant* 5 *miles, is ordered to take station on a bearing* 360° *from the flagship at a distance of* 5 *miles. Her available speed is* 20 *knots. What course should the destroyer steer, and how long will she take to change station?*

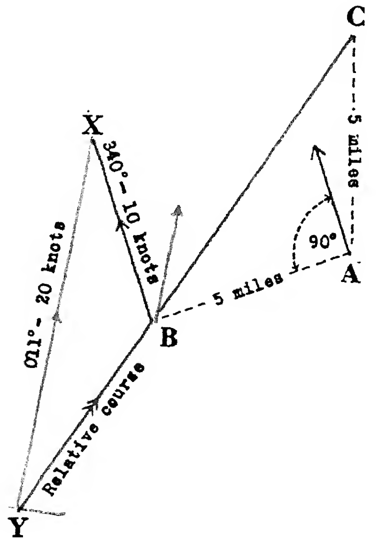


FIGURE 124.

Plot the flagship's position, A; the destroyer's original position, B, and the final position of the destroyer, C.

Join BC, which will be the course the destroyer must make good relative to A.

Plot the flagship's course and speed from B. (BX)

With centre X and radius representing 20 knots on the same scale, strike an arc cutting CB produced in Y. Join YX.

In the speed triangle BXY:

BX=flagship's course and speed.
YX=destroyer's course and speed.
YB=destroyer's relative course and relative speed.

Therefore YX gives the course B must steer to make good YB, and the length of YB on the speed scale gives the relative speed. The relative distance, BC, divided by the relative speed, YB, gives the time it will take to change station.

Answer. Course to steer: 011°.
Time taken: 39 minutes.

Example 2. Changing Station on a Moving Ship and Using a Given Course.

In figure 125, *the flagship* (A) *is steering* 145° *at* 15 *knots. A cruiser* (B) *bearing* 120°—6 *miles from the flagship, is ordered to take station* 8 *miles on a bearing* 100°. *Her captain decides to steer* 115°. *At what speed must the cruiser proceed and how long will she take to change station?*

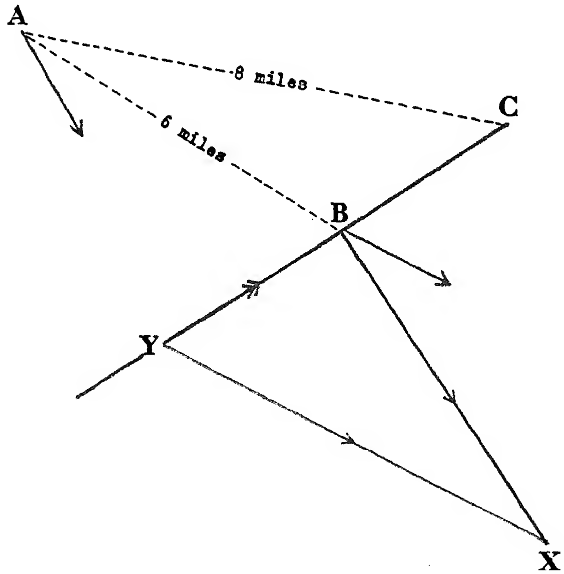


FIGURE 125.

Plot the flagship, A; the cruiser's original position, B, and the cruiser's final position, C.

Join BC, which will be the course the cruiser must make good relative to A.

Plot the flagship's course and speed from B. (BX)

Plot the cruiser's course, 115°, reversed from X, cutting CB produced in Y.

In the speed triangle BXY :

BX=flagship's course and speed.
YX=cruiser's course and speed.
YB=cruiser's relative course and relative speed.

Therefore the length YX on the speed scale gives the speed at which the cruiser must proceed. The relative distance, BC, divided by the relative speed, YB, gives the time it will take to get into station.

Answer. Speed of cruiser : 18 knots.
Time taken : 21 minutes. (3′·1 at 9 knots.)

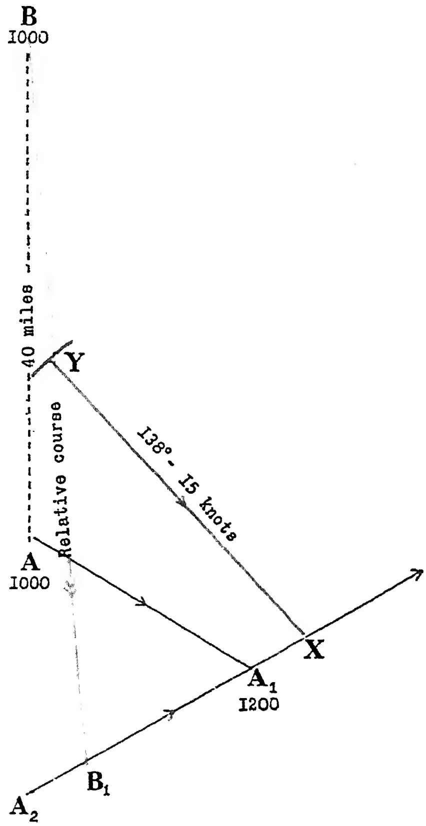


FIGURE 126.

Example 3. Changing Station on a Ship that Alters Course during the Manœuvre.

In figure 126, *the flagship* (A) *is steering* 120° *at* 10 *knots. At* 1000, *a cruiser* (B), *stationed* 40 *miles* 000° *from the flagship, receives the following signal : ' Take station* 5 *miles ahead. Intend to alter course to* 060° *at noon.'*

If the cruiser proceeds at 15 *knots, what course must she steer, and at what time will she be in station?*

Plot the cruiser's position at 1000, B.

Plot the flagship's positions at 1000 and noon, A and A_1 respectively.

From the flagship's noon position, A_1, plot the imaginary position she would have been in at 1000 if she had steered 060° from 1000 to noon, A_2.

From A_2, plot the cruiser's new station 5 miles ahead, B_1.

Join BB_1. This will be the course the cruiser must make good.

From B_1 plot the flagship's new course and speed for one hour, B_1X.

With centre X and radius representing the cruiser's speed, 15 knots, cut BB_1 in Y.

In the speed triangle B_1XY:

B_1X=flagship's course and speed
YX=cruiser's course and speed
YB_1=cruiser's relative course and speed

Therefore YX gives the cruiser's course to steer to make good BB_1, and the length YB_1 on the speed scale gives the cruiser's relative speed. The relative distance, BB_1, divided by the relative speed, YB_1, gives the time she takes to change station.

Answer. Course to steer: 138°.
Time of arrival: 1332 (58 miles at 16·4 knots).

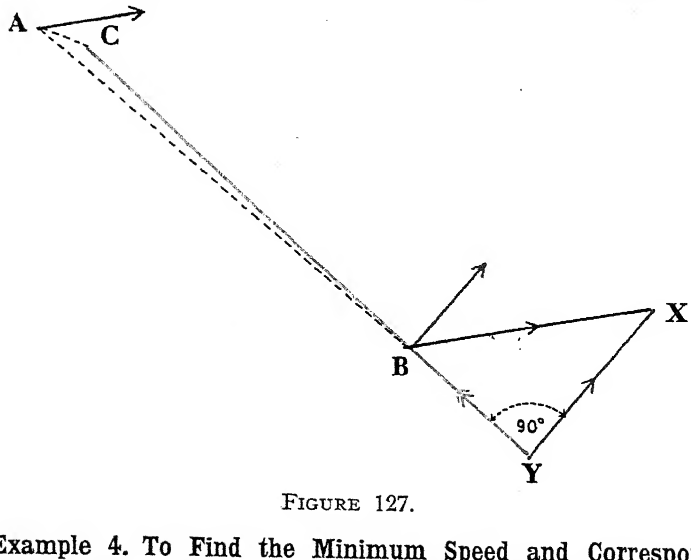


FIGURE 127.

Example 4. To Find the Minimum Speed and Corresponding Course Necessary to Reach a Certain Station, and the Time Required to Complete the Manœuvre.

In figure 127 *a submarine* (B) *observes an enemy ship* (A) *bearing* 310° *distant* 4 *miles. The enemy's course is estimated at* 082° *and her speed at* 12 *knots. It is desired to run submerged to a position*

for attack, ½ mile, 30° on her starboard bow, and to proceed at the lowest possible speed. At what course and speed should the submarine proceed?

Plot the enemy ship, *A*; the submarine's original position, *B*, and the submarine's final position, *C*.

Join *BC*, the course to be made good.

Plot the enemy's course and speed from *B*. (*BX*)

Clearly, if the submarine's speed is to be a minimum, the third side of the speed triangle must be as short as possible. The shortest distance is the perpendicular from *X* on to *CB* produced.

Draw this perpendicular, *XY*.

In the speed triangle *BXY*:

BX=enemy's course and speed.
YX=submarine's course and speed.
YB=submarine's relative course and speed.

Answer. Course to steer: 042°.
Speed: 9¼ knots.

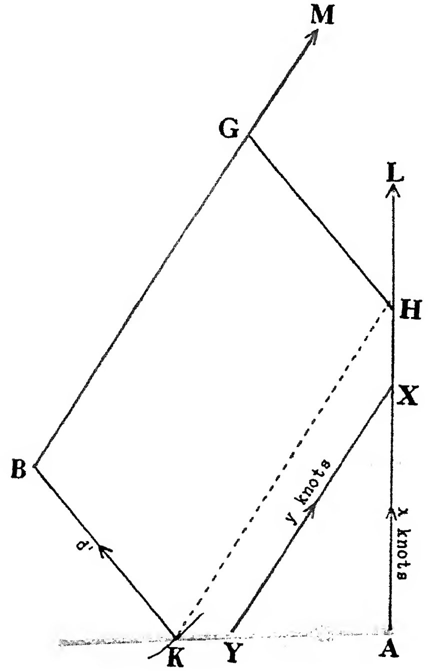


FIGURE 128.

Example 5. To Find the Time at which Two Ships, Steaming Different Courses at Different Speeds, will be a Certain Distance Apart.

In figure 128, *A* is a ship steering a course *AL* at *x* knots. *B* is a ship steering a course *BM* at *y* knots.

From A lay off AX representing A's course and speed.
From X lay off YX representing B's course and speed.
Join AY, and AY is the relative course and speed of A.
Let d be the required distance in miles. With centre B and radius equivalent to d miles, describe an arc cutting AY produced in K.
When A has travelled the distance AK at the relative speed AY, the ships will be the required distance apart.
From K draw a line parallel to BM, cutting AL in H.
Then through H draw a line parallel to BK cutting BM in G.
H and G will be the actual positions of A and B respectively, when they are d miles apart.

NOTE. If the arc of radius equivalent to d miles does not cut AY produced, the two ships will not get within d miles of each other on their present courses and speeds.

Example 6. Opening and Closing on the Same Bearing.

In figure 129 *a cruiser* (B) *stationed* 2 *miles on the port bow of a flagship* (A) *steering* 328° *at* 11 *knots, is ordered to open to* 10 *miles and to preserve the bearing. The cruiser's available speed is* 17 *knots. Having opened to* 10 *miles, she is ordered to close to* 4 *miles on the same bearing.*

Find the course to open, the course to close, and the time taken.

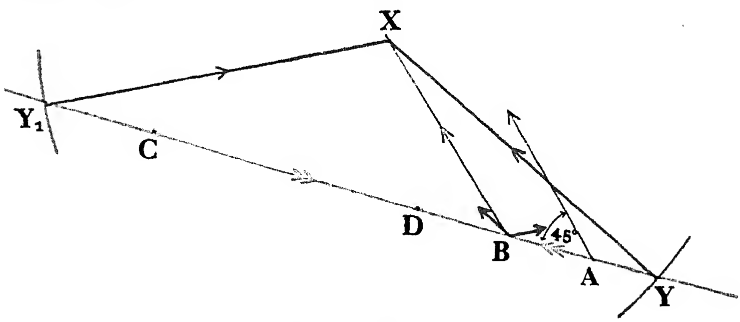


FIGURE 129.

Plot the flagship, A, and the cruiser's original position, B.
C is the cruiser's position when she has opened to 10 miles.
D is the cruiser's position when she has closed to 4 miles.
Join CB, which will be the course the cruiser must make good relative to A when opening and closing.
Plot the flagship's course and speed from B. (BX)
With centre X and radius representing the cruiser's speed, cut BC produced at Y and Y_1.
In the speed triangles BXY and BXY_1:

BX=flagship's course and speed.
YX=cruiser's course and speed to open.
Y_1X=cruiser's course and speed to close.
YB=cruiser's relative course and speed to open.
Y_1B=cruiser's relative course and speed to close.

YX gives the course *out*, Y_1X the course *in*. The time taken to open is 8 miles at a speed of YB, and the time taken to close is 6 miles at a speed of Y_1B.

Answer. Course to open : 310°.
Time taken : 1 hour 4 mins.
Course to close : 076°.
Time taken : $15\frac{1}{2}$ mins.

Example 7. To Get Within a Given Distance as Quickly as Possible. (AG—in)

In figure 130, A is a ship steering a course AL. Another ship, B, wishes to close to within d miles of A as quickly as possible. Clearly, if A were stationary then B would steer straight towards A. But A is moving. B must therefore steer for some point C, on the

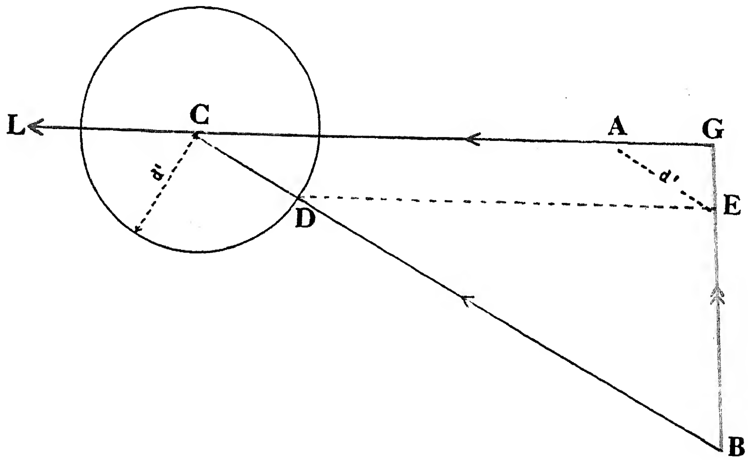


FIGURE 130.

line AL, at which A will arrive later, this point being so placed that when A arrives at C, B is at D, d miles from C.

B will then have steamed the shortest distance to reach her objective, and A will be directly ahead of her when she arrives within d miles.

From D lay off DE, parallel to and equal to CA, representing A's course and distance steamed.

BD is B's course and distance steamed during the same period, and BE is the relative course and distance made good.

Produce BE, cutting A's course at G. Then if B closes the point G on a steady bearing, she will fulfil the requirements.

To find the length AG.

Join EA.

Then EA is parallel to BD.

In the triangles AGE and DEB:

$$\frac{AG}{AE}=\frac{DE}{DB}=\frac{A\text{'s speed}}{B\text{'s speed}}$$

AE is the distance to which B wishes to close.

$$\therefore \qquad AG=\text{distance to be closed to}\times\frac{\text{other ship's speed}}{\text{own speed}}$$

If, therefore, the distance AG is laid off *astern* of A and a course is steered so as to make good a relative track BG, then B will close A to a distance d miles as quickly as possible.

In figure 131, *a cruiser* (B) *stationed* 10 *miles on the port beam of the flagship* (A) *steering* 270° *at* 12 *knots, is ordered to close to* 3 *miles as quickly as possible. If the cruiser's available speed is* 18 *knots, what course must she steer, and at what time will she arrive in position?*

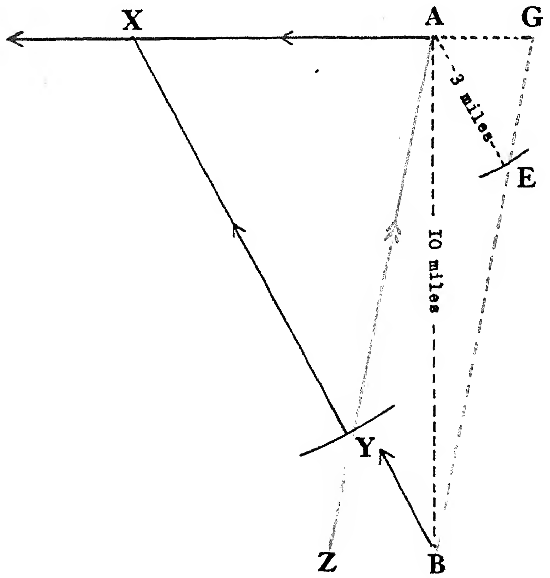


FIGURE 131.

Plot the flagship's original position, A, and the cruiser's original position, B.

$$AG=\text{distance to be closed to}\times\frac{\text{flagship's speed}}{\text{cruiser's speed}}$$

$$=3\times\frac{12}{18}=2\text{ miles}$$

Plot the point G, 2 miles astern of the flagship. Join BG. BG will be the cruiser's relative track on closing.

Through A, draw AZ parallel to BG, the course to be made good.

Plot the flagship's course and speed from A. (AX)

With centre X and radius representing the cruiser's available speed, cut AZ in Y.

In the speed triangle AXY:

AX=flagship's course and speed.
YX=cruiser's course and speed.
YA=cruiser's relative course and speed.

To find the time taken.

With centre A and radius the distance to be closed, 3 miles, cut GB in E. Then BE will be the relative distance to be steamed, at the relative speed YA.

Answer. Course to steer: 330°.
Time taken: 28 minutes. (7′·5 at 15·8 knots.)

Example 8. To Open to a Given Distance as quickly as Possible. (AG—out)

In figure 132, A is a ship steering a course AL. Another ship, B, wishes to open to d miles from A, as quickly as possible.

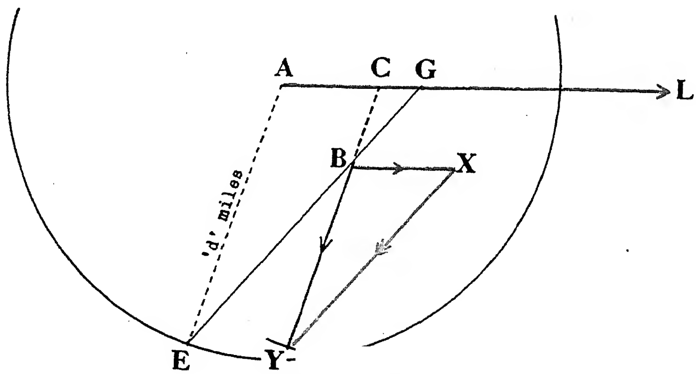


FIGURE 132.

In essentials, this is exactly the same problem as Example 7. B must steer such a course that, when she reaches the required range, A will be exactly astern of her.

Clearly, if A were stationary then B would steer directly away from A. But A is moving. B must therefore steer away from some point C, on the line AL, at which A will arrive later, this point being so placed that when A reaches it, B will be at Y, d miles distant, steering a course BY.

Construct BX, parallel and equal to AC.

Then XY is the relative track to be made good.

Through B draw EBG, parallel to XY. Then B must open on a steady bearing from G, and will have opened to d miles when she has gone a relative distance BE, at a relative speed of XY.

To find the length AG.

In the triangles BYX and AEG:

$$\frac{AG}{AE}=\frac{BX}{BY}=\frac{A\text{'s speed}}{B\text{'s speed}}$$

$$\therefore\quad AG=\text{distance to be opened to}\times\frac{\text{other ship's speed}}{\text{own ship's speed}}$$

AG must be laid off *ahead* of A.

If, therefore, the distance AG is laid off *ahead* of A, and a course is steered so as to make good a relative track, BE, then B will open from A to a distance of d miles as quickly as possible.

Example 9. To Open to a Given Distance as Quickly as Possible. (AG—out)

In figure 133, *a destroyer* (B) *which has carried out a torpedo attack from a position* 3 *miles on the port beam of an enemy ship* (A), *steering* 270° *at* 20 *knots, wishes to open beyond the range of* A*'s secondary armament as quickly as possible. If the destroyer's available speed is* 30 *knots and the range of the enemy's secondary armament is* 12,000 *yards, what course must she steer, and at what time will she arrive in position?*

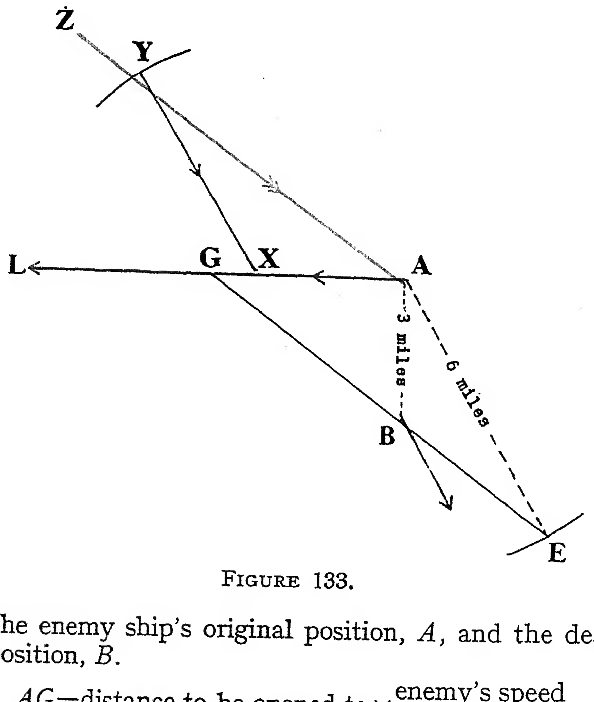


FIGURE 133.

Plot the enemy ship's original position, A, and the destroyer's original position, B.

$$AG=\text{distance to be opened to}\times\frac{\text{enemy's speed}}{\text{own speed}}$$

$$=6\times\frac{20}{30}=4\text{ miles.}$$

Plot the point G, 4 miles ahead of the enemy. Join BG.

GB produced will be the destroyer's relative track when opening.

Through A draw AZ parallel to GB, the course to make good.

Plot the enemy's course and speed from A. (AX)

With centre X and radius representing the destroyer's available speed, cut AZ in Y.

In the speed triangle AXY:

AX=enemy's course and speed.
YX=destroyer's course and speed.
YA=destroyer's relative course and speed.

To find the time taken.

With centre A and radius equivalent to the range of A's secondary armament, 6 miles, cut GB produced in E. Then BE will be the relative distance to be steamed at the relative speed YA.

Answer. Course to steer: 151°.
Time taken: 5 minutes. (3′·7 at $43\frac{1}{2}$ knots.)

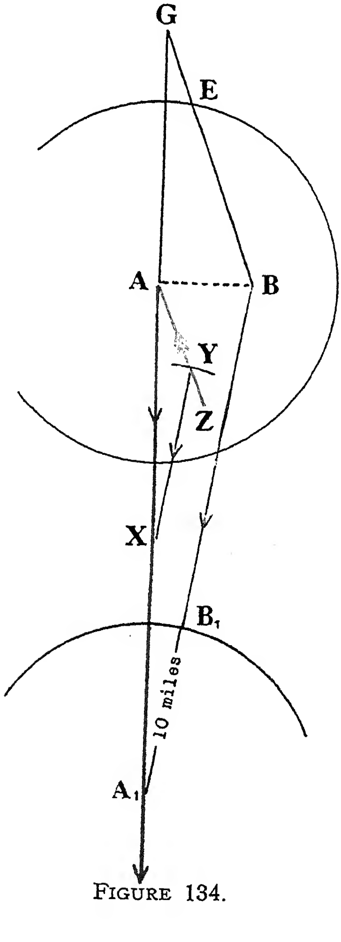


FIGURE 134.

Example 10. A Slower Ship Remaining Within a Given Range of a Faster Ship for as Long as Possible.

In figure 134, *a cruiser* (A) *steering* 180° *at* 30 *knots bears* 270°—10,000 *yards from a battleship* (B). *The battleship, with available speed* 20 *knots, wishes to keep the cruiser under fire as long as possible. Visibility* 10 *miles.*

If the cruiser does not alter course, what course should the battleship steer, and how long will the cruiser be under fire?

Plot the cruiser's original position, *A*, and the battleship's original position, *B*.

The problem is exactly the same as the '*AG*—in' (Example 7) except that *B* starts inside the distance and loses distance, instead of starting outside the distance and closing.

The distance *AG* is laid off astern of *A*, and *B* steers the course *YX* to make good the relative course *BG*. (*YA*)

When *B* arrives at *E*, on the relative course, *BG*, she will actually be at the point B_1, and *A* will be at A_1, 10 miles ahead of her.

Answer. Course to steer: 190°.
Time under fire: 57 minutes. (10′·5 at 11 knots.)

Example 11. Crossing a Danger Area.

A ship, shown in figure 135, *wishes to cross a patrol belt from* A *to* B *as quickly as possible.*

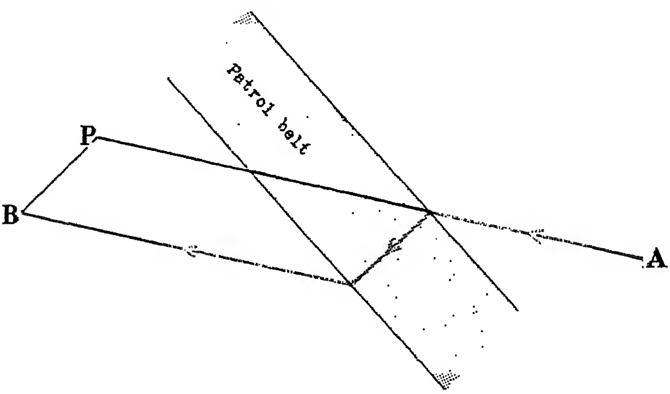


FIGURE 135.

From *B* lay off *BP*, the width of the zone, on the course to be steered through it. Join *AP*, which will be the course to steer on each side of the zone.

NOTE. This construction makes the angle of incidence to the patrol belt equal to the angle of departure from the belt.

Example 12. Scouting in a Given Direction and Returning in a Given Time.

In figure 136, *a cruiser in company with the flagship* (A) *at* 0600, *is ordered to scout in a direction* 240° *at* 20 *knots, and to rejoin the fleet at* 1800. *Course and speed of the fleet* 270°—15 *knots.*

At what time must the cruiser alter course to rejoin, and what will be her course ?

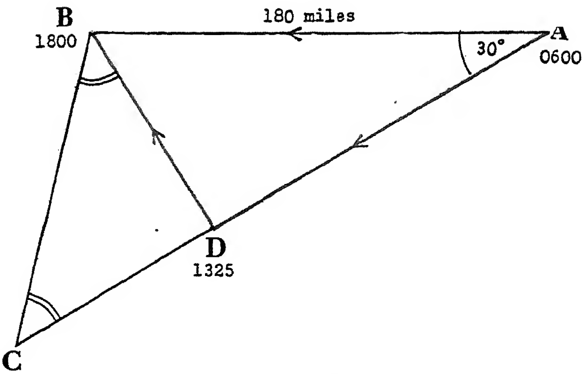


FIGURE 136.

Plot the flagship's positions at 0600 and 1800, A and B respectively.

From A lay off AC in a direction 240°, and equal to 12 hours' steaming at 20 knots.

Join BC, and draw BD so that $\angle CBD$ is equal to $\angle BCA$.

Then BD is equal to DC, and D will be the turning point.

Measure AD to find the time to alter course.

Answer. Time to alter course : 1330. (150′ at 20 knots.)
Course to rejoin : 327°.

Example 13. Proceeding to a Base and Rejoining at a Given Time.

In figure 137, *a cruiser in company with a fleet steering* 100° *at* 10 *knots, is ordered at* 0800 *to proceed to a base that bears* 180°, *distant* 40 *miles, and to rejoin at* 1500. *What is the latest time she can part company if she proceeds to and from the base at* 20 *knots, and remains at the base for one hour ?*

Plot the flagship's positions at 0800 and 1500, A and B respectively.

Plot the base, C. Measure BC. (74′·25)

Calculate the time the cruiser will take to steam from C to B at 20 knots. (3^h43^m)

That is, the cruiser must leave the base at 1117 and, if she is to remain at the base for one hour, she must arrive there at 1017.

Plot the fleet's position at 1017, D.
Join DC.
Construct a speed triangle DXY where :

DX = fleet's course and speed.
YX = cruiser's course and speed.
YD = cruiser's relative course and speed.

Draw CH parallel to XY.
Then H will be the point at which to part company.
AH, by measurement, equals 3 miles.

Answer.—The cruiser must part company at 0818. (3′ at 10 knots.)

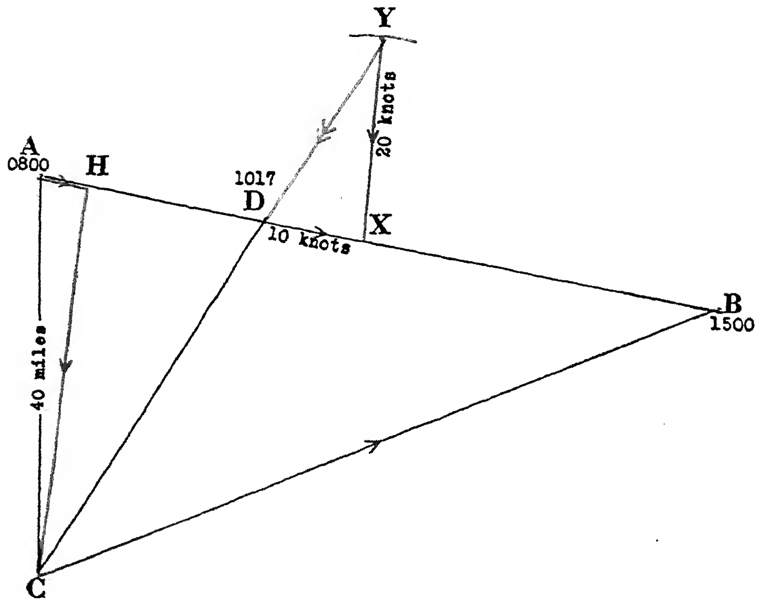


FIGURE 137.

Example 14. Scouting to a Given Line and Rejoining to a Given Distance as Quickly as Possible.

In figure 138, *a cruiser in company with the fleet, and with available speed* 25 *knots, at* 0800 *is ordered to scout as quickly as possible to a certain line,* CD, *running* 090°–270°, *and* 40 *miles*—000° *from the flagship's present position,* (A). *On reaching the line the cruiser is to return to visibility distance* (10 *miles*) *of the flagship as soon as possible. Course and speed of the fleet,* 250°—15 *knots. What course must the cruiser steer, and at what time will she turn to rejoin? At what time will she sight the flagship?*

Plot the flagship's position at 0800, A, and the patrol line, CD.

From A, drop a perpendicular AE on to the line CD, and produce it to B so that AE is equal to EB. (B is sometimes called the *mirrored position*.)

The cruiser can be considered to be at B, and ordered to close to within 10 miles of the flagship as soon as possible: thus the problem becomes an ordinary 'AG—in', similar to Example 7.

Plot the position G and construct the speed triangle AXY in which YA is parallel to $B.G$.

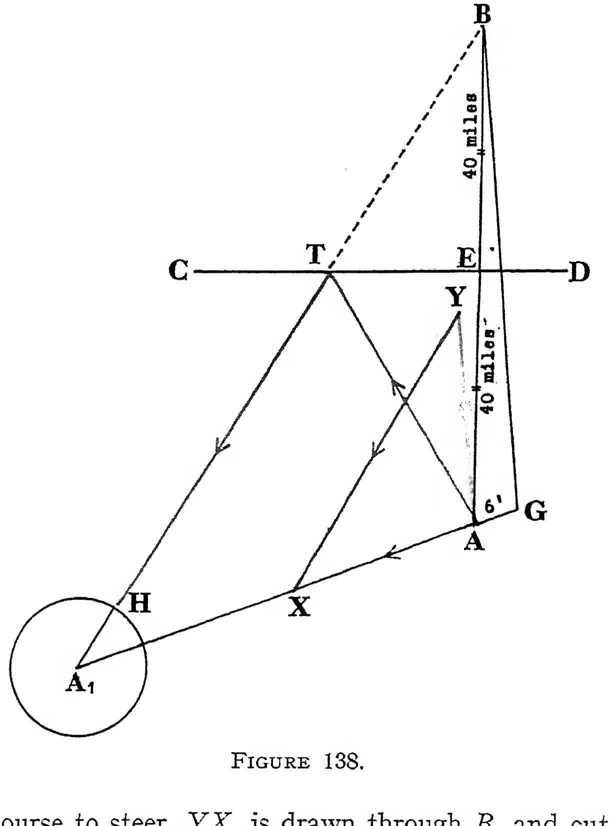


FIGURE 138.

The course to steer, YX, is drawn through B, and cuts CD at T, and the flagship's course at A_1.

Join AT.

Then AT is the course out, and TA_1 is the course to rejoin.

The flagship will be at A_1 when the cruiser is at H.

Answer. Cruiser's course *out*: 330°.
Cruiser's course to rejoin: 211°.
Cruiser's time to turn: 0953. (47′ at 25 knots.)
Cruiser will sight the flagship at 1222. (109′ at 25 knots.)

Example 15. Scouting. To Find the Shortest Distance to a Given Line, Rejoining the Fleet at a Given Time.

(a) *At* 0800 *the fleet* (A), *shown in figure* 139, *is steering* 130° *at* 12 *knots in latitude* 48°*N*. *A cruiser in company is ordered to scout to latitude* 48°30′*N*., *and rejoin at* 1300. *What courses must the cruiser steer, and what will be the least speed required?*

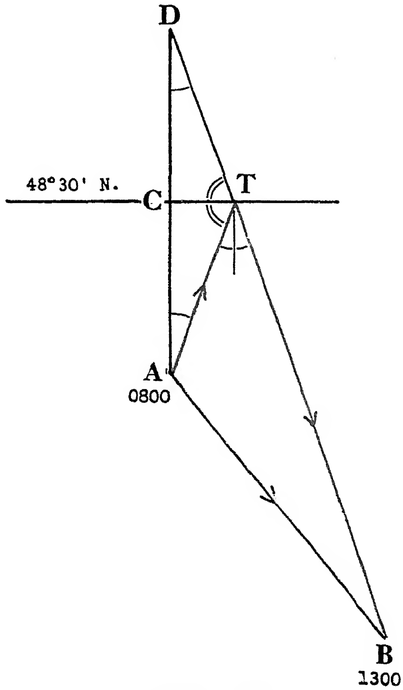


FIGURE 139.

Plot the flagship's positions at 0800 and 1300, A and B respectively.

Plot the parallel of 48°30′N.

Clearly the cruiser's angle of incidence to the line must be made equal to the angle of her departure from the line. Therefore from A drop a perpendicular to the line, cutting it at C.

Produce AC to D, so that AC is equal to CD. (D is the *mirrored position*.)

Join DB, and let this line cut the parallel of 48°30′ in T.

Then AT will be the course out, and TB will be the course to rejoin.

The speed at which it will be necessary to proceed is found by measuring $AT+TB$.

Answer. Course *out*: 025°.
Course to rejoin: 155°.
Speed: 21·6 knots. (108′ in 5 hours.)

(b) *At* 0800 *the fleet* (A), *in figure* 140, *is steering* 175° *at* 12 *knots. A cruiser in company is ordered to scout to a certain line CD, running* 315°–135°, *and* 30 *miles*, 045° *from the flagship, and rejoin as soon as possible. Cruiser's available speed* 20 *knots. What courses must the cruiser steer and at what time will she rejoin ?*

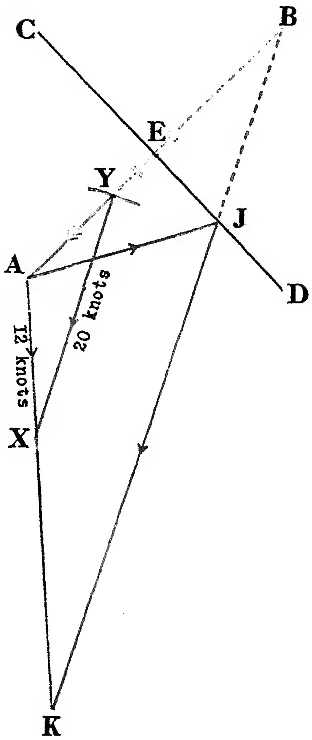


FIGURE 140.

Plot the flagship's position at 0800, A, and the line CD.

From A, drop a perpendicular AE on the line CD, and produce it to B, so that AE is equal to EB. (B is the *mirrored position.*)

The cruiser can be considered to be at B when it will be necessary for her to close the flagship on a constant bearing.

Plot the flagship's course and speed, AX.

With centre X and radius representing the cruiser's speed, cut AE in Y.

In the speed triangle AXY:

AX=flagship's course and speed.

YX=cruiser's course and speed to close on a constant bearing.

YA=cruiser's relative course and speed to close on a constant bearing.

Through B draw BK parallel to YX, cutting CD in J.

The cruiser will, therefore, steer the course AJ and on arrival at J will alter to the course JK, rejoining the fleet at position K.

The time of rejoining is found by the time taken to steam the distance $AJ+JK$ at 20 knots.

Answer. Cruiser's course *out*: 072°.
Cruiser's course to rejoin: 198°.
Time of rejoining: 1354. (118′ at 20 knots.)

Example 16. Scouting on a Given Bearing at a Given Speed and Returning in a Given Time on the same Bearing.

At 1200 *a cruiser* (A), *in figure* 141, *in company with the fleet, is ordered to scout on a bearing* 145° *from the fleet at* 26½ *knots, and rejoin by* 1600 *on the same bearing. Course and speed of the fleet* 090°—12 *knots.*

What courses must the cruiser steer and at what time will she turn to rejoin?

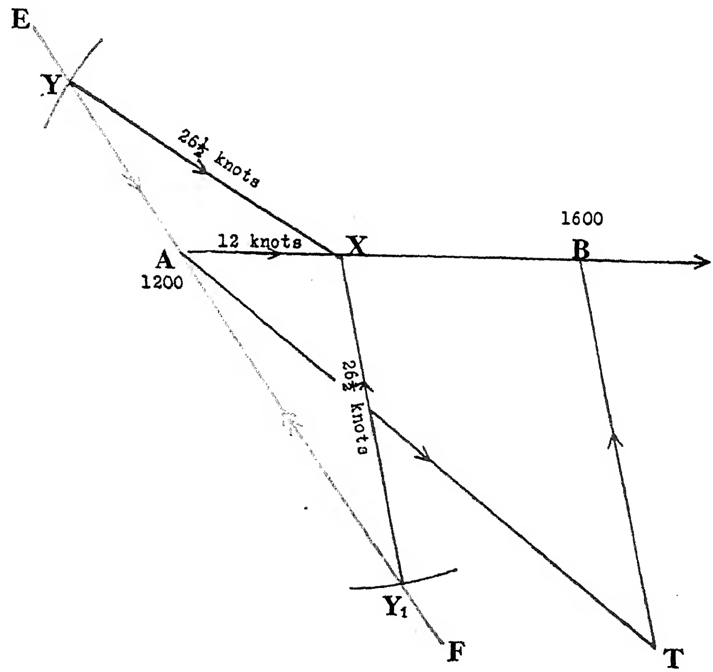


FIGURE 141.

This problem is similar to example 15 (*b*) except that the bearing is given.

Plot the flagship's course and speed. (AX)

Through A draw the bearing, 145° and 325°. (EF)

With centre X and radius representing the cruiser's speed, cut EF in Y and Y_1.

In the speed triangles AXY and AXY_1:

AX=flagship's course and speed.
YX=cruiser's course and speed *out*.
Y_1X=cruiser's course and speed to rejoin.
YA=cruiser's relative course and speed *out*.
Y_1A=cruiser's relative course and speed to rejoin.

Plot the flagship's position at 1600, B.

From B draw BT parallel to Y_1X, the cruiser's rejoining course.

From A draw AT parallel to YX, the cruiser's course *out*.

T will be the turning point, and the distance AT at $26\frac{1}{2}$ knots will give the time of turning to rejoin.

Answer. Cruiser's course *out*: 123°.
Cruiser's course to rejoin: 347°.
Time of turning to rejoin: 1433. (67′·5 at $26\frac{1}{2}$ knots.)

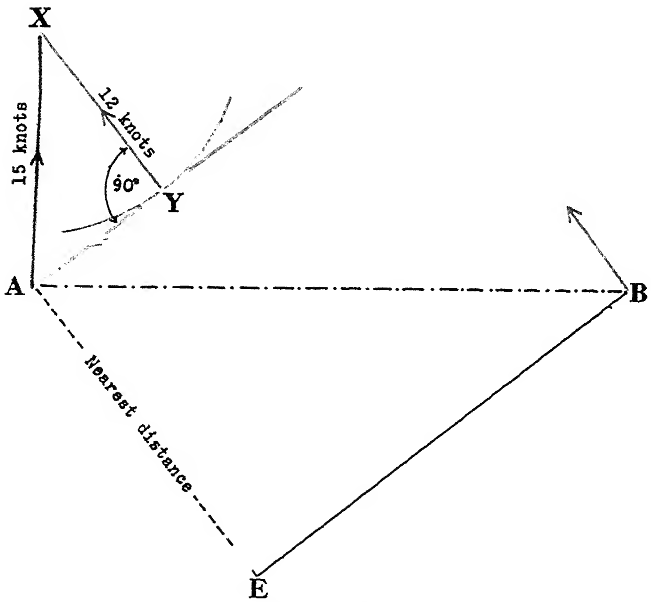


FIGURE 142.

Example 17. To Find the Shortest Distance to which a Ship can Approach Another Ship that is Proceeding at a Greater Speed.

(a) *In figure* 142 *a sloop* (*B*) *with available speed* 12 *knots, sights a merchant ship* (A) *bearing* 270°—10 *miles. The sloop estimates the other ship's course and speed as* 000°, 15 *knots, and wishes to close her. What course must the sloop steer?*

The slower ship must, if she cannot preserve the bearing, steer a course at right-angles to the virtual course.

Plot the merchant ship, A, and the sloop, B.

Plot the merchant ship's course and speed. (AX)

With centre X and radius representing the sloop's speed, strike an arc.

Draw AZ, the tangent to the arc, and from X drop a perpendicular on AZ, cutting AZ in Y.

In the speed triangle AXY:

AX=merchant ship's course and speed.
YX=sloop's course and speed.
YA=sloop's relative course and speed.

It is clear from this triangle, in which the angle XYA is equal to 90°, that YA is the best relative course that the sloop can make good.

To make good YA, the sloop must steer the course YX, and the shortest distance to which she can approach A is the distance AE.

Answer. The sloop must steer 322°.

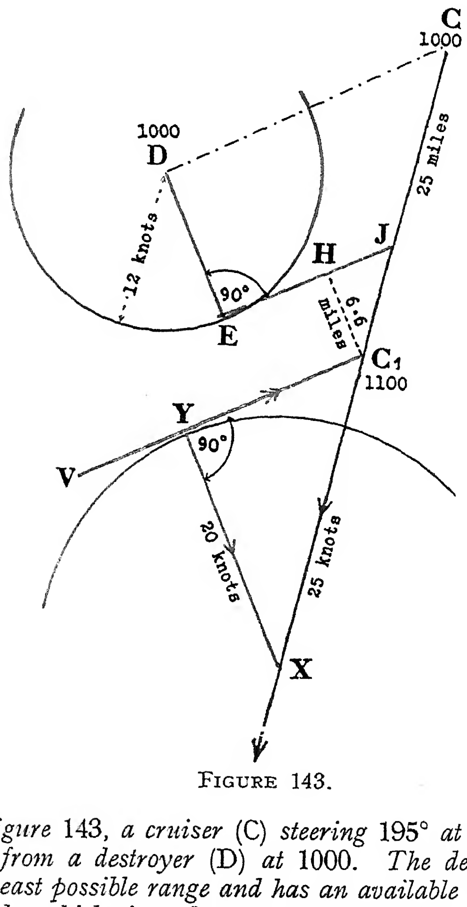


FIGURE 143.

(b) *In figure* 143, *a cruiser* (C) *steering* 195° *at* 25 *knots is* 065° —25 *miles from a destroyer* (D) *at* 1000. *The destroyer wishes to close to the least possible range and has an available speed of* 12 *knots until* 1100, *by which time she can raise steam for* 20 *knots. What course must the destroyer steer? What is the least range to which she can close, and at what time will she arrive at this range?*

Plot the destroyer's position at 1000, D, and the cruiser's positions at 1000 and 1100, C and C_1.

From C_1 plot the cruiser's course and speed for one hour. (C_1X)

With centre X, draw a circle of radius representing the destroyer's speed at 1100. (20 knots.)

From C_1 draw the tangent to this speed circle. (C_1V)

Clearly VC_1 will be the best relative course for the destroyer to make good after 1100.

From X drop a perpendicular meeting VC_1 at Y.

In the speed triangle C_1XY:

C_1X=cruiser's course and speed.
YX=destroyer's course and speed.
YC_1=destroyer's relative course and speed.

With centre D, draw a circle of radius representing the destroyer's speed at 1000. (12 knots.)

Draw EJ, parallel to VC_1 and tangential to the 12-knot speed circle.

Clearly the best course for the destroyer to steer from 1000 to 1100 is DE, perpendicular to EJ and parallel to YX.

The destroyer will, therefore, be in position E at 1100 when the cruiser is at C_1.

The destroyer will be nearest to the cruiser at H where C_1H is perpendicular to VC_1, and she will reach this position at some time after 1100, given by the time taken to steam EH at the relative speed YC_1.

Answer. Course to steer $157\frac{1}{2}°$.
Shortest distance $6'{\cdot}6$ at 1138.

Example 18. Avoiding Action. A Ship of Greater Speed Keeping Outside a Certain Range of a Ship of Lesser Speed, that may Steer any Course.

In figure 144, *a cruiser* (B) *steaming at* 25 *knots sights an enemy battleship* (A) *bearing* 270°—15 *miles, steaming* 15 *knots. The cruiser wishes to pass to the south-west of the battleship. What is the most westerly course she can steer to keep outside a range of* 10 *miles from the battleship?*

Plot the battleship, A, and the cruiser, B.

With centre A, draw a 10-mile range-circle.

The cruiser must make good a relative course BC, tangential to the 10-mile circle.

The battleship's best course is at right-angles to the cruiser's relative course BC.

Plot the battleship's course and speed. (AX)

Through A draw B_1C_1, parallel to the cruiser's relative course BC.

With centre X and radius representing the cruiser's speed, cut B_1C_1 in Y.

In the speed triangle AXY:

AX=battleship's course and speed.

YX=cruiser's course and speed.

YA=cruiser's relative course and speed.

Therefore YX is the cruiser's most westerly course.

Answer. The cruiser must steer 192°.

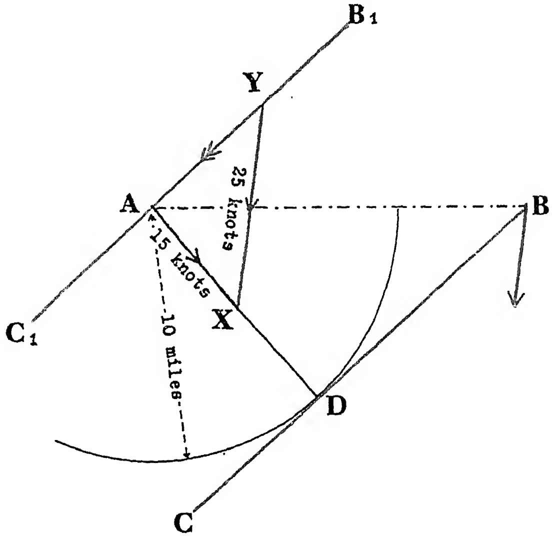


FIGURE 144.

Example 19. Avoiding Action. A Ship of Lesser Speed Escaping from a Ship of Greater Speed.

In figure 145 *a sloop* (A) *steaming* 15 *knots bears* 270°—15 *miles from a merchant ship* (B) *steaming* 10 *knots. The merchant ship is escaping from the sloop at dusk. If the sloop steers a steady course* 090°, *what are the best courses for the merchant ship to steer, and what will be the least distance apart of the ships on these courses?*

Plot the sloop, A, and the merchant ship, B.

Plot the sloop's course and speed. (AX)

With centre X describe a circle of radius representing the speed of the merchant ship.

Draw AC and AD tangential to the 10-mile circle.

AC and AD will be the two best relative courses for the merchant ship to make good.

Drop perpendiculars from X, cutting AC at Y and AD at Y_1.

In the speed triangles AXY and AXY_1:

AX=sloop's course and speed.

AY and AY_1=merchant ship's relative courses and speeds.

YX=merchant ship's best course to the south.

Y_1X=merchant ship's best course to the north.

If the merchant ship makes good AY or AY_1, clearly the least distance apart of the ships will be BM or BN, parallel to XY and XY_1.

Answer. Merchant ship's best course to the north : 042°.
Merchant ship's best course to the south : 138°.
Least distance apart of the ships 10 miles.

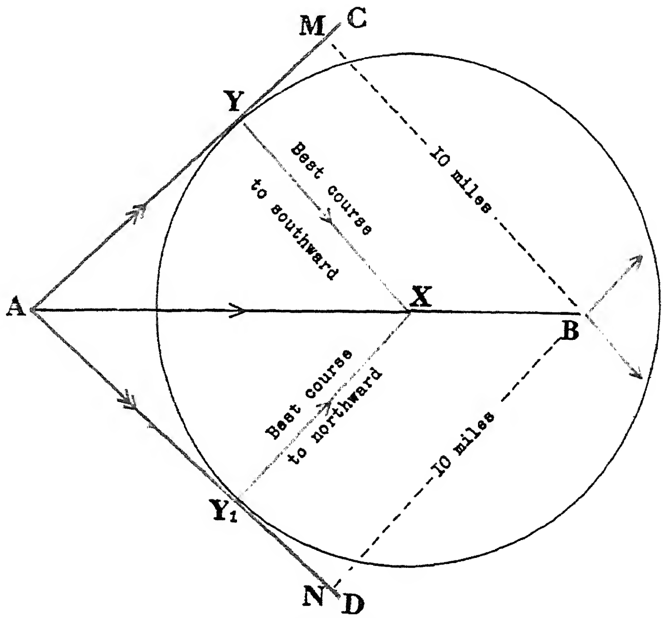


FIGURE 145.

Example 20. Avoiding Action. Typical Problem.

In figure 146, *a cruiser* (C) *is steering* 270° *at* 20 *knots. At* 1000 *she receives information that an enemy submarine* (S), *with a speed of* 9 *knots, is* 270°—20 *miles from her. The cruiser wishes to proceed westward as quickly as possible, and decides to avoid the submarine by* 6 *miles, passing north of her.*

What course must the cruiser steer, and when can she alter course back to 270°.

There are two actions open to the submarine, and the cruiser must guard against both of them. The submarine can be considered as :

(1) steering the best course to intercept the cruiser.
(2) steering the best course to prevent the cruiser from altering course back to the west.

To find the amount that the course must be altered.

Plot the cruiser, C, and the submarine, S.

With centre S draw a circle of radius 6 miles.

In order to keep 6 miles from the submarine, the cruiser must make good the relative course CL, tangential to the 6-mile circle. (This assumes that the submarine will steer at full speed at right-angles to CL.)

From S drop a perpendicular meeting CL at Y, and produce SY to X, where YX equals the course and speed of the submarine.

With centre X and radius representing the cruiser's speed, cut YC in A.

In the speed triangle AXY:

AX=cruiser's course and speed.
AY=cruiser's relative course and speed.
YX=submarine's course and speed.

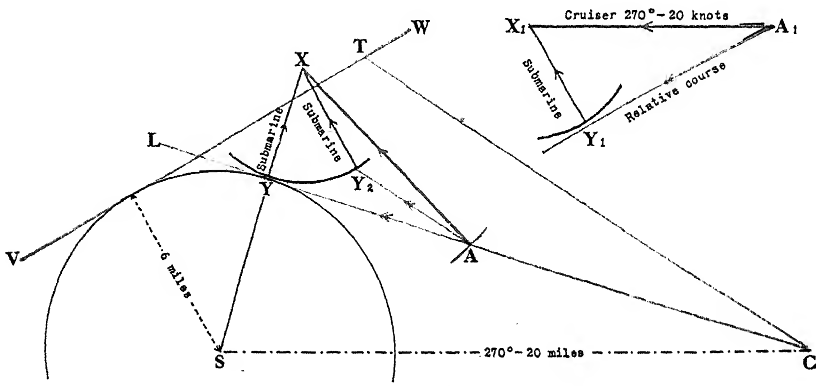


FIGURE 146.

To find the time the cruiser can alter course back to the west.

It must now be assumed that the submarine will steer at right-angles to the cruiser's relative course when the cruiser steers 270°. This will be her best action to prevent the cruiser from turning to the west.

Construct a separate speed triangle $A_1X_1Y_1$, where:

A_1X_1=cruiser's course and speed (on course 270°).
A_1Y_1=cruiser's relative course and speed.
Y_1X_1=submarine's course and speed.

Draw VW parallel to A_1Y_1 and tangential to the 6-mile circle round S.

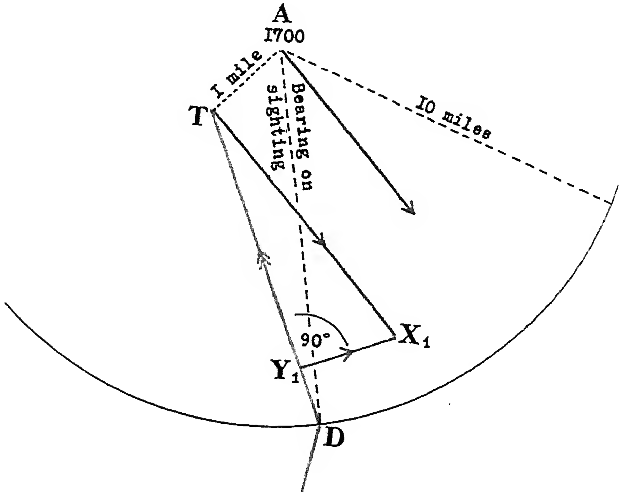
If the submarine had been steering a course Y_1X_1 throughout, the cruiser would not have made good the relative course CY. To find the relative course she would have made good, construct another speed triangle AXY_2, where:

Y_2X (parallel to Y_1X_1)=submarine's course and speed.
AX=cruiser's course and speed.
AY_2=cruiser's relative course and speed.

Draw CT parallel to AY_2, cutting WV at T.

The cruiser must, therefore, make good the relative track CT at the relative speed AY_2 before she can alter course back to 270°.

Answer. The cruiser must alter course to 315°, and can resume her original course of 270° at 1130. (17′·8 at 11·8 knots.)



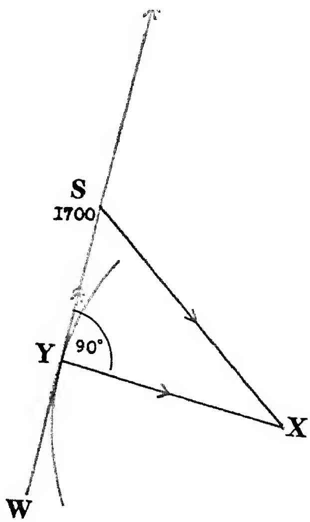


FIGURE 147.

Example 21. Submarine Attack. Typical Problem.

In figure 147, *a submarine* (S) *receives a report that a battleship* (A) *steering* 135° *at* 15 *knots, bears* 000°—20 *miles from her at* 1700.

The submarine wishes to proceed to an attacking position, one mile on the starboard beam of the battleship, steaming as slowly as possible while submerged. The visibility is 10 *miles, and the submarine will dive when the battleship is sighted. The submarine's maximum speeds are :*

on the surface, 13 *knots.*
submerged, 9 *knots.*

Required :

(a) *the submarine's course and speed on the surface.*
(b) *the time the submarine will dive.*
(c) *the bearing on which it is expected to sight the battleship.*
(d) *the submarine's course and speed when submerged.*
(e) *the time of arrival in the position to attack.*

Plot the battleship's position at 1700, A, and the submarine's position at 1700, S.

Plot the submarine's attack position relative to the battleship, T.

From S plot the battleship's course and speed, SX.

With centre X and radius representing the submarine's surface speed, draw a speed circle. The best relative course for the submarine to make good is WS, tangential to the submarine's speed circle.

From X drop a perpendicular cutting SW at Y.

In the speed triangle SXY :

SX=battleship's course and speed.
YX=submarine's course and speed on the surface.
YS=submarine's relative course and speed on the surface.

With centre A and radius the visibility, draw a visibility circle.

Produce YS, the submarine's relative course and speed on the surface, cutting the visibility circle at D.

D will be the point at which the submarine must dive.

Join DT, the course to be made good in order to reach the attack position when the submarine is submerged.

From T plot the battleship's course and speed, TX_1.

From X_1 drop a perpendicular meeting DT at Y_1.

In the speed triangle TX_1Y_1 :

TX_1=battleship's course and speed.
Y_1X_1=submarine's course and speed when submerged.
Y_1T=submarine's relative course and speed when submerged.

The time of arrival at D can be found by dividing the relative distance, SD, by the submarine's relative surface speed, YS.

The time of arrival at T can be found by dividing the relative distance, DT, by the submarine's relative submerged speed, Y_1T.

Answer. (*a*) Submarine's course and speed on the surface 105°—13 knots.
(*b*) The submarine will dive at 1826.
(*c*) The battleship should be sighted bearing $344\frac{1}{2}$°.
(*d*) Submarine's course and speed when submerged $068\frac{1}{2}$°—6 knots.
(*e*) Submarine will arrive in the attack position at 1908.

GUNNERY PROBLEMS AND TARGET WORK

The following terms are used in target work.

The Line of Sight is the line joining the firing ship to the target.

Inclination is the angle, right or left, that the course of the target makes with the line of sight produced beyond the track of the target.

Rate and Deflection. Rate and deflection are the two components of the relative track, the former being the component resolved *along* the line of sight, and the latter being the component resolved *at right-angles* to the line of sight.

Since the direction of the line of sight is generally altering, although the relative track remains the same, it follows that the rate and deflection must be continually changing.

To open or close at a steady rate the bearing must remain constant, unless the course or speed is being continually altered, or both are being altered together.

When the direction of the line of sight is at right-angles to the virtual track, the rate at that instant must be nil.

To find the rate and deflection at any instant, it is necessary to draw only the relative course and speed through the position of the firing ship, and drop a perpendicular from its end on the line of sight.

NOTE. (1) Deflection is **Right** when the guns have to point to the right of the target and vice versa.
(2) An **Opening** or **Closing** rate of 100 yards per minute is equal to 3 knots.

In the following examples :
F will denote the firing ship.
T will denote the target.
the course and speed of the firing ship will be coloured Red.
the course and speed of the target will be coloured Blue.
the relative course and speed of the firing ship will be coloured Green.

Example 22. To Find the Rate and Deflection.

In figure 148, *a firing ship* (F) *steering* 270° *at* 15 *knots, bears* 165°—6 *miles from a target* (*T*) *steering* 250° *at* 9 *knots. What are the rate and deflection ?*

Plot the firing ship and the target, F and T respectively.
From F plot the course and speed of the firing ship, FX.
From X lay back the course and speed of the target, YX.
In the speed triangle FYX:

YX = target's course and speed.
FX = firing ship's course and speed.
FY = firing ship's relative course and speed.

From Y drop a perpendicular on the line of sight, cutting FT at Q. Then:

FQ is the rate (4·8 knots) 160 yards per minute *closing*.
YQ is the deflection, 5·5 knots *right*.

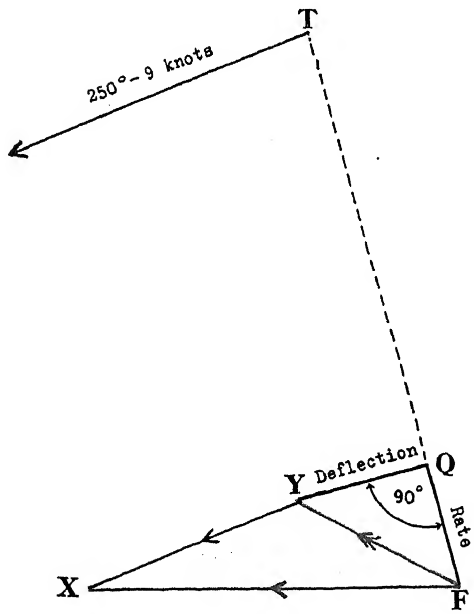


FIGURE 148.

Example 23. *In figure* 149, *a ship* (F) *is ordered to carry out the following procedure:*

(1) *Open fire from a position* 21,000 *yards*—40° *abaft the starboard beam of a target* (T) *steering* 090° *at* 8 *knots.*
(2) *Steer such a course that the rate:*
   (a) *is* 300 *yards per minute, closing, at the moment of opening fire.*
   (b) *will decrease to* 100 *yards per minute, closing, when the ship is on the target's starboard beam.*
(3) *Continue steering this course until the rate becomes nil.*

(4) *When the rate is nil, alter to such a course that the rate will be 300 yards per minute, opening, when the ship is 40° before the starboard beam of the target. This will be the moment of ceasing fire.*

(5) *Maintain a constant speed throughout the run.*

Required :

(a) *the course and speed of the firing ship when opening fire.*

(b) *the range and bearing of the target when the rate is nil.*

(c) *the number of degrees it will be necessary to alter course when the rate is nil.*

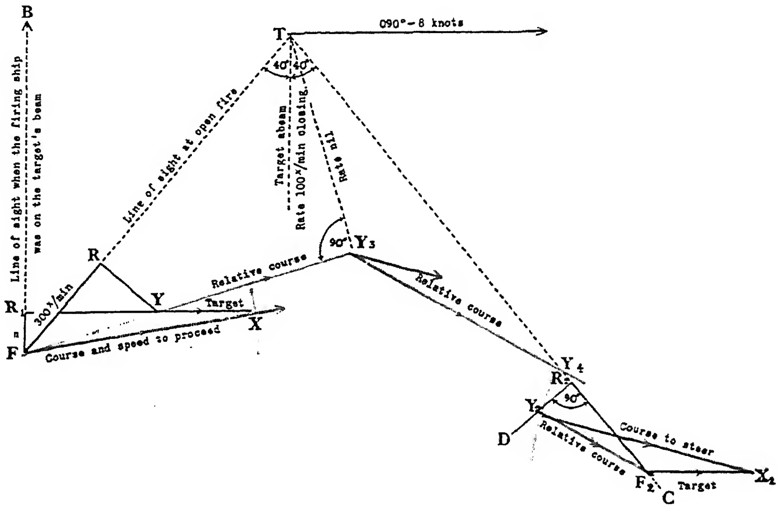


FIGURE 149.

To find the course to steer on opening fire.

Plot the position of the firing ship, F, and the target, T.

FT represents the line of sight on opening fire.

From F draw FB, representing the line of sight when the firing ship is on the target's beam.

From F measure along the two lines of sight, FT and FB, the rates of 300 yards per minute closing, and 100 yards per minute closing, FR and FR_1, respectively.

From R and R_1 construct perpendiculars. The intersection of these perpendiculars will give the first relative course, FY.

NOTE. RY will be the deflection at the moment of opening fire.
R_1Y will be the deflection when the firing ship is on the target's beam.

From Y plot the target's course and speed, YX.

Join FX.

In the speed triangle FXY :

YX=target's course and speed.

FX=firing ship's course and speed at the moment of opening fire.

FY=firing ship's relative course and speed.

To find the point when the rate will be nil.

The rate will be nil when the target bears at right-angles to the relative course, FY, that is, at the point Y_3. The time when the firing ship will arrive at Y_3 is found by dividing the relative distance, FY_3, by the relative speed, FY.

To find the course to steer when the rate is nil.

An unusual construction is used for this speed triangle.

Draw the line of sight when the firing ship is 40° before the beam of the target, TC.

Take any point on this line, R_2.

Through R_2 draw a perpendicular to the line of sight, R_2D.

From R_2 measure along the line of sight, TC, the required rate of 300 yards per minute opening, R_2F_2.

From F_2 plot the target's course and speed, F_2X_2.

With centre X_2 and radius representing the speed of the firing ship (given by FX in the original speed triangle), cut R_2D in Y_2.

In the speed triangle $F_2X_2Y_2$:

F_2X_2=target's course and speed.

Y_2X_2=firing ship's course and speed to fulfil the necessary requirements.

Y_2F_2=firing ship's relative course and speed.

The firing ship, therefore, will alter course at Y_3, and the time of ceasing fire can be found by dividing the relative distance, Y_3Y_4, by the relative speed, Y_2F_2.

Answer. Course and speed of the firing ship on opening fire: 081°—18¾ knots.

Range and bearing of the target when the rate is nil: 11,800 yards—344°.

When the rate is nil the firing ship will alter course to 099°.

Example 24. *In figure* 150, *a ship* (F) *is in position* 16,000 *yards*—070° *from a target* (T) *steering* 180° *at* 6 *knots.*

Firing will be to starboard and the ship is ordered to carry out the following procedure :

(1) *Open fire with a rate of* 400 *yards per minute, closing.*

(2) *Steer such a course that the rate will have decreased to* 100 *yards per minute after the run has been in progress for ten minutes.*

What must be the course and speed of the firing ship ?

Plot the positions of the firing ship F, and the target T, at the moment of opening fire.

FT represents the line of sight at the moment of opening fire.

From F measure along the line of sight, FT, the distance FR, equal to a rate of 400 yards per minute closing for 10 minutes. (2 miles.)

From R draw a perpendicular RC.

With centre F and radius a distance equal to a rate of 100 yards per minute closing for 10 minutes ($\frac{1}{2}$ mile), describe a circle.

With a set square, make an angle TYD equal to 90°, so that Y is a point on RC, and YD is tangential to the 100 yards per minute rate-circle.

Join FY, and this will be the relative course and distance for 10 minutes.

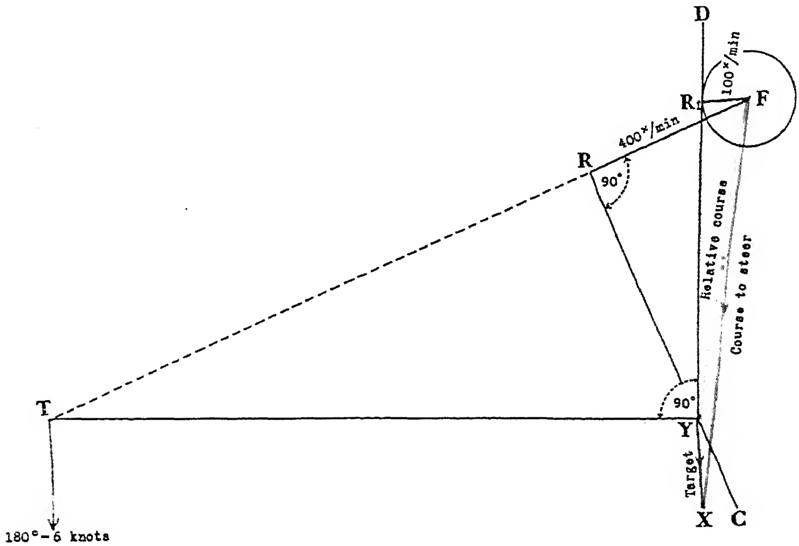


FIGURE 150.

From Y plot the target's course and speed for 10 minutes, YX.

Join FX.

In the speed triangle YXF:

YX=target's course and speed for 10 minutes.
FX=firing ship's course and speed for 10 minutes.
FY=firing ship's relative course and speed for 10 minutes.

NOTE. It is seen that the rate at the moment of opening fire is 400 yards per minute closing. Also R_1F is parallel to TY, the line of sight after 10 minutes. Therefore the length R_1F is a measure of the rate at this moment, and the necessary requirements are fulfilled.

Answer. Course and speed of the firing ship 190°—27 knots.

TORPEDO-FIRING PROBLEMS

These may be either problems of attack or defence, and in general they can be solved on a Battenberg course indicator. If they are solved by this means, the ship that is being attacked should always be put in the centre of the instrument.

Attack Problems. It may be necessary to obtain the following information:

(1) The course and speed required to proceed to the firing position. (An ordinary changing-station problem.)
(2) The extreme firing range (E.F.R.) and whether the enemy is inside it.
(3) The director angle, and the deflection for firing.
(4) The number of degrees the enemy must alter course away to outrange the torpedoes.
(5) When the torpedoes will cross the enemy's track.
(6) How much the enemy must alter course to comb the track of the torpedoes.

In all these problems it is necessary to estimate the enemy's course and speed, and on this estimation it is possible to obtain the relative course of the torpedoes, the speed and running range of which are known. From this information the required director triangle can be drawn.

Defence Problems. It may be necessary to obtain the following information:

(1) Is the ship inside the enemy's torpedo range?
(2) How much must the course be altered, away or towards, in order to comb the torpedo tracks?
(3) When will the torpedoes cross the ship's track?
(4) How much must the course be altered away to outrange the torpedoes?

In these problems the speed of the torpedoes must be estimated, but the course and speed of the target ship are known, and also, on the assumption that the torpedoes will hit the ship if no avoiding action is taken, the relative course of the torpedoes.

Example 25. Attacking Problem.

In figure 151, *a destroyer* (D) *steaming* 28 *knots, wishes to attack a battleship* (B) *bearing* 000°—14 *miles from her. The battleship's course and speed are estimated as* 212°—20 *knots. The destroyer's torpedoes are set to run* 13,000 *yards at* 25 *knots, and she wishes to make a track angle of* 90° *and have an over-run of* 25 *per cent.*

Required:

(a) *the destroyer's course to proceed to the attack.*
(b) *the time taken by the destroyer to reach the attacking position.*

(c) *the number of degrees that the destroyer must swing to fire on reaching the attacking position if her torpedo tubes are trained RED* 70°.

Plot the battleship's position, B, and the destroyer's position, D.

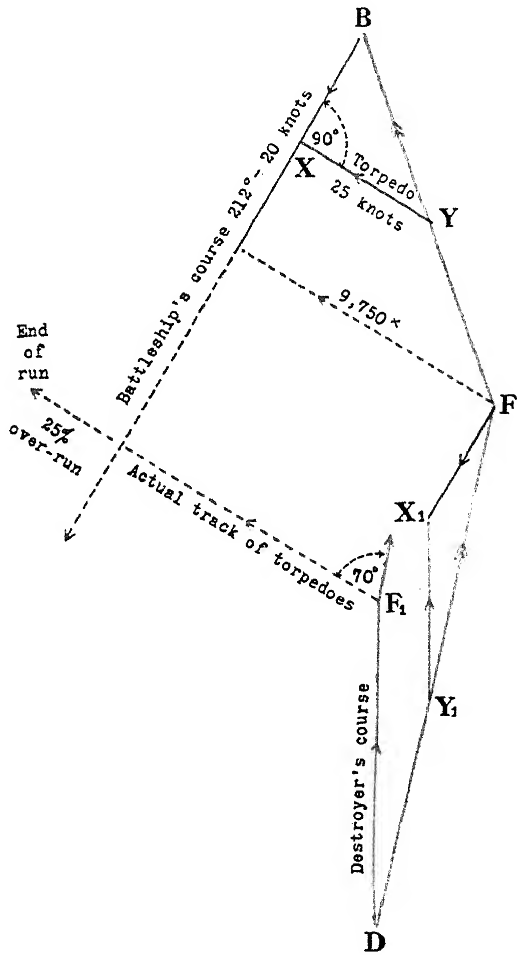


FIGURE 151.

At B construct a director triangle, making the track angle 90°.

In the director triangle BXY:

BX=battleship's course and speed.
YX=track of torpedo.
YB=relative track of torpedo.

The firing position must lie on the relative track of the torpedoes at a point which is 13,000 yards less 25 per cent., that is, 9,750 yards from the battleship's track.

Plot the relative firing point, F, on BY produced.

Join DF, which is the relative track the destroyer must make good.

From F plot the battleship's course and speed for one hour, FX_1.

With centre X_1 and radius representing the destroyer's speed, cut FD in Y_1.

In the speed triangle FX_1Y_1 :

FX_1=battleship's course and speed.
Y_1X_1=destroyer's course and speed.
Y_1F=destroyer's relative course and speed.

To arrive at the firing position, the destroyer must steam the relative distance, DF, at the relative speed, Y_1F. (8·3 miles at 47 knots.)

She will, therefore, have to proceed from D on a course 002° for $10\frac{3}{4}$ minutes.

It is seen from the figure that it will be necessary to swing 10° to starboard before firing torpedoes.

Answer. 1. Attacking course 002°.
2. Destroyer will take $10\frac{3}{4}$ minutes to reach the firing position.
3. Destroyer must swing 10° to starboard before firing.

Example 26. Avoiding Problems.

The battleship under fire in example 25, wishes to avoid the torpedoes fired by the destroyer. How many degrees must she alter course at the moment of firing to ensure that the torpedoes pass 2000 yards from her? (*Figure* 152.)

With centre B draw a circle of radius 2000 yards.

The battleship must alter course so that the torpedoes make a relative track tangential to this circle.

Draw the tangents, FM and FN.

From F plot the torpedoes' track for one hour, FX.

With centre X and radius representing the battleship's speed, cut FM in Y and FN in Y_1.

In the speed triangles FXY and FXY_1 :

YX and Y_1X=battleship's courses and speeds.
FX=track of torpedoes.
FY and FY_1=relative tracks of torpedoes.

From these triangles it is seen that the battleship can turn :

(1) 24° *away* from the destroyer, in which event the torpedoes will pass 2000 yards away and cross her track astern.

(2) 24° *towards* the destroyer, in which event the torpedoes will pass 2000 yards away and cross her track ahead.

Answer. The battleship must alter course to 188° or 236°.

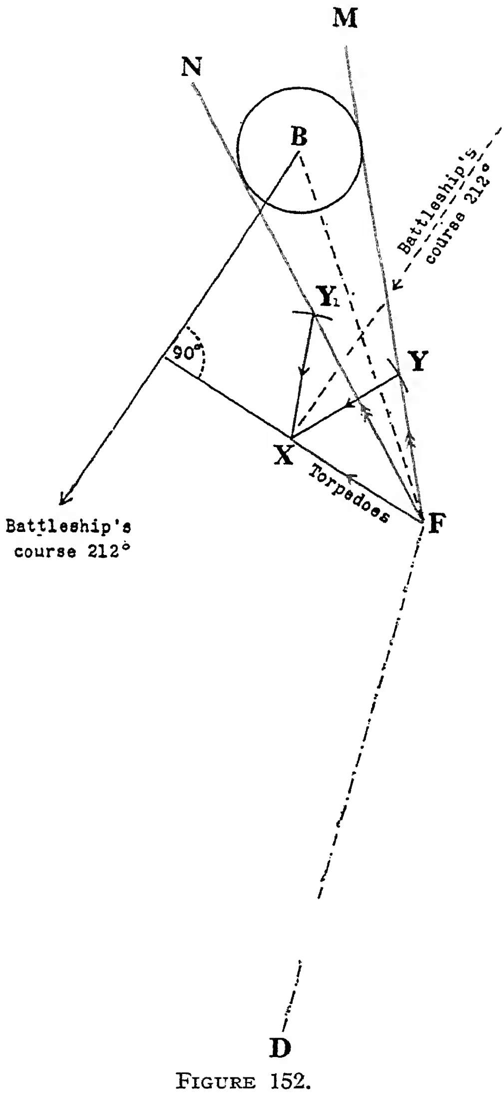


FIGURE 152.

Example 27. Turning a Definite Amount to Avoid Torpedoes.

In figure 153 *a squadron of* 5 *battleships* (S_1 *to* S_5) *is in single line ahead, three cables apart, steering* 040° *at* 18 *knots.*

An enemy destroyer flotilla (D) *fires torpedoes from a position* 45°—10,000 *yards on the starboard bow of the leading ship.*

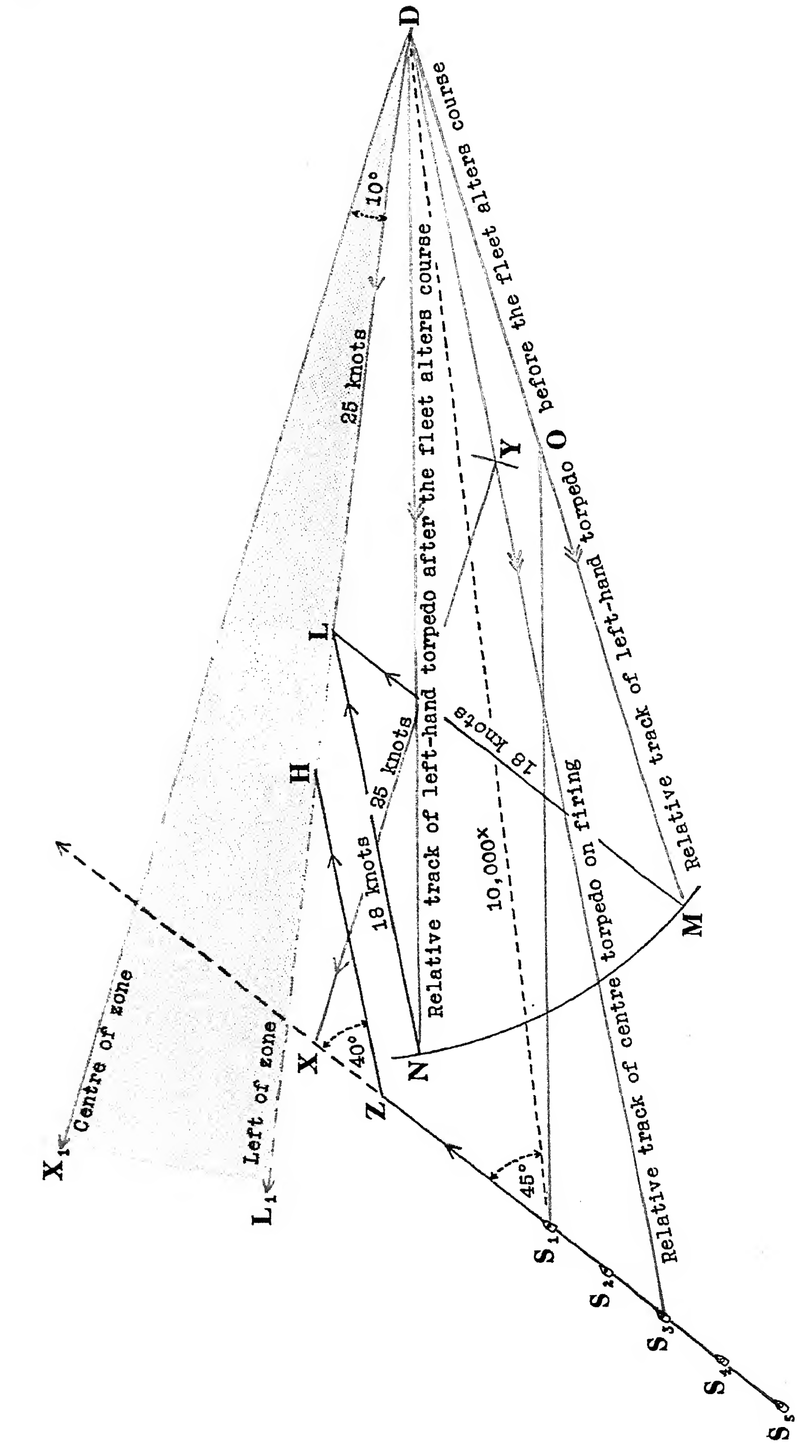

D
10°
25 knots
Relative track of left-hand torpedo after the fleet alters course
before the fleet alters course
Y
O
Relative track of left-hand torpedo
L
H
35 knots
18 knots
18 knots
10,000x
Relative track of centre torpedo on firing
M
N
X
40°
Z
45°
Centre of zone
Left of zone
X_1
L_1
Relative track of centre
S_1
S_2
S_3
S_4
S_5

FIGURE 53

It is assumed that :

(1) *the speed of torpedoes will be 25 knots.*
(2) *the point of aim will be the centre battleship.*
(3) *the spread of the torpedo zone will be 20°.*

The Admiral decides to alter course 40° towards the destroyers.

What is the latest time to alter course in order to avoid the torpedoes ?

Plot the position of the battle squadron, S_1 to S_5, and the destroyer flotilla, D.

Join DS_3.

DS_3 represents the relative track of the centre torpedo of the zone.

From S_3 plot the squadron's course for one hour, S_3X.

With centre X and radius representing the torpedo speed, cut S_3D in Y.

In the speed triangle S_3XY :

S_3X=squadron's course and speed.
YX=course and speed of the centre torpedo of the zone.
YS_3=relative course and speed of the centre torpedo of the zone.

Draw DX_1 parallel to YX, and DL_1, 10° to the left of DX_1.
Then :

DX_1 is the actual track of the centre torpedo of the zone.
DL_1 is the actual track of the most southerly torpedo of the zone.

Construct a speed triangle DLM, where :

DL=torpedo's course and speed along the track DL_1.
ML=squadron's course and speed.
DM=torpedo's relative course and speed to make good the track DL_1.

Construct a speed triangle DLN, where :

DL=torpedo's course and speed along the track DL_1.
NL=squadron's course and speed after it has altered course 40° towards the flotilla.
DN=torpedo's relative course and speed after the squadron has altered course.

Draw S_1O parallel to DN and cutting DM at O.

Clearly the squadron must alter course to 080° (40° to starboard) after the most southerly torpedo has run a relative distance, DO, at a relative speed, DM, at which time the leading ship, S_1, will have arrived at position Z.

Answer. The squadron must alter course after $2\frac{3}{4}$ minutes. (1′·75 at 38 knots.)

NOTE. It is seen in the figure that if the squadron alters course after $2\frac{3}{4}$ minutes, the leading ship, S_1, will just touch the left of the torpedo zone at H.

CHAPTER XIX

CHRONOMETERS AND WATCHES

(See Chapter XI in Volume I)

The marine chronometer is, in its essentials, simply an enlarged watch that owes its superior timekeeping qualities to one or two mechanical refinements embodied in it, and to the care taken in shielding it from various sources of error to which the ordinary watch is exposed. Its mechanism is by no means complicated, although its construction demands the most accurate workmanship and its adjustment requires a high degree of skill.

General Principle. The motive power of a chronometer is supplied by a coiled steel spring, called the *mainspring*, which transmits its energy through a train of wheels and pinions, so arranged that for one revolution of the wheel on which the mainspring acts, the last wheel of the train makes a large number of revolutions. In order that the motion of the last wheel, which is called the *escape wheel*, may be uniform, the rotation is checked by a device called an *escapement*, which acts on the teeth in such a way that they are permitted to escape one by one at equal intervals of time. These intervals are measured by the oscillations of a governing device called a *balance*, swinging to and fro under the action of a spring. This balance causes the escapement to release a tooth of the escape wheel at every vibration, or *beat*, and the escape wheel, in turn, imparts to the balance sufficient of the energy it receives from the mainspring, by means of the train of wheels, to keep it swinging. The remainder of the energy stored in the mainspring is expended in overcoming the friction and inertia of the train (which has to be started from rest at every beat) and in rotating the hands, which are connected with the train by means of a separate set of wheels and pinions, called the *motion work*, so as to indicate the time measured by the chronometer.

If the balance controlled the escapement with absolute accuracy, so that the intervals at which it released the teeth of the escape wheel were exactly equal in all circumstances, the chronometer to which it was fitted would be a perfect timekeeper. Owing to various unavoidable sources of error, this ideal is unattainable, although in a first-class instrument it is closely approached.

BRIEF DESCRIPTION OF A CHRONOMETER

The Framework and Case. The whole of the mechanism is enclosed in a brass case with a glass lid over the dial. This case is

suspended in gimbals in a wooden box provided with two lids, an inner one of glass, allowing the dial to be read with this glass lid closed, and an outer one of wood. The gimbals can be locked when necessary, as, for example, when the instrument is being transported. The fact of the instrument's being, in the ordinary course, freely suspended in the gimbals ensures that the face is horizontal, and that in consequence the weight of each moving part is supported by the end of the lower pivot, and not, as would occur in an inclined or vertical position, on the sides of both pivots. This ensures that the friction at these points is constant and a minimum. At the bottom of the brass case is a hole through which the key is inserted to wind the chronometer. This hole is normally kept closed by a revolving shutter, retained in place by a spring. The object of this shutter, as of the glass over the dial and the double lids fitted to the wooden box, is to ensure that dust and damp are, as far as possible, prevented from finding their way into the mechanism.

The Mainspring and Driving Mechanism. The mainspring is contained in a brass barrel, the outer end of the spring being fixed to the rim of the barrel and the inner end to the axle of the barrel. On the outside of the barrel is coiled a slender steel chain called the *fusee chain*, which leads from the barrel to a conical drum called the *fusee*. One end of the chain is hooked into the circumference of the fusee at its larger end and the other into the rim of the barrel.

When the chronometer is wound, the key is applied to the fusee, and the fusee, being rotated, winds the chain of the barrel on the fusee, and so, by revolving the barrel, winds up the mainspring.

The mainspring, when wound, tends to revolve the barrel, and, by means of the fusee chain, the fusee. The tension that it exerts diminishes as it unwinds, but this is compensated by the construction of the groove on the conical-shaped fusee in which the fusee chain runs.

It must be realized that the action of winding the chronometer turns the fusee barrel in the reverse direction to that in which it normally drives the clock. For this reason, to enable the chronometer to continue running when being wound, a maintaining spring is fitted. This spring is normally kept compressed to its fullest extent by the mainspring. When the chronometer is being wound, the maintaining spring comes into action and continues to drive the train of wheels. On the completion of the winding, the mainspring takes charge and the maintaining spring is compressed and remains ready for action when the chronometer is wound the next time.

To prevent any damage by overwinding, a device called the *stopwork* is fitted, which, when the winding is completed, prevents the fusee from being further revolved.

An *Up and Down* indicator, graduated from 0 to 56 hours, is fitted on the dial of the chronometer. An arm, which is connected to the fusee, moves over the graduated arc on the dial and shows the time the chronometer has run since it was wound.

THE PRINCIPAL CAUSE OF ERROR IN TIMEKEEPING

The first, and by far the most important, source of error is the variation of temperature to which the chronometer is exposed. It is not too much to say that, if this could be eliminated, the performance of a first-class chronometer would be practically equal to that of a good astronomical clock on shore, a standard that is far from being attained at present. Variation in temperature affects only the balance and balance spring. Such causes of error as influence the other portions of the mechanism are comparatively trivial, and will be briefly discussed later.

The effect of a rise of temperature upon the timekeeping qualities of a balance is twofold: it diminishes the strength of the balance spring, and, by causing the balance to expand, increases its moment of inertia, both of which changes increase the period of vibration of the balance. The effect of the former is roughly five times as great as that of the latter, and the total alteration produced in an uncompensated balance has been found to be a change of over 6 seconds in the daily rate for each degree of increase or decrease in temperature.

Compensation for Temperature. On account of the large alteration in rate which would be caused by a slight change of temperature, it is necessary to compensate for the effects produced on the balance and balance spring by such changes.

The method generally adopted at the present time is to mount the balance weights on laminated strips formed with an outer layer of brass and an inner layer of steel, fused together. The necessary compensation is obtained by making use of the different coefficients of expansion of the two metals.

This compensation is, in actual fact, correct at only two temperatures. Between these two temperatures a chronometer so compensated will gain, and outside them it will lose. In a well-adjusted instrument, however, the error at the temperature midway between the two standard temperatures (which are generally taken as 45° and 90°F.) does not exceed 2 seconds per day.

The discovery of invar, an alloy with a negligible coefficient of expansion, and of elinvar, an alloy with elastic constants that are practically independent of temperature, has made possible the construction of a balance and balance spring that enable the chronometers to remain almost unaffected by change of temperature.

Testing Chronometers at the Royal Observatory. Chronometers, before being purchased for the Royal Navy, are subjected to very severe tests at the Royal Observatory at Greenwich, in

order to determine their performance at various temperatures. These tests extend over 29 weeks and comprise observations of the rate at temperatures up to 98°F.

After repair and before re-issue on service, chronometers are subjected to similar tests at the Royal Observatory.

The limits within which the daily rate of a chronometer should lie, before the chronometer is supplied to one of H.M. ships, are given in Volume I and in the preface to *Form S 384, Daily Comparisons and Errors of Chronometers and Watches.*

MINOR SOURCES OF ERROR

Isochronism. The motion of a balance is, theoretically, isochronous—that is to say, it describes long or short arcs in equal times—yet in practice, owing to friction and the impossibility of making a perfect spring, this does not always occur. Unless a spring is isochronous, errors in timekeeping will occur whenever the arc described by the balance varies. The extent of this arc depends upon the impulse transmitted to the balance by the mainspring, by means of the escapement, and although the use of a fusee renders this impulse practically constant as far as the force of the mainspring is concerned, the proportion of that force transmitted to the escapement varies with the friction in the train, which in turn depends on the age of the oil lubricating its pivots.

The length of the balance spring required to render the action of any particular chronometer isochronous can be found only by experiment, and usually the main difficulty is to get the short arcs slow enough.

It is found that all chronometers, when first set going, tend to accelerate their rates slightly for a few months, after which they settle down. The cause of this is obscure. It possibly results from a change in the molecular structure of the balance spring. Since an alteration in the length of a balance spring effects its isochronism, chronometers are never provided with regulators, such as are fitted in watches for adjusting them to keep mean time.

The Effect of Age upon the Rate. The effect of age on a chronometer is to produce a change in the viscosity of the oil, a deposit of dirt on the various parts of the mechanism, and a slight wear between the moving parts. These tend to decrease the arc of swing of the balance and so produce a slight acceleration.

Abnormal Variations in the Daily Rate. In spite of the compensation of a chronometer for temperature, variations in the rate sometimes occur. These variations are caused by:

(1) atmospheric conditions.
(2) magnetism.
(3) motion of the ship.
(4) damp.

WATCHES

The Mechanism of Chronometer Watches. Chronometer watches differ somewhat in constructional details from chronometers, although the general principles of their working are precisely the same. These watches are capable, with intelligent and careful handling, of giving results practically equal to those of a chronometer.

The constructional differences are, briefly, as follows :

(1) elimination of the fusee.
(2) use of the ' lever ' escapement.
(3) employment of a different form of compensation balance.

These will be referred to in the above order. They apply equally to the mechanism of chronometer, deck and pocket watches, but deck and pocket watches are inferior in quality to a chronometer watch and do not receive such careful adjustment.

Elimination of the Fusee. The size of a fusee prevents its use in a watch. For this reason a *going barrel* is fitted to practically every watch now purchased by the Admiralty. In this construction the mainspring is extremely long, and the barrel containing it is toothed on the outside and forms the great wheel. The use of a very long spring, much longer in proportion than that of a chronometer, renders the power acting on the train almost constant over the period for which the watch is designed to go, and so the complication of a fusee is not necessary. The going barrel also removes the necessity for a maintaining spring.

The Lever Escapement. This escapement is fitted to all watches now purchased by the Admiralty. For accuracy in timekeeping it is inferior only to the chronometer escapement, and as used in watches it possesses several practical advantages over the latter.

NOTE. The ideal escapement is that fitted to chronometers. It has, however, the disadvantage that it can be stopped by rotating or twisting the chronometer, and this is the reason for the precautions, detailed in Volume I, that have to be taken when chronometers are transported. A watch, however, must be able to withstand any movement and must, therefore, be given a different type of escapement.

The Balance and Balance Spring. These are somewhat different from those of a chronometer. The balance spring is usually spiral. The balance is provided with two brass and steel laminæ, as in the chronometer, but these do not carry any weights. Instead a number of large-headed screws are fitted in a series of holes tapped in the laminæ, and adjustment for position error can be made by shifting the positions of pairs of screws towards or from the ends of their respective laminæ.

Position Error. Since a chronometer watch is not slung in gimbals, it is tested for position error before being issued. A good

quality watch, well-adjusted, will have practically no position error, but to afford its timekeeping qualities the fairest scope, and to reduce, as far as possible, the weight of its mechanism, it should be kept dial up and never in any position other than horizontal.

NOTES ON REPAIRING SHIP'S CLOCKS

The navigating officer is responsible for winding and regulating the ship's clocks. When a clock is keeping irregular time or breaks down, the normal procedure is to return it to the central store, even though the defect may be a minor fault which would be revealed after a few minutes' careful inspection.

It may happen that several clocks become defective at a time when there are no others available to replace them and when there may be no opportunity for drawing new stores, or for sending them for expert repair, for some considerable time.

In these circumstances the navigating officer may easily repair the defective clocks himself because, although the work requires care and patience, it does not require a high standard of mechanical knowledge.

Clock Mechanism. Nearly all clocks are fundamentally the same and consist of:

(1) a mainspring.
(2) a balance and escapement.
(3) a chain of gear wheels to drive the hands.

The escapement is worked directly off the mainspring and its rate of movement is controlled by the balance. The hands are operated by a train of wheels. The minute hand is fitted over the axle of, and thus driven by, the second wheel in the train from the mainspring. The hour hand is driven by 12 to 1 gearing from the minute hand. Both the hour and minute hands are friction driven.

Types of Clocks. There are two types of clocks in general use in H.M. ships.

(1) *Pattern* 306. This clock has a large dial and a black metal case with a brass rim. It is fitted with a ' second ' hand.

(2) *Pattern* 304. This clock is smaller and more compact than pattern 306. It has a chromium rim to the dial and is not fitted with a ' second ' hand.

Investigating Breakdowns and Faulty Running. When it is decided to examine a faulty clock, there are several points to be checked before it can be dismantled.

(1) Make certain that the clock has been wound.
(2) Check that the hands do not foul against:

  (*a*) the face of the dial.
  (*b*) the glass cover.
  (*c*) one another.

Having dealt with these points, proceed to dismantle a pattern 306 clock in the following sequence:

(1) Remove the 'second' hand by gently pulling it off.

(2) Remove the pin securing the hour and minute hands and lift them off.

(3) Unscrew the dial, which is secured by three small screws, and lift it off.

(4) Remove the clock mechanism which is secured to a wooden base by four screws.

The mechanism can now be examined. The fault may be in the balance or escapement mechanism, the mainspring and wheels, or in the regulator.

Most failures occur in the balance and escapement mechanism. This part of the clock should therefore be examined first.

1. Balance and Escapement Mechanism. Failure may be caused by:

(*a*) *Loose balance bearings.* The balance bearings consist of two square-headed adjustable screws. If the balance is loose, screw up the bearings but take care that the balance is able to swing freely.

(*b*) *The balance spring's (or hairspring's) working loose at one end.* The outer end of the hairspring is secured by a small metal wedge. If the wedge has become loose it must be gently forced home. When this has been done, it is necessary to make certain that the hairspring is in balance. If the clock beats with an even tick the spring is in balance: if not, the wedge must be removed and the hairspring lengthened or shortened and the wedge replaced. Repeat this procedure until the beat is even.

NOTE. An alternative method to removing the wedge is to make the adjustment by moving, with a screwdriver, the brass ring round the balance staff, called the 'collet', to which the inner end of the hairspring is secured. This method is not recommended because it will affect the regulator, as described in para. 2 (*c*) below.

(*c*) *The hairspring's being out of balance.* This will be shown by the clock's beating with an uneven tick. Proceed as explained in para. (*b*) above.

(*d*) *Dust and congealed oil on the bearings, hairspring, or escapement stops.* Carry out the procedure for cleaning a clock, as described later in this section.

2. Mainspring and Wheels. The robust construction of these parts of the mechanism is usually proof against failures, but the following faults may occur:

(*a*) *The mainspring overwound by careless winding.* If the mainspring is not broken or has not become unshipped, fit the winding key and take the strain of the spring; then, with the point of a screwdriver, free the retaining-spring catch, and ease the tension on the mainspring by allowing the winding key to come back half a turn.

NOTE. It is not advisable to attempt to dismantle the mainspring.

(*b*) *Wheel teeth damaged or wheels out of mesh, caused by dropping or severely jolting the clock.* If this has occurred and the need is urgent, it may be possible for a skilled artificer to make a new wheel.

(*c*) *Failure of the regulator.* It sometimes happens that when the regulator is hard over to fast (or slow) the clock continues to lose (or gain). The regulator consists of an adjustable clamp through which the hairspring passes and its action is merely to lengthen or shorten the effective length of the hairspring.

When this fault occurs it means that the length of the hairspring must be adjusted. Set the regulator to its central position, and remove the wedge securing the outer end of the hairspring. Then according as the clock is gaining or losing, increase or decrease the length of the spring. Replace the wedge.

When the length of the hairspring has been adjusted, it will probably be found to be out of balance, as described in para. 1 (*c*) above. If this occurs the wedge should not be touched but the adjustment should be made by rotating the brass collet as described in the note to para. 1 (*b*) above.

Cleaning a Pattern 306 Clock.

(1) Dismantle the clock in the way already described.

(2) Fit the winding key and take the strain of the mainspring; then with the point of a screwdriver free the retaining-spring catch and ease the tension on the spring by allowing the winding key to come back half a turn. This is most important because if it is not done and an attempt is made to remove the balance mechanism, the clock will probably be wrecked. It will be necessary to ease all the tension of the mainspring, and this procedure must therefore be repeated by half turns.

(3) Ease back the four plate screws.

(4) Take out the wedge securing the outer end of the hairspring, unreave the spring from the regulator and remove the balance and the spring.

(5) Remove the escapement and escape wheel by unscrewing the lower bearing.

(6) The remainder of the mechanism can now be removed, but this will usually be unnecessary unless it is extremely oily and dirty.

(7) Clean the wheel teeth, pivots and pallets of the escapement with a tooth brush and a piece of fine linen.

(8) Clean the pivot holes in the plates either with a match sharpened to a fine point or with a needle.

(9) Lubricate, with fine typewriter oil, the pallets and the small steel pin on the balance staff, using the match or needle.

(10) Replace all parts, starting from the mainspring and working towards the balance, closing the plates as the work proceeds. After

the plates are closed, replace the balance, using the adjustable bearings.

NOTE. It is necessary to take great care when the balance is handled because the spring is easily bent, and when bent, it can be straightened only by an expert.

(11) Oil, very lightly, all the bearings by the method described in (9) above.

Pattern 304 Clock. This clock has a more delicate mechanism and is of finer workmanship than the pattern 306 clock. Although the pattern 304 clock is not so liable to break down as the pattern 306, the foregoing remarks concerning pattern 306 clocks apply, in general, to it.

Dismantle the clock in the following sequence:

(1) Unscrew the four small screws in the circumference of the chromium dial and withdraw the whole clock from its case.

(2) Remove the hands. Both hands are held by friction.

(3) Unscrew the four small screws on the side of the aluminium case and withdraw the mechanism, including the key.

NOTE. Do not unscrew the two large screws on the back that secure the mechanism.

(4) Unscrew the three screws securing the brass plate to the three metal lugs on the back of the clock.

Cleaning the Pattern 304 Clock. If the balance and escapement mechanism are dirty and require cleaning, proceed in the following sequence:

(1) Hold the mechanism so that one finger presses against the fourth wheel and stops the clock.

(2) Unscrew the two steel screws securing the steel bed of the balance and escapement mechanism to the brass bed of the clock, and remove the combined balance and escapement mechanism.

(3) Ease the tension of the mainspring by easing the finger pressure on the fourth wheel and allowing the wheels to revolve slowly until the mainspring is completely unwound.

(4) Dip the combined balance and escapement mechanism into petrol.

(5) Replace the mechanism.

CHAPTER XX

THE ERROR OF THE CHRONOMETER

The error of the chronometer can be found by several methods, all of which involve comparison with a deck watch since the chronometer itself cannot be moved. These methods are based on :

(1) W/T time signals, details of which are given in the *Admiralty List of Wireless Signals*. (Volume I gives a list of stations and their functions, and Volume II describes the methods of transmission and the general regulations.)

(2) the chronometer-depot clock.

(3) the telegraphic time signal at 1000 G.M.T. received by post offices in the British Isles.

(4) visual time signals, details of which are given in the *Light Lists*. (This method, however, is falling into disuse.)

(5) sextant observations of the Sun in the following order of importance :

- (*a*) equal altitudes of the Sun.
- (*b*) the mean of two sets of altitudes of the Sun, one taken before and the other taken after meridian passage.
- (*c*) the mean of two sets of altitudes taken either before or after meridian passage.

Similar observations of stars provide slightly more accurate results, for reasons that will be explained, but they are difficult to take, particularly without a sextant stand, and a sextant stand is seldom available for a navigating officer. It is therefore recommended that the following observations, given in order of importance, should be considered only if the observations (*a*) to (*c*) cannot be taken :

- (*d*) stars of equal or similar altitudes east and west of the meridian, taken as far as possible at the same time.
- (*e*) stars of dissimilar altitudes east and west of the meridian.
- (*f*) the mean of two sets of altitudes of a star east or west of the meridian.

Comparison with the Deck-Watch Time. In methods (1) and (4) the deck watch can be compared with the chronometers immediately before and after the time signal is taken. In the other methods the deck watch must be landed, and from the comparisons made before and after landing, a *mean comparison* corresponding to the

time at which the deck-watch error is obtained must be calculated. The procedure for making both direct and mean comparisons is described in Chapter XI of Volume I.

Error of the Chronometer by Astronomical Observation. The G.M.T. of an observation can be expressed in terms of the hour angle. Thus:

$$\text{G.M.T.} = \text{H.A.T.S.} - \text{E} \pm \text{longitude}$$

or

$$\text{G.M.T.} = \text{H.A.}\ast + \text{R.A.}\ast - \text{R} \pm \text{longitude}$$

—according as the heavenly body is the Sun or not. In these equations, the only unknown quantity on the right-hand side is the hour angle, and this can be calculated by the methods explained in Volume II if the latitude, longitude and altitude are known.

Since any inaccuracy in the altitude affects the error of the chronometer and therefore every position of the ship obtained with that chronometer, it is essential that the altitude should be measured as accurately as possible within the limits imposed by sextant observation. For this reason an artificial horizon is employed instead of the sea-horizon, which is always suspect on account of refraction.

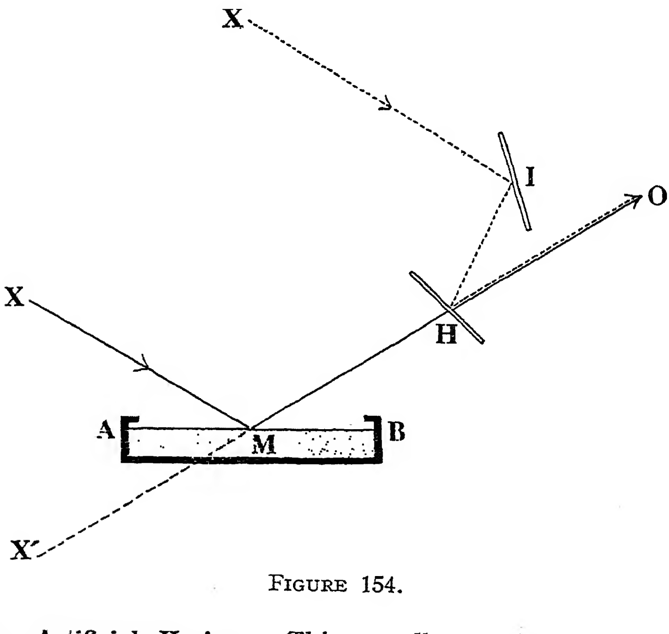


FIGURE 154.

The Artificial Horizon. This usually consists of a shallow rectangular trough filled with mercury, and the heavenly body is observed by reflection in the horizontal mirror that the surface of the mercury forms.

In figure 154, X is a heavenly body, one ray from which is reflected at I, the index glass of the sextant, and H, the horizon

glass, and the other at M on the surface of the mercury. XI and XM are parallel, and after reflection their final path is HO. An observer at O therefore sees an image of X in the direction OX′, and he measures the angle XMX′.

Since, by the optical law of reflection, the angle XMA is equal to the angle OMB, the angle XMA is equal to the angle X′MA. The angle measured in the artificial horizon, when corrected for the instrumental errors of the sextant, is therefore twice the actual altitude of the heavenly body, and there is no dip because there is no height of eye.

To prevent dust from settling on the surface of the mercury and to avoid the disturbing effects of wind, the trough is protected by a roof of plate glass, shaped as indicated in figure 155.

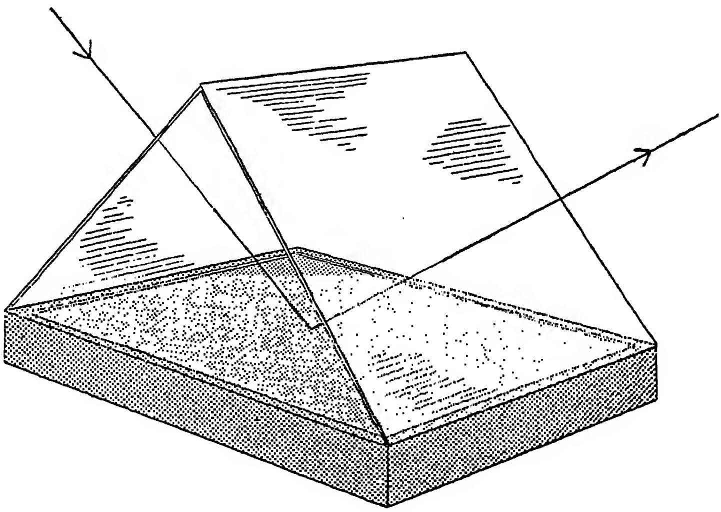


FIGURE 155.

The two sheets of glass are held in position by a metal frame (which is not shown in the figure) and the fit is sufficiently loose to avoid any possibility of the warping that would result on account of the unequal coefficients of expansion if the glass were held fast by the metal.

Every effort is made to grind the surfaces of the glass plates so that they are parallel, but there is always the possibility that they are not, and a small refractive error results. It is therefore advisable at times to mark one side of the roof with white paint to distinguish it from the other, and to take half the observations with the mark on the right and half with the mark on the left.

Observations of a heavenly body cannot be taken in an artificial horizon when the altitude is less than 15°, because the incoming ray fouls the edge of the mercury trough.

Precautions to be Taken when an Artificial Horizon is Used. These precautions are conveniently summarized in the order in which they must be taken.

(1) The place at which observations are to be made should be on solid ground, remote from traffic and sheltered, as far as possible, from the wind. Nearby traffic causes the mercury to tremble and makes observation impossible. The place must also give an unrestricted field of view so that the heavenly body may be observed at the required altitude on both sides of the meridian, and it should not be in the immediate vicinity of mountains because the presence of a great land mass causes the direction of gravity to deviate slightly from the vertical, and the surface of the mercury, which settles at right-angles to the direction of gravity, is therefore no longer horizontal. Since the position of the place must be known exactly, time is saved by selecting one that is already marked on the chart. Otherwise the position must be plotted by sextant angles.

(2) The trough must be thoroughly cleaned and pointed in the direction required, with the roof over it except at one end, before the mercury is poured in at this free end. To prevent the scum and impurities from escaping into the trough, it is advisable to place a finger over the hole of the bottle and to invert the bottle long enough for the scum to rise through the mercury. The mercury that enters the trough when the finger is removed is then clear, and if care is taken not to drain the bottle, the scum does not reach the trough. If it does, the surface of the mercury is clouded. Once the mercury has been satisfactorily poured into the trough, the roof can be adjusted to cover both ends.

(3) When the actual observations are being taken, the eye should be placed so that the image of the heavenly body appears in the centre of the trough.

Observations of the Sun in an Artificial Horizon. A star has no appreciable diameter, and its altitude is measured when the direct and reflected images coincide. (Both images are actually reflected, but it is convenient to refer to that which is seen by a single reflection in the mercury as the direct image to distinguish it from that which is doubly reflected by the sextant mirrors.) The extent of the Sun's diameter, however, makes accurate coincidence of the two images of the Sun's disc impossible, and the observer is compelled to make the two images touch.

In figure 156, S is the Sun, the upper and lower limbs of which to an observer O are U and L. In the artificial horizon, their images are U' and L'.

If the observer takes the altitude of the Sun's lower limb in the artificial horizon (denoted by Obs. Alt. $\underline{\underline{\odot}}$) the angle measured is LOL', and :

$$\begin{aligned} \angle LOL' &= \angle LML' \\ &= 2\angle LMA \\ &= 2 \text{ Obs. Alt. } \underline{\odot} \end{aligned}$$

Similarly, when he takes the altitude of the Sun's upper limb (denoted by Obs. Alt. $\overline{\odot}$) :

$$\begin{aligned}\angle UOU' &= \angle UMU' \\ &= 2\angle UMA \\ &= 2 \text{ Obs. Alt. } \overline{\odot}\end{aligned}$$

Whichever limb is observed, the measured angle (denoted by D.Alt.) is therefore twice the actual altitude of the particular limb.

When the observations are taken in the forenoon, that is when the altitude is increasing, it is seen from figure 156 that if the lower

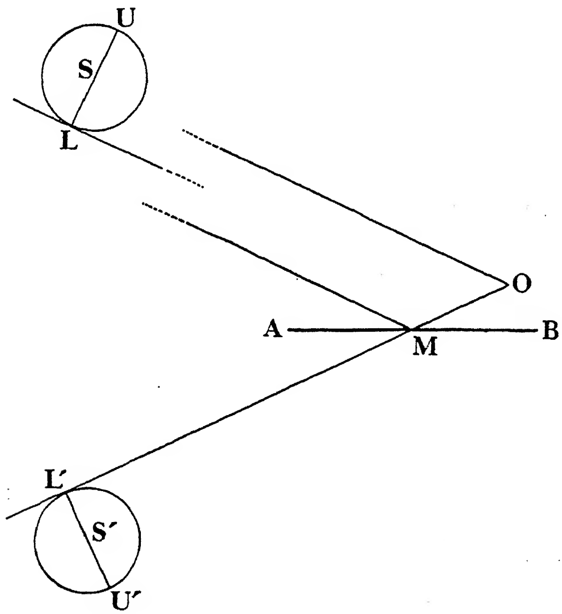


FIGURE 156.

limb is observed, the direct image L' and the reflected image L are separating, and that if the upper limb is observed, the two images are closing. In the afternoon, when the altitude is decreasing, the situation is reversed. In summary, this may be stated :

Forenoon	Closing Suns	U.L.
	Opening Suns	L.L.
Afternoon	Closing Suns	L.L.
	Opening Suns	U.L.

In the ordinary sextant the telescope inverts the object seen, and the image that moves in the field of view when the index bar is moved is the *reflected* image. The reflected image can thus

be decided, and once this is done it is known which limb of the Sun is being observed because :

(1) when the reflected image is above the direct, the upper limb is being observed.
(2) when the reflected image is below the direct, the lower limb is being observed.

Experience has shown that the altitude can be measured with greater accuracy when the images are opening than when they are closing. Altitudes of the Sun should therefore be obtained by observing the lower limb in the forenoon and the upper limb in the afternoon, except when equal altitudes are taken.

Adjustment of the Sextant. The sextant should be carefully adjusted. All side error should be removed, and the centering error should be checked if it is thought that the figure given in the Kew certificate is no longer correct. One method of doing this is to ascertain the latitude by the meridian altitudes of stars north and south. The centering error is then the difference in the latitudes obtained, and is correct for that part of the scale which is the mean of the double altitudes. For example :

The following latitudes were obtained from meridian altitudes by sextant observation in an artificial horizon :

11°18′41″·9S. *from α Triangulæ Australis*
11°18′52″·7S. *from δ Cygni.*

What was the centering error of the sextant, and for what part of the scale was it correct?

The difference of the latitudes is 10″·8.

The declination of α Triangulæ Australis is 68°54′·9S., and of δ Cygni 44°58′·8N. The double altitudes are therefore :

Star S.		*Star N.*	
Lat. S.	11°18′·7	Lat. S.	11°18′·9
Dec. S.	68°54′·9	Dec. N.	44°58′·8
Z.D.	57°36′·2	Z.D.	56°17′·7
Alt.	32°23′·8	Alt.	33°42′·3
D. Alt.	64°47′·6	D. Alt.	67°24′·6

The sum of these double altitudes is 132°12′·2, and the part of the scale for which the centering error is correct is therefore 66°06′·1.

The latitude of the observer is the mean of the latitudes found ; that is 11°18′47″·3. The latitude obtained from δ Cygni, being 11°18′52″·7S., is thus 5″·4 too large, so that the double altitude is 10″·8 too small. The centering error is therefore positive to the sextant altitude.

Index Error. The index error should be checked. If it is checked by the Sun and the altitude of the Sun is low, the diameter should be measured between the right and left limbs and not between

the upper and lower limbs. By doing this, the effect of refraction is avoided.

Procedure for Taking Observations. When the Sun is observed, the inverting telescope of the highest power should be used, because the bigger the Sun's images appear in the telescope, the better is the contact of the limbs that can be observed. The images should be brought together before the telescope is screwed into position.

The actual observations should be taken at equal intervals of altitude. Usually an interval of 10′ suffices, but if the heavenly body is changing its altitude rapidly, it may be necessary to take an interval of 20′. This procedure at once shows whether the observations are reliable because, if the intervals of arc are the same, the intervals of time should be practically the same.

A number of observations—not less than seven—should be taken in order to ensure accuracy. The choice of an odd number enables the mean altitude to be found at a glance.

When a strongly illuminated object is seen against a dark background, it appears larger than it really is, an optical illusion known as the *error of irradiation.* This error may be eliminated by taking two sets of observations, one of the upper limb and the other of the lower limb, the observations being taken alternately and the eye-piece used being the darkest through which the body can be clearly distinguished.

Twilight is the best time for observing stars. Later, when it is dark, the time-keeper must use a light, and this is apt to disturb the observer. To avoid observing the wrong star, it is best to set twice the calculated altitude on the sextant and to place the sextant as close as possible to the artificial horizon.

The Sextant Level. This is an instrument that greatly assists in the identification of the chosen star. It consists of a spirit level attached to the index arm of the sextant, parallel to the plane of the sextant.

Figure 157 shows how the level is used.

When it is properly in position, it is parallel to the surface of the artificial horizon, for which reason :

$$\begin{aligned}\Omega &= \angle BCA\\ &= 180^\circ-(\alpha+ABC)\\ &= \theta\end{aligned}$$

Since θ, which is the angle between the horizon glass and the line of sight of the telescope is constant, Ω is constant for any altitude.

In order to attach the level in the correct position, a double altitude of the Sun should be taken in the artificial horizon, and the level clamped to the arm when the two images have been brought into coincidence and the bubble of the level has been brought into the centre of its run by an adjusting screw.

If the double altitude of a star is now set on the sextant and the sextant is held so that the bubble of the level is horizontal, the star should be seen in the artificial horizon.

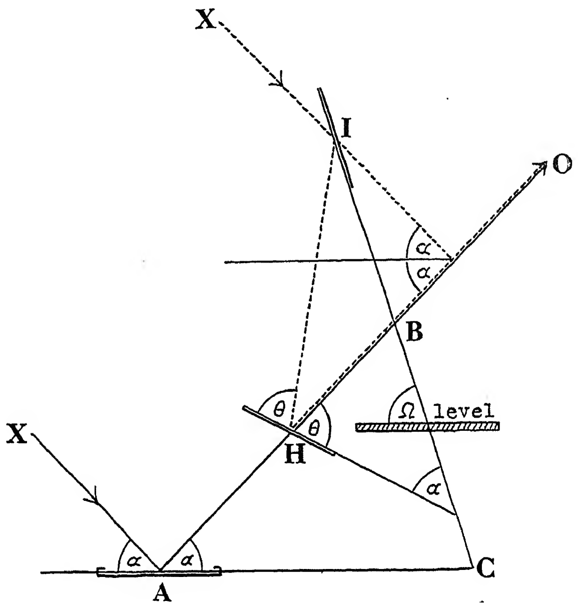


FIGURE 157.

The Sextant Stand. This consists of a pillar about 18 inches high, mounted on three legs and fitted with counterpoised weights to balance the sextant, and it greatly facilitates the taking of observations in an artificial horizon.

Both sextant levels and sextant stands may be obtained on loan to H.M. ships by demand to the Hydrographer, either directly or through the nearest Naval Store Officer.

The Working of Sights. The position of the observation point should be ascertained as accurately as possible because any error in the latitude and longitude will, like an error in the altitude, give rise to an error in the hour angle. For the same reason, all the elements and their corrections must be taken from the *Nautical Almanac* with an accuracy commensurate with that to which the sight can be worked.

In actual survey work, where seven-figure logarithms and the astrolabe, which is a more accurate instrument than a sextant, are used, right ascensions, declinations and the equation of time are taken from the standard edition of the *Nautical Almanac*, and interpolation is carried out by Bessel's coefficients (explained in Chapter IX). In navigational surveys, however, where five-figure logarithms are used, the same accuracy in taking out the elements is unnecessary because there is no point in expressing quantities

with an accuracy greater than that given by the logarithms or by the reading of the sextant. At the same time they should not be expressed with less accuracy.

If the accuracy with which a sextant can be read by an experienced observer is taken as $0'\cdot1$, then the quantities used must be of that accuracy. For this reason the quantities relating to the Sun can be taken from the abridged *Nautical Almanac*, but those relating to the stars and the planets should be taken from the standard edition. In the abridged *Nautical Almanac* the ephemeris of the Sun is correct to the nearest $0'\cdot1$ because the equation of time is given to $0^s\cdot1$, but that of the stars and planets is not because right ascensions are given to the nearest 1^s, that is to $0'\cdot25$. It is possible to obtain the right ascensions of stars correct to an extra figure from the main list of stars in the abridged edition, but if the standard edition is available, it is clearly more convenient and desirable to use that.

Error of the Chronometer by the Altitude of the Sun. The method of finding the error of the chronometer by one set of observations of the Sun is made clear by the steps in the procedure that must be followed.

(1) Mean the altitudes and deck-watch times.
(2) Work out the approximate G.M.T. of the sights, using the approximate deck-watch error.
(3) For this G.M.T., look out the Sun's declination and the quantity E in the abridged *Nautical Almanac*.
(4) Correct the sextant double-altitude for index error and centering error, and the observed altitude for refraction, semi-diameter and parallax, and obtain the true zenith distance to the nearest second of arc.
(5) By means of the ' half log haversine ' formula, find the hour angle to the nearest tenth of a second in time.
(6) Apply E and the longitude to the hour angle and obtain the G.M.T. and the deck-watch error.
(7) Find the error of the deck watch on the chronometer, using the method of mean comparisons if necessary, and so obtain the error of the chronometer.

On the 5*th May* 1937 *at about* 1045 *B.S.T. in latitude* 50°47′59″*N. longitude* 1°06′18″*W., the Sun's upper limb was observed in the artificial horizon. Barometer,* 1015 *mb. Thermometer,* 58° *F. Index error,* +2′10″. *Centering error,* —20″. *The approximate error of the deck watch was* 40^s *slow on G.M.T.*

Comparisons :	*Before Landing*	*After Landing*
	h m s	h m s
Chronometer A	9 49 00	10 15 30
Deck Watch	9 31 36	9 58 06
	17 24	17 24

(1) *Observations :*

Sext. D.Alt.			Deck-Watch Times		
°	′	″	h	m	s
92	10	00	9	43	00·5
92	20	00	9	43	41·5
92	30	00	9	44	22·5
92	40	00	9	45	03·0
92	50	00	9	45	46·5
93	00	00	9	46	28·0
93	10	00	9	47	09·0
92	40	00	7 | 7×9	315	31·0

(2)

	h	m	s
	9	45	04·4
			40·0 slow
Approximate G.M.T.	9	45	44·4

(3) From the abridged *Nautical Almanac :*

Declination at 10^h	16°11′·4N.
Change in $14^m15^s·6$	0′·2
Declination at $9^h45^m44^s·4$	16°11′·2N.

E : $12^h03^m21^s·2$

(4) From *Inman's Tables :*

	° ′ ″
Sext. D. Alt.	92 40 00
Index Error	+2 10
Centering Error	−20
Obs. D. Alt.	92 41 50
Obs. Alt.	46 20 55
Refraction	−54
	46 20 01
Semi-diameter	−15 54
	46 04 07
Parallax	+6
True Altitude	46 04 13
True Zenith Distance	43 55 47

(5) By the 'half log haversine' formula, in which the secants of the latitude and declination are used instead of the cosecants of the polar distances :

	° ′ ″	
Lat.	50 47 59	0·199 26
Dec.	16 11 12	0·017 57
l—d	34 36 47	
ZX	43 55 47	
+	78 32 34	4·801 40
—	9 19 00	3·909 63
H.A.T.S.	$21^h44^m38^s{\cdot}7$	8·927 86

		h m s
(6)	H.A.T.S.	21 44 38·7
	E	12 03 21·2
	L.M.T.	9 41 17·5
	Long. W.	4 25·2
	G.M.T.	9 45 42·7
	D.W.T.	9 45 04·4
	Error	0 00 38·3 slow

	m s	
(7) Deck Watch on A	17 24·0 slow	(A—D.W.)
Deck Watch on G.M.T.	00 38·3 slow	(G.M.T.—D.W.)
∴ A on G.M.T.	16 45·7 fast	(A—G.M.T.)

Error of the Chronometer by the Altitude of a Star or Planet. The steps by which the error of the chronometer is obtained from the altitude of a star or planet are, in principle, the same as those followed when the Sun is observed. The difference in actual detail results from the different quantities used. Thus, the star's declination (to the nearest 1″) and right ascension (to the nearest $0^s{\cdot}1$) are taken from the standard edition of the *Nautical Almanac*, and R (to the nearest $0^s{\cdot}1$) is taken from the abridged edition. Also, if a planet is observed, its horizontal parallax must be taken from the standard edition, and the altitude corrected for parallax.

In the example that follows, the error of the deck watch is found by observations of stars of similar altitude, east and west of the meridian.

On the 31*st March* 1937 *at about* 1930 (—3) *in latitude* 19°59′04″*N. longitude* 40°08′40″*E., Aldebaran was observed west of the meridian and Denebola east of it. Barometer,* 1030 *mb. Thermometer,* 80°*F. Index error,* +30″. *Centering error,* +10″. *The approximate error of the deck watch on G.M.T. was* 8^m *fast.*

Observations :

ALDEBARAN (W)			DENEBOLA (E)
Sext.D.Alt.	D.W.T.	D.W.T.	Sext. D. Alt.
° ′ ″	h m s	h m s	° ′ ″
87 40 00	16 38 59·4	17 19 40·4	85 05 00
87 20 00	16 39 39·2	17 20 25·6	85 25 00
87 00 00	16 40 25·0	17 21 10·2	85 45 00
86 40 00	16 41 04·4	17 21 54·0	86 05 00
86 20 00	16 42 48·0	17 22 39·2	86 25 00
86 00 00	16 43 30·4	17 23 24·0	86 45 00
85 40 00	16 44 09·4	17 24 06·4	87 05 00
86 40 00	7)7.16 287 215·8	7)7.17 150 199·8	86 05 00
	16 41 30·8	17 21 54·3	
	08 00 Error	08 00	
	16 33 31 Approx. G.M.T.	17 13 54	

	° ′ ″		° ′ ″	
	86 40 00	Sext. D. Alt.	86 05 00	
Refrn	+40	I.E.+C.E.	+40	Refrn
1′02″				1′03″
+1	86 40 40	Obs. D. Alt.	86 05 40	+1
—4	43 20 20	Obs. Alt.	43 02 50	—4
0′59″	—59	Refraction	—01 00	1′00″
	43 19 21	True Alt.	43 01 50	
	46 40 39	T.Z.D.	46 58 10	

From the standard edition of the *Nautical Almanac*.

16°23′03″N.	Declination	14°55′09″N.
$4^h32^m19^s{\cdot}4$	Right Ascension	$11^h45^m53^s{\cdot}8$

From the abridged edition:

	$12^h34^m30^s.9$	R	$12^h34^m37^s.6$	
	° ′ ″		° ′ ″	
0·026 97	19 59 04	Lat. N.	19 59 04	0·026 97
0·018 00	16 23 03	Dec. N.	14 55 09	0·014 89
	3 36 01	$l-d$	5 03 55	
	46 40 39	ZX	46 58 10	
4·628 20	50 16 40		52 02 05	4·642 11
4·564 82	43 04 38		41 54 15	4·553 38
9·237 99	h m s		h m s	9·237 35
	3 16 36·7	H.A.	20 43 32·6	
	4 32 19·4	R.A.	11 45 53·8	
	7 48 56·1	R.A.M.	32 29 26·4	
	12 34 30·9	R	12 34 37·6	
	19 14 25·2	L.M.T.	19 54 48·8	
	2 40 34·7	Long. E.	2 40 34·7	
	16 33 50·5	G.M.T.	17 14 14·1	
	16 41 30·8	D.W.T.	17 21 54·3	
	00 07 40·3	Error fast	00 07 40·2	

The mean error of the deck watch on G.M.T. was therefore $7^m\ 40^s.25$ fast.

Errors Involved in Altitudes Taken on One Side of the Meridian. These errors are, in summary:

(1) *Instrumental Error.* This includes all unknown errors of the sextant, and its effect cannot be eliminated.

(2) *Shade Error.* If the heavenly body is too bright for observation with the unprotected eye, shade error may arise because the direct and reflected rays pass through different shades, and the shades may have different refractive errors. It can be avoided by using a dark eye-piece in the telescope.

(3) *Roof Error.* This occurs if the faces of the glass used in the roof of the artificial horizon are not parallel. It can be eliminated by reversing the roof when half the set of observations has been taken.

(4) *Error of Irradiation.* As already stated, this error can be avoided by taking two sets of observations, one of the heavenly body's upper limb and one of its lower, and by using the darkest eye-piece available.

(5) *Error resulting from Abnormal Refraction.* This cannot be eliminated.

(6) *Personal Error.* This, being a peculiarity of the observer, cannot be eliminated, but, with experience, an observer may ascertain its approximate value and allow for it.

Errors Involved in the Mean of Altitudes Taken East and West of the Meridian. The actual errors involved in each set of observations are the same as those given in the previous section, but when the mean of the results of the two sets is taken, the instrumental error practically vanishes, because its effect on the errors of the deck watch is approximately equal and opposite ; and for the same reason the effect of abnormal refraction is eliminated if the atmospheric conditions do not change during the interval between the taking of the two sets. Since these conditions are more likely to change during the day than during the night, the observation of stars of equal or similar altitudes east and west of the meridian is probably more accurate than the observation of the Sun before and after meridian passage. It is unlikely, however, that the gain in accuracy will be sufficient to outweigh the inaccuracy inherent in observation with the ordinary sextant, and, for the additional reason that the Sun is more easily observed than a star, observation of the Sun before and after meridian passage should be regarded as a standard astronomical method of finding the error of the chronometer.

When observations are taken on both sides of the meridian, it is unnecessary to reverse the roof of the artificial horizon when half the set has been taken, but the observer should take care that the mark on the roof is on the same side throughout the observations.

Error of the Chronometer by Equal Altitudes. By equal altitudes are meant observations of the same heavenly body when its altitude west of the meridian is exactly equal to its altitude east of the meridian. Observations of this type, it is seen, avoid as far as possible instrumental errors and errors arising from abnormal refraction.

The principle involved in finding the error of the chronometer by observation of equal altitudes is the same as that involved in finding the longitude by the methods described in Chapter XII. Since the Earth revolves at a uniform rate, a heavenly body will have equal altitudes each side of the meridian at equal intervals of time from meridian passage, and the mean of the times of these equal altitudes therefore gives the time of the meridian passage.

This is strictly true when stars are observed because the declinations of stars are practically constant, and the chronometer error is found by taking the difference between the mean of the times shown by the chronometer and the calculated time of meridian

passage. Thus, if t_1 is the time by the chronometer at the first observation, and t_2 the time at the second, then $\frac{1}{2}(t_1+t_2)$ is the chronometer time of meridian passage, and this time, when compared with the true time of meridian passage, which is (R.A.✱ —R.A.M.S.), gives the error of the chronometer at the time of meridian passage.

But the declination of the Sun is constantly changing, and the altitudes of the Sun at equal intervals from meridian passage are therefore not the same, as explained in Chapter XII. It is thus necessary to find a correction, based on this change of declination, that can be applied to the middle time between the observations of equal altitude and so reduce that time to apparent noon. This correction is called the *equation of equal altitudes.*

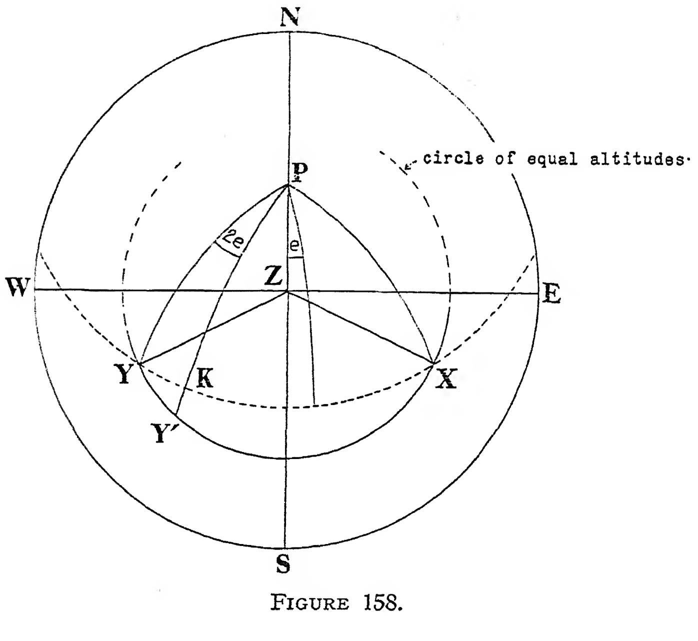


FIGURE 158.

Formula for the Equation of Equal Altitudes. If the polar distance of the heavenly body is increasing, the interval between meridian passage and the second altitude is less than the interval between meridian passage and the first.

In figure 158, X and Y are the positions of the Sun at which equal altitudes would occur if the Sun's declination did not change. If, however, the polar distance increases during the interval, equal altitudes occur when the Sun is at X and Y', and KY', denoted by $\triangle p$, is the change in declination between sights.

If the error in the hour angle of the second sight, the angle YPY', is denoted by $2e$, the correction to be applied to the mean

of the deck-watch times is e, so that the deck-watch time of meridian passage is given by:

$$\tfrac{1}{2}(t_1+t_2)\pm e$$

—where the plus sign is taken if the polar distance is increasing and the minus sign if it is decreasing, and e is given its proper sign.

If the triangle KYY' is regarded as plane, it follows that:

$$\begin{aligned}KY&=KY' \cot KYY'\\ &=KY' \cot (90^\circ-ZYK)\\ &=KY' \cot PYZ\end{aligned}$$

But KY is the distance along a parallel. Therefore:

$$\begin{aligned}KY&=\text{d'long}_{yy'} \cot (\text{lat. } Y)\\ &=2e \cos d\end{aligned}$$

Hence, by equating these two values of KY:

$$2e \cos d=\triangle p \cot PYZ$$

i.e. $$2e=\triangle p \cot PYZ \sec d$$

If e is now expressed in seconds of time and $\triangle p$ in seconds of arc, and the angle PXZ is substituted for the angle PYZ:

$$e=\frac{\triangle p}{30} \cot PXZ \sec d \qquad \text{(in seconds)}$$

This is the equation of equal altitudes.

If dp represents the change of declination in half the elapsed time, the equation may be written:

$$e=\frac{dp}{15} \cot PXZ \sec d \qquad \text{(in seconds)}$$

The time shown by the chronometer at the instant when the Sun is on the meridian is thus:

$$\tfrac{1}{2}(t_1+t_2)+\frac{dp}{15} \cot PXZ \sec d \quad \text{(in seconds)}$$

—where dp is positive when the polar distance is increasing and negative when it is decreasing, and cot PXZ is given its proper sign.

Practical Form of the Equation of Equal Altitudes. In practice it is convenient to express the equation of equal altitudes in terms of half the elapsed time T. This can be done by means of the four-part formula.

In figure 159 the angle ZPX is half the elapsed time, and, by the four-part formula:

$$\cot c \sin p-\cot PXZ \sin \tfrac{1}{2}T=\cos p \cos \tfrac{1}{2}T$$

This, divided by $\sin p \sin \tfrac{1}{2}T$, gives:

$$\cot PXZ \operatorname{cosec} p=\cot c \operatorname{cosec} \tfrac{1}{2}T-\cot p \cot \tfrac{1}{2}T$$

But, from the equation of equal altitudes:

$$e=\frac{dp}{15}\cot PXZ \operatorname{cosec} p$$

Hence, by substitution:

$$e=\frac{dp}{15}(\cot c \operatorname{cosec} \tfrac{1}{2}T-\cot p \cot \tfrac{1}{2}T)$$

As before, the correct signs for the trigonometrical functions of the angles must be used, and when this is done, the chronometer

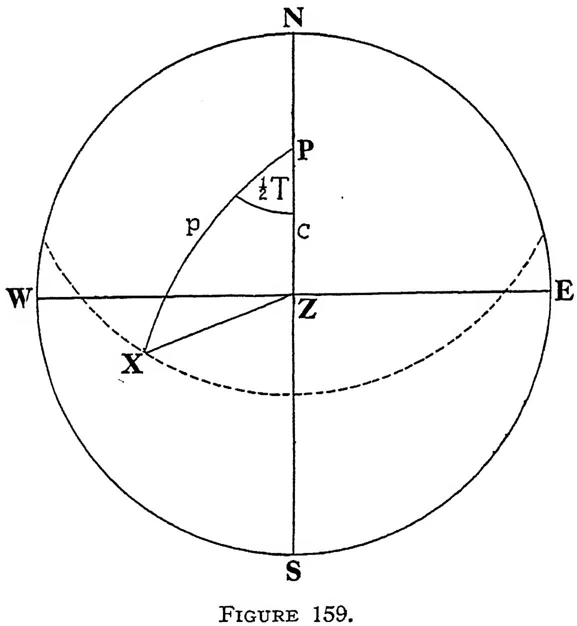


FIGURE 159.

time of apparent noon can be found by applying the value of e obtained to the middle chronometer-time according to the rule:

Add algebraically if the polar distance is increasing.
Subtract algebraically if the polar distance is decreasing.

—because, if the Sun, for example, is coming towards the observer as the result of its change in declination, the altitude is still increasing when the Sun reaches the meridian, and maximum altitude occurs after meridian altitude. Therefore, when the polar distance is decreasing, e must be subtracted from the middle chronometer-time, which is the time of maximum altitude, to give the time of meridian altitude, which is the chronometer time when the hour angle of the Sun is $0^h0^m0^s$.

If observations are made each side of the observer's lower

meridian, so that the error is found at midnight, the equation of equal altitudes becomes :

$$e=\frac{dp}{15}(-\cot c \operatorname{cosec} \tfrac{1}{2}T-\cot p \cot \tfrac{1}{2}T)$$

—and, in calculating the change in declination during the period $\frac{1}{2}T$, the interpolation is based on the G.M.T. of local midnight.

Change of Refraction Between Sights. Refraction depends on temperature and barometric pressure. Also the afternoon temperature is usually much higher than the forenoon temperature, and the barometric pressure is subject to small fluctuations. The error introduced into the equation of equal altitudes by assuming that the refraction does not change may therefore be appreciable.

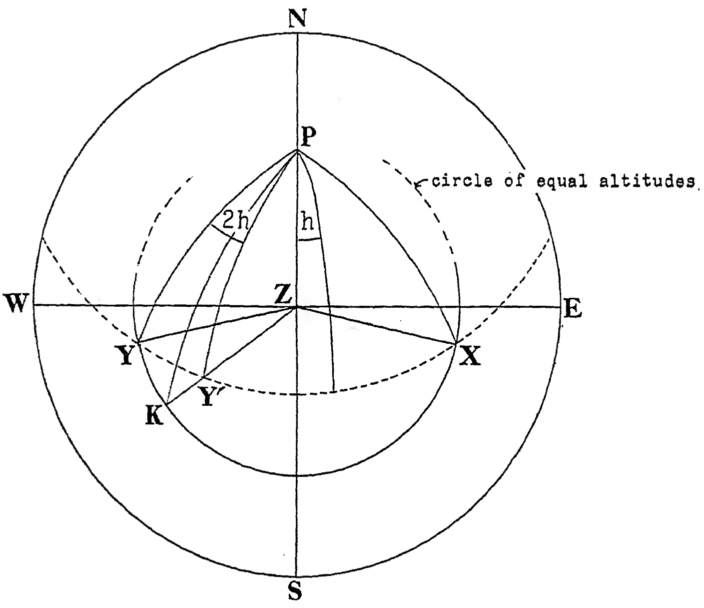


FIGURE 160.

A change in refraction causes the second set of observations to be taken at the wrong time. The observations are actually taken so that the mean altitude in the afternoon is equal to the mean altitude in the forenoon, but, when the effect of a varying refraction is allowed for, the true altitudes differ by an amount equal to the change in the refraction during the elapsed time. If the refraction is *less* in the afternoon, the true position of the Sun in the afternoon observations is nearer the zenith than it would be if the refraction remained unaltered. That is, the Sun is observed a few seconds before it should have been.

In figure 160, X and Y are the positions of the Sun at which equal altitudes would be observed if there was no change in refrac-

tion and declination, and Y' is the position when the refraction changes by the amount KY' (denoted by r) but the declination remains unaltered.

If $2h$ is the error in the hour angle of the afternoon sight, h is the correction to the mid-time. Also, since the triangle KYY' may be considered plane:

$$\sin KYY' = \frac{KY'}{YY'} = \frac{r}{2h \cos d}$$

But the angle KYY' is equal to $(90° - ZYY')$ or PXZ. Hence:

$$h = \frac{r}{2} \operatorname{cosec} PXZ \sec d$$

If h is now expressed in seconds of time, and r in seconds of arc:

$$h = \frac{r}{30} \operatorname{cosec} PXZ \sec d$$

The evaluation of this formula involves the calculation of the angle PXZ. This, however, may be avoided since, from the spherical triangle PZX:

$$\frac{\sin PXZ}{\sin PZX} = \frac{\sin PZ}{\sin PX}$$

i.e.
$$\frac{\sin PXZ}{\sin (az.)} = \frac{\cos l}{\cos d}$$

or
$$\operatorname{cosec} PXZ \sec d = \operatorname{cosec} (az.) \sec l$$

The formula thus becomes:

$$h = \frac{r}{30} \operatorname{cosec} (az.) \sec l$$

—and the complete expression for the deck-watch time of true apparent noon is:

$$\frac{t_1 + t_2}{2} \pm e \pm \frac{r}{30} \operatorname{cosec} (az.) \sec l$$

—where the plus signs are taken if the polar distance is increasing and the refraction is less in the afternoon than in the forenoon, and the minus signs are taken if the polar distance is decreasing and the refraction is greater, and e is given its proper sign.

Limiting Positions of the Heavenly Body Observed. In order that the chronometer error may be found from astronomical observations, the altitude of the heavenly body observed must be such that the heavenly body can be seen in the artificial horizon, and the rate at which the altitude is changing must be sufficiently large to ensure accuracy in observation.

Limits of Altitude. These are, with an artificial horizon of the type described on pages 292 and 293, not less than 15° when the roof is off, and not less than 18° when the roof is on. Also, in

order that the double altitude may be measured, the actual altitude must not be greater than 60° if a sextant is used.

Rate of Change in Altitude. The rate of change in altitude is greatest when the heavenly body is on the prime vertical. At any instant it can be found from the formula :

$$\text{change per minute of time} = 15' \cos l \sin (az.)$$

or

$$\text{change per second of time} = \tfrac{1}{4}' \cos l \sin (az.)$$

Experience shows that the altitude should not be changing at a rate less than 5′ (that is, 10′ of double altitude on the sextant) in 40 seconds, if satisfactory sights are to be taken. This means that :

$$\tfrac{1}{4} \cos l \sin (az.) \not< \tfrac{1}{8}$$

or

$$\sin (az.) \not< \tfrac{1}{2} \sec l$$

In the latitude of England and higher latitudes, the Sun will not fulfil this condition during winter, and at mid-winter its altitude is too small to be measured in an artificial horizon even at meridian passage.

Inman's Tables include a table giving the limiting times between which the Sun's altitude lies between 15° and 60°, and the change in altitude is not less than 5′ in 35^s for latitudes up to 55°. In latitudes higher than 55°09′, no heavenly body can change its altitude at a greater speed.

Method of Using the Equation of Equal Altitudes. The method of using this equation is adequately explained by the steps which must be followed when the error of the chronometer is found.

(1) Decide the most suitable times at which to take sights.

(2) Find the G.M.T. of apparent noon by taking the Greenwich hour angle of the Sun at apparent noon and applying E, which is found in the abridged edition of the *Nautical Almanac* by successive approximation. (Two approximations are sufficient.)

(3) Apply the approximate error of chronometer A and find the chronometer time of apparent noon ; at which time compare chronometer A with the deck watch and chronometers B and C.

(4) Take the mean of each set of deck-watch times.

(5) From comparisons before and after landing, find the error of the deck watch on chronometer A at the mid-time of the forenoon and afternoon observations, and thus obtain the mid-time of the observations by chronometer A.

(6) Find the elapsed time T between the two observations, and express $\tfrac{1}{2}T$ in hours and decimals of an hour.

NOTE. The equation of equal altitudes is unaffected, for practical accuracy, if the quantity $\tfrac{1}{2}T$ is altered by a few seconds, and it is thus sufficient to express the quantity to two decimal places. Also, when the logarithmic cosecants and cotangents are taken out, decimals of a second may be neglected.

(7) From the abridged *Nautical Almanac* take out the Sun's declination at apparent noon, and obtain the quantity dp.

(8) Calculate e, the equation of equal altitudes.

(9) If the refraction has changed between the forenoon and afternoon observations, work out the difference and apply it to e.

(10) Obtain the chronometer time of apparent noon by applying e to the mid chronometer-time of sights.

(11) Apply the G.M.T. of apparent noon to the chronometer time of apparent noon, and so obtain the error of chronometer A on G.M.T.; the comparisons in (3) then give the errors of B and C.

When equal altitudes of a star are taken, the only correction to the mid chronometer-time is the correction for the change in refraction, and this is usually negligible. The error of the chronometer is thus the difference between the mid chronometer-time and the G.M.T. of meridian passage.

In the following example of chronometer error by equal altitudes of the Sun, the steps are indicated by the bracketed figures.

On the 26*th May* 1937 *in latitude* 5°49′15″*S.*, *longitude* 11°48′15″*E.*, *equal altitudes of the Sun's lower limb were observed in the artificial horizon. Barometer, A.M.* 1014 *mb; P.M.* 1022 *mb. Thermometer, A.M.* 63° *F.; P.M.* 73° *F. Approximate error of chronometer A on G.M.T.* $19^{m}20^{s}\cdot0$ *slow.*

(1) *Inman's Tables* show that if the observations are made about 2^{h} before and after meridian passage, the Sun will have a suitable altitude for measurement in an artificial horizon, and a rate of change in altitude sufficiently rapid to give accurate results.

(2) *Time of Apparent Noon.*

	h m s		*Second Approximation* h m s	
H.A.T.S.	24 00 00			
Long. E.	47 13			
G.H.A.T.S.	23 12 47		23 12 47·0	
E ($12^{h}00^{m}$)	12 03 09	($11^{h}10^{m}$)	12 03 09·3	
G.M.T.	11 09 38		11 09 37·7	
		Chronometer A	19 20·0	fast (approx.)
		Approximate time of Noon by Chronometer A	10 50 17·7	

(3) *Comparisons at Apparent Noon.*

	h m s		h m s		h m s
D.W.	11 10 48·7	A	10 51 00·0	A	10 52 00·0
A	10 50 00·0	B	6 23 23·4	C	8 00 28·6
A	00 20 48·7 slow	B	4 27 36·6 slow	C	2 51 31·4 slow

(4) *From the observations:*

Mean D.W.T. A.M. $8^h43^m02^s{\cdot}6$ P.M. $13^h38^m51^s{\cdot}0$ Sext. Alt. $\underline{\odot}$ 76°00′00″

(5) *Comparisons A.M.*

	h m s		h m
Before landing D.W.	8 06 48·2	On return D.W.	10 13 48·0
A	7 46 00·0		9 53 00·0
∴ A is slow on D.W.	0 20 48·2	A slow on D.W.	0 20 48·0

Comparisons P.M.

	h m s		h m s
Before landing D.W.	12 20 47·6	On return D.W.	14 18 47·4
A	12 00 00·0	A	13 58 00·0
∴ A is slow on D.W.	0 20 47·6	A is slow on D.W.	0 20 47·4

	h m s	
At 08^h06^m D.W.T., A was	0 20 48·2	slow on D.W.
At 10^h13^m	0 20 48·0	slow ,,
∴ at 08^h43^m ,, ,,	0 20 48·1	slow ,,
Mean D.W.T.	8 43 02·6	
∴ time by chronometer A	8 22 14·5	
At 12^h20^m D.W.T., A was	0 20 47·6	slow on D.W.
At 14^h18^m ,, ,,	0 20 47·4	slow ,,
∴ at 13^h38^m ,, ,,	0 20 47·5	slow ,,
Mean D.W.T.	13 38 51·0	
∴ time of chronometer A	13 18 03·5	

(6) Elapsed Time $(T) = 13^h18^m03^s{\cdot}5 - 8^h22^m14^s{\cdot}5$

$= 4^h55^m49^s{\cdot}0$

∴ $\tfrac{1}{2}T = 2^h27^m54^s{\cdot}5$

$= 2^h{\cdot}46$

(7) Declination at 11^h36^m G.M.T. is 21°05′·8N.
Change in declination in 24^h is 624″.
Therefore *dp*, the change in $2^h{\cdot}46$, is 64″.

(8) *To find the value of e :*

$dp=64$	1·806 18		
15	1·176 09		
$dp/15$	0·630 09		
cosec $\frac{1}{2}$T	0·220 74 (+)	cot $\frac{1}{2}$T	0·123 25 (+)
cot c	9·452 35 (+)	cot p	9·586 36 (−)
	9·673 09 (+)		9·709 61 (−)
$dp/15$	0·630 09 (+)	$dp/15$	0·630 09 (+)
$+2^s{\cdot}01$	0·303 18 (+)	$-2^s{\cdot}19$	0·339 70 (−)

(The bracketed signs against the separate quantities are the proper trigonometrical signs that govern the signs of the products and quotients.)

Therefore, since the polar distance is increasing :

$$e=+2^s{\cdot}01-(-2^s{\cdot}19)$$
$$=+4^s{\cdot}20$$

(9) *Refraction.*

A.M.	1′15″·0	P.M.	1′15″·0
Barometer 1014 mb.	00″·0	1022 mb.	+00″·5
Thermometer 63°F.	00″·0	73°F.	−04″·0
	1′15″·0		1′11″·5

The afternoon refraction is thus 3″·5 less, and the correction must be added to *e*.

Correction for Refraction.

The azimuth, from the tables, is S.134°35′E.

$h=3{\cdot}5$	0·544 07
cosec (*az.*)	0·147 38
sec l	0·016 77
	0·708 22
30	1·477 12
$+0^s{\cdot}17$	9·231 12

Corrected value of e is $(4^s{\cdot}20+0^s{\cdot}17)$ or $+4^s{\cdot}37$.

	h	m	s
(10) Time of forenoon observations by chronometer A,	8	22	14·5
,, afternoon ,, ,, ,, ,,	13	18	03·5
	21	40	18·0
Mid-time of observations ,, ,, ,,	10	50	09·0
Corrected value of *e*			+4·4
Time shown by chronometer A at noon	10	50	13·4
G.M.T. at noon	11	09	37·7
Chronometer A is therefore slow on G.M.T. by	00	19	24·3

	h	m	s	
Chronometer B is	4	27	36·6	slow on A
∴ ,, ,,	4	47	00·9	slow on G.M.T.
Chronometer C is	2	51	31·4	slow on A
∴ ,, ,,	3	10	55·7	slow on G.M.T.

Summary of Comparisons. *For Equal Altitudes of the Sun.* The times at which comparisons between chronometer A and the deck watch must be made when the chronometer error is found, are :

(1) before landing in the forenoon.
(2) after returning in the forenoon.
(3) at apparent noon.
(4) before landing in the afternoon.
(5) after returning in the afternoon.

At apparent noon, chronometers B and C must be compared with chronometer A.

Comparisons (1), (2), (4) and (5) are necessary in order to obtain the mean comparisons in the forenoon and afternoon. Comparison (3) is required because equal altitudes give the error at apparent noon.

If the observations are made in the afternoon of one day and the forenoon of the next, comparison (3) must be made at apparent midnight.

For Altitudes on Both Sides of the Meridian.

The comparisons required are the same as those required for equal altitudes of the Sun.

For Altitudes on One Side of the Meridian.

Chronometer A must be compared with the deck watch before landing and after returning, and with chronometers B and C.

For Altitudes of Stars East and West of the Meridian.

The comparisons required are the same as those required for altitudes on one side of the meridian.

Lunar Method of Finding the Approximate G.M.T. The methods so far discussed give the exact G.M.T. An approximate G.M.T., however, can be found by observing the Moon.

The Moon, the right ascension of which changes rapidly, is continually passing stars, the right ascensions of which are fixed. If the difference between the right ascensions of the Moon and a particular star (which is also the difference between their hour angles) is calculated at some convenient moment, the right ascension of the Moon at this moment can be found and compared with the right ascension given in the *Nautical Almanac*. A simple proportion sum then gives the G.M.T.

The rules that follow govern and explain the procedure.

(1) The Moon and the star, when observed, should be at a considerable distance from the meridian. The nearer they are to the prime vertical, the more accurate is the result.

(2) If abnormal refraction is suspected, the altitude of the star should not be taken until it is almost the same as the Moon's.

(3) If some time elapses between the taking of the two altitudes—the Moon, for example, might be taken during the day, and the star at twilight—the refraction must be corrected for each altitude to allow for the difference in the barometer and thermometer readings.

(4) Since the method is based on the passage of the Moon across a star, allowance must be made for the interval between the actual taking of the sights. If, for example, the star is observed after the Moon, the interval between the observations is the interval by which the star's position lay to the east of the position it occupied when the Moon was observed. This interval requires to be corrected for the rate at which the star crosses the sky.

(5) The Moon's hour angle, and therefore the G.M.T. of the instant at which the Moon is observed, are obtained in two steps: first by using the Moon's declination corrected for a time that is in error by the amount of the difference between the assumed and calculated G.M.Ts; then by using the Moon's declination corrected for the more accurate G.M.T. If necessary, a third approximation should be made.

(6) The same observer, using the same sextant, should take both sights.

CHAPTER XXI

THE MAGNETIC COMPASS

This chapter is an extension, to cover all magnetic compasses, of Chapter X of Volume I which describes the practical correction of a well-placed compass. It should, therefore, be read in conjunction with Volume I. Further information, however, is given in *The Theory of the Deviations of the Magnetic Compass in Iron Ships*, which can be demanded from the Admiralty Compass Observatory, Slough.

Symbols. To conform with the *Theory of the Deviations of the Magnetic Compass in Iron Ships*, 1936 (B.R. 101/37), the following symbols are used throughout this chapter :

H	denotes	the Earth's directive force in the horizontal plane on shore.
H′	,,	the directive force at the compass needles on any particular direction of the ship's head.
Z	,,	the vertical component of the Earth's lines of force on shore.
Z′	,,	the vertical component of the Earth's lines of force at the compass needles on any particular direction of the ship's head.
θ (theta)	,,	the angle of dip.
δ (delta)	,,	the deviation.
ζ (zeta)	,,	the magnetic course.
ζ'	,,	the compass course.
λ (lambda)	,,	the ratio of the mean directive force at the compass to the Earth's directive force on shore at the place.
λ_2	,,	the ratio of the mean directive force at the compass, with the spheres in place, to the Earth's directive force on shore at the place.
'i'	,,	the angle of heel.
μ (mu)	,,	the ratio of the mean vertical force acting on the compass to the Earth's vertical force on shore at the place.
μ_2	,,	the 'ship's multiplier', that is, the ratio of the vertical force acting on the compass after heeling error has been corrected, to the Earth's vertical force at the place.
M	,,	the number of degrees the spheres are slewed from the athwartship line.

The Ship's Permanent Magnetism. During building and fitting out, partly on account of the vibration set up by hammering, riveting, etc., and partly on account of the inclusion of guns, turrets, etc., a ship becomes a combination of permanent magnets. Thus at any particular position there is a magnetic field dependent in strength and direction on the combination of the various local fields in the vicinity. The effect of this permanent magnetic field in a compass position can be considered as that of three forces acting on the compass needle in directions mutually at right-angles.

These forces, called P, Q and R, are fully described and illustrated in Volume I.

In general it is found that the standard compass, which is usually high in the forepart of the ship, has a —P. This probably results from the fact that the permanent magnetism induced by Z (the Earth's vertical field) while the ship is building and fitting out is tending the whole time, regardless of the direction of the ship's head, to make a blue upper part to the ship, the pole of which is likely to be abaft the standard compass position. If the compass has been placed in accordance with the rules (given in Volume I) to be observed when a standard compass position is chosen, it is probably outside the influence of local fields and, therefore, it will respond to the general trend of the magnetic field in the ship, which, as explained above, is likely to be that described as a —P.

The component P at a well-placed standard compass in a warship built in the United Kingdom, is generally greater than Q. The probable reason for this is the fact that in these latitudes the magnetising force Z is greater than H.

At between-deck compasses the relative values of P and Q depend entirely on local fields.

Correcting Magnets. Binnacles are constructed to comply with the following rules :

(1) *Fore-and-aft magnets.* The vertical athwartship plane through the centre of the compass must always pass through the centre of every fore-and-aft correcting magnet.

(2) *Athwartship magnets.* The vertical fore-and-aft plane passing through the centre of the compass must always pass through the centre of every athwartship magnet.

NOTE. Horizontal magnets should not be brought closer than twice their length to the compass needles.

The Effect of a Change in the Ship's Magnetic Latitude on the Deviation Caused by P and Q. The pulls exerted by P and Q will be the same everywhere, but the amount of deviation they cause will vary with the directive force on the compass needles. As described in Volume I, this directive force varies directly with the horizontal intensity (H) of the Earth's field.

A greater deviation, caused by P and Q, may therefore be

expected when the ship proceeds to higher latitudes, where H decreases, and vice versa.

This deviation, if small, varies as $\frac{1}{H}$.

Example 1. P causes a maximum deviation of 7°E. when the ship's head is east (compass), and Q causes a maximum deviation of 6°E. when the ship's head is south (compass).

What is the deviation caused by P and Q when the ship's head is S.20°W. (compass) ?

As explained in Volume I:

$$\text{Deviation caused by P and Q} = P \sin \zeta' + Q \cos \zeta'$$

On ship's head east (compass) : $P \sin \zeta' = \quad P = +7.$

On ship's head south (compass) : $Q \cos \zeta' = -Q = +6.$

On ship's head S.20°W. (compass) deviation caused by P and Q $= \quad 7 \sin 200° - 6 \cos 200°.$

$$= -7 \sin 20° + 6 \cos 20°.$$
$$= -2{\cdot}4 + 5{\cdot}6.$$
$$= +3{\cdot}2 \text{ or } 3°{\cdot}2\text{E}.$$

Example 2. The maximum deviation at Quebec ($H = 0{\cdot}14$) caused by P and Q was 5°W.

What will be the maximum deviation caused by P and Q at Bombay ($H = 0{\cdot}36$) ?

H is greater at Bombay and, therefore, the deviation is less.

$$\text{Deviation at Bombay} = \text{deviation at Quebec} \times \frac{\text{H at Quebec}}{\text{H at Bombay}}$$

$$= \frac{5 \times 0{\cdot}14}{0{\cdot}36}\text{W}.$$
$$= 2°\text{W}.$$

INDUCED MAGNETISM

The effects of the soft iron of a ship in all circumstances, that is, for any direction of the ship's head, geographical position and movement from an upright position, can be represented diagrammatically by nine soft-iron rods. These rods are considered to have length but no thickness.

The various positions of these rods are shown in figure 161.

Rods 'a', 'b', 'c' when induced by the Earth's field have poles in the fore-and-aft line, that is, directly *before* or *abaft* the compass.

Rods 'd', 'e', 'f' when induced by the Earth's field have poles directly to *starboard* or *port* of the compass.

Rods 'g', 'h', 'k' when induced by the Earth's field have poles directly *above* or *below* the compass.

Rods 'a', 'd', 'g' are in, or parallel to, the fore-and-aft line.

Rods 'b', 'e', 'h' are in, or parallel to, the athwartship line.

Rods 'c', 'f', 'k' are vertical.

DIAGRAM SHOWING THE POSITION OF THE NINE SOFT-IRON RODS WHICH REPRESENT THE WHOLE OF THE SOFT IRON OF A SHIP IN ITS ACTION ON THE COMPASS.

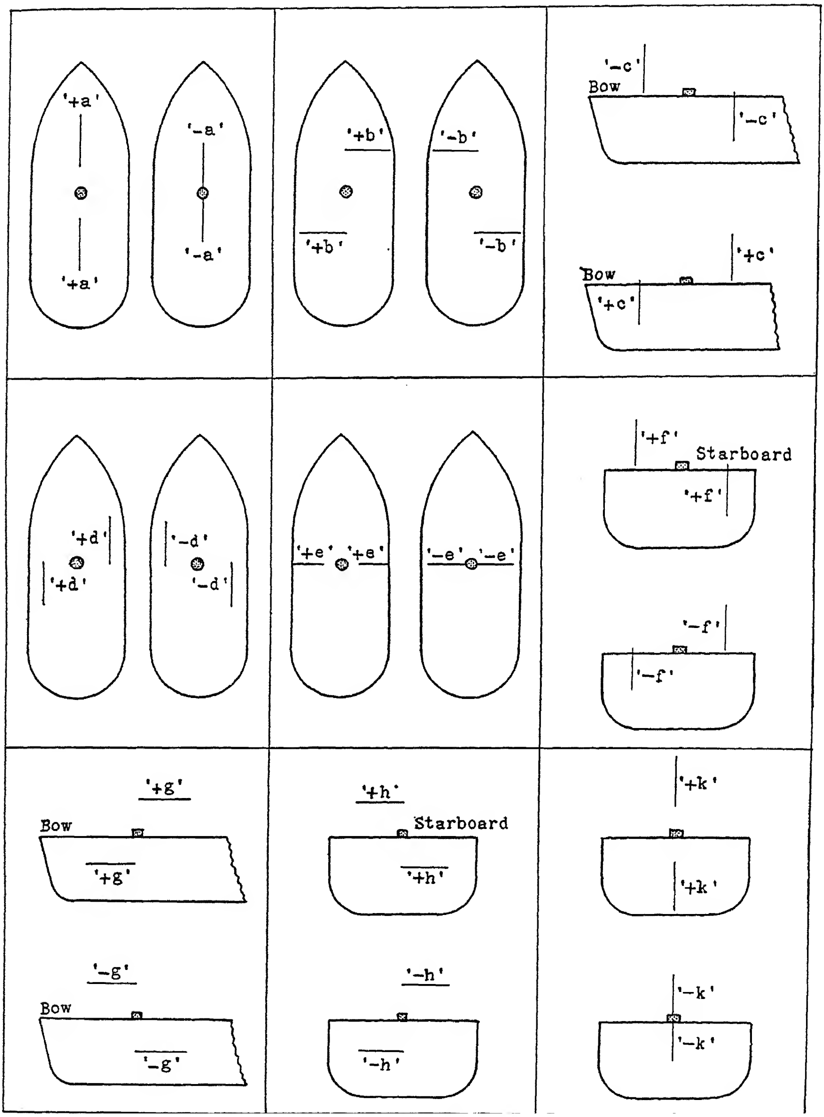


FIGURE 161.

NOTE. Two positions are shown for each rod. In practice, one rod may completely represent a particular effect; for example, the effect of the vertical soft iron of a ship's foremast and funnels is represented by one 'c' rod below and abaft the compass.

Similarly with an island-type aircraft carrier where the compass is placed to one side of the fore-and-aft line, single rods will represent the 'b', 'd', 'f' and 'h' effects. This is explained in detail later in the chapter.

Values of Rods. The values of these rods are based on the proportions of the Earth's force that they represent, when fully induced, at the compass position.

Thus an 'a' rod when fully induced exerts a force on the compass needle, when the ship is upright, of 'a'H; and a 'c' rod when fully induced, when the ship is upright, exerts a force of 'c'Z.

When the ship's head is ζ', and the deviation is δ, the horizontal rods are at an angle to the magnetic meridian and the Earth's inducing force.

Thus an 'a' rod, with the ship heading ζ' and deviation δ, is at an angle ζ, the magnetic course, to the Earth's inducing force. Its force on the compass is then equal to 'a'H $\cos\zeta$. When the ship's head is north, magnetic, it is fully induced and when the ship's head is east, magnetic, it is not induced at all.

Signs of Rods. All the rods are of two types, plus (+) or minus (—).

Rods that are: STARBOARD and BEFORE, STARBOARD and BELOW, BEFORE and BELOW — are $+^{\text{VE}}$ types

The remaining rods, including all rods that pass through the centre of the compass, are negative.

Rods Found at a Well-Placed Compass. Only the rods 'a', 'e', 'c' and 'k' are found to be required to represent the forces at a well-chosen position for the compass.

The magnetisation, correction, and deviation curves caused by these rods are fully described in Volume I. It remains, however, to describe the effect on the deviations caused by 'a', 'e' and 'c' when a ship changes her magnetic latitude.

Effect on Deviation caused by 'a' and 'e' when the Ship Changes Her Magnetic Latitude. The magnetisation of the rods and the directive force on the compass needles both equally depend, when the ship is upright, on the horizontal intensity of the Earth's field.

As the directive force increases, the pulls of the rods increase, and vice versa. In fact, the deviations caused remain constant.

Example 1. The deviation caused by 'a' and 'e' rods is 4° E. when the ship's head is S.20°W. (compass).

What will be the deviation caused by 'a' and 'e' rods when the ship's head is N.60°E. (compass)?

The deviation varies as $\sin 2\zeta'$. Hence:

$$+4=+k\sin 40^\circ$$

i.e.

$$k=\frac{4}{\sin 40^\circ}$$

Therefore on N.60°E. (compass):

$$\text{deviation}=+k\sin 120^\circ$$

$$=\frac{+4\sin 120^\circ}{\sin 40^\circ}$$

$$=5^\circ{\cdot}5\text{E.}$$

Example 2. If the deviation caused by 'a' and 'e' rods is 6°W. when the ship's head is N.40°W. (compass), what will be the maximum deviation caused ?

The maximum deviation caused by 'a' and 'e' rods occurs on any quadrantal point. Also, since the deviation varies as sin $2\zeta'$:

$$-6° = -k \sin 80°$$

The deviation on the quadrantal points is therefore :

$$\frac{-6 \sin 90°}{-\sin 80°} \quad \text{and} \quad \frac{-6 \sin 270°}{-\sin 80°}$$

i.e. 6°·2E. and 6°·2W.

The Correction of 'a' and 'e' Rods by Spheres. It has already been shown how the spheres produce '—a' and '+e' rods ; it must be noted, however, that the deviation caused by these rods is not

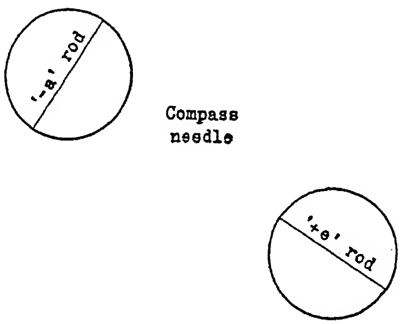


FIGURE 162.

the same, because the '—a' rod produced is lying broadside to the compass needles and the '+e' rod is lying end on to them, as shown in figure 162.

Hence the '+e' rod produces twice the effect of the '—a' rod.

When the final '—e' rod equals the final '—a' rod, the spheres must have put in twice as much '+e' effect as they have '—a' effect.

Example. A ship has uncorrected '—a' and '—e' rods of value 0·08 and 0·35 respectively.

What rods do the spheres produce to correct the deviation by 'a' and 'e'?

Uncorrected '—a' = 0·08
,, '—e' = 0·35

Difference 0·27

If this difference is wholly corrected by fitting spheres, the ratio of the ' +e ' and ' −a ' put in will be 2 to 1.

$\therefore$ ' −a ' put in by spheres $=0{\cdot}27\times\frac{1}{3}=0{\cdot}09$
' +e ' ,, ,, $=0{\cdot}27\times\frac{2}{3}=0{\cdot}18$
Final ' −a ' $=(-0{\cdot}08-0{\cdot}09)=-0{\cdot}17$
Final ' −e ' $=(-0{\cdot}35+0{\cdot}18)=-0{\cdot}17$

Effect on Deviation Caused by a ' c ' Rod when the Ship Changes her Magnetic Latitude. The effect of ' c ' depends on the Earth's vertical force Z, and therefore varies directly as Z, but the effect it has on the compass needles depends on the strength of the directive force on the compass needles which varies directly as the Earth's horizontal force H.

The effect of ' c ' will, therefore, vary directly as Z and inversely as H, that is, as Z/H or tan (dip).

As the ship moves to higher latitudes, Z increases and H decreases. Hence the deviation caused by ' c ' increases. As the ship moves to lower latitudes, the deviation decreases and becomes nil at the magnetic equator.

Example 1. The maximum deviation caused by a ' −c ' rod in a ship at Bombay is 10°E. (H=0·36 ; Z=0·25).

What will be the maximum deviation caused by this ' −c ' rod at Simonstown (H=0·16 ; Z=−0·32) ?

The deviation varies as $\frac{Z}{H}$. Hence, at Bombay :

$$+10=k\frac{0{\cdot}25}{0{\cdot}36}$$

At Simonstown the deviation is therefore :

$$\frac{-0{\cdot}32}{0{\cdot}16}\cdot k$$
$$\frac{-0{\cdot}32}{0{\cdot}16}\times\frac{10\times0{\cdot}36}{0{\cdot}25}$$
$$=\underline{-29\text{ or }29^\circ\text{W.}}$$

Example 2. What will be the deviation caused by this '−c' rod in the same ship when the ship's head is N.40°E. (compass) at Bombay?

A ' −c ' causes westerly deviation in the east semicircle in north magnetic latitude, as explained in Volume I. Also the deviation varies as sin ζ', and the maximum deviation occurs when ζ' is equal to 90°. Hence :

$$-10=k\sin 90^\circ$$

The deviation on N.40°E. (compass) is therefore :

$$\frac{-10\sin 40^\circ}{\sin 90^\circ}$$
$$=-6{\cdot}4\text{ or }6^\circ{\cdot}4\text{W.}$$

Example 3. What will be the deviation caused by this '$-$c' rod in the same ship, on the same course, at Simonstown?

At Bombay, the deviation on N.40°E. (compass) is $-6°\cdot4$. Hence:

$$-6{\cdot}4 = k\frac{0{\cdot}25}{0{\cdot}36}$$

Also, at Simonstown, the deviation is equal to $\dfrac{-0{\cdot}32}{0{\cdot}16}k$.

The deviation on N.40°E. (compass) at Simonstown is therefore:

$$\frac{-0{\cdot}32}{0{\cdot}16} \times \frac{-6{\cdot}4 \times 0{\cdot}36}{0{\cdot}25}$$

$$= +18{\cdot}4 \text{ or } 18°{\cdot}4\text{E.}$$

Rods Found at a Badly-Placed Compass. At a compass placed out of the ship's fore-and-aft line or in a ship unsymmetrically built, the induced magnetism is not regular and to represent the forces in the compass position it is necessary to introduce rods 'b', 'd' and 'f'.

NOTE. In general only small values of 'g' and 'h' may be expected unless the compass is situated over the extremities of frames or beams.

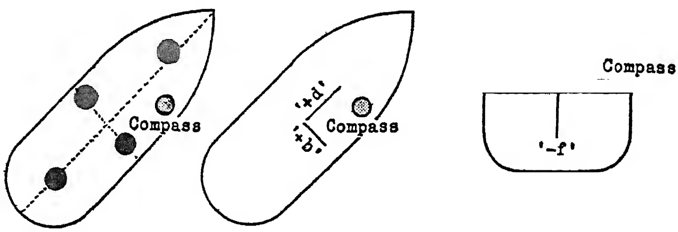


FIGURE 163.

The Signs of the Rods. (1) At a compass placed to starboard of the fore-and-aft line, as shown in figure 163, the signs are:

'+b' as there is more horizontal athwartship iron abaft and to port of the compass.

'+d' as there is more horizontal fore-and-aft iron abaft and to port of the compass.

'$-$f' as there is more vertical iron to port and below the compass.

(2) At a compass placed to port of the fore-and-aft line.

It can be seen that the opposite effects to those shown in figure 163 will occur and, therefore, '$-$b', '$-$d' and '+f' rods will be found.

(3) At a compass placed, for instance, near a washdeck locker as shown in figure 164.

The expected signs of the rods can be determined in the same way.

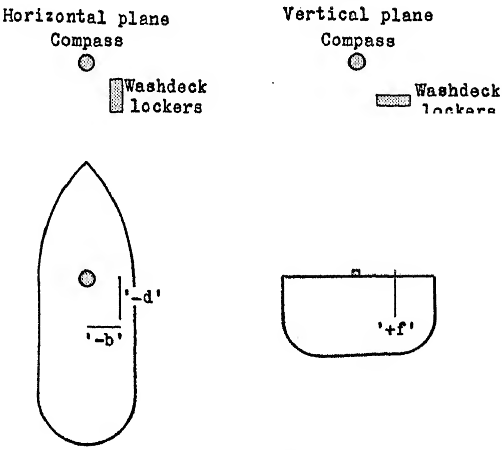


FIGURE 164.

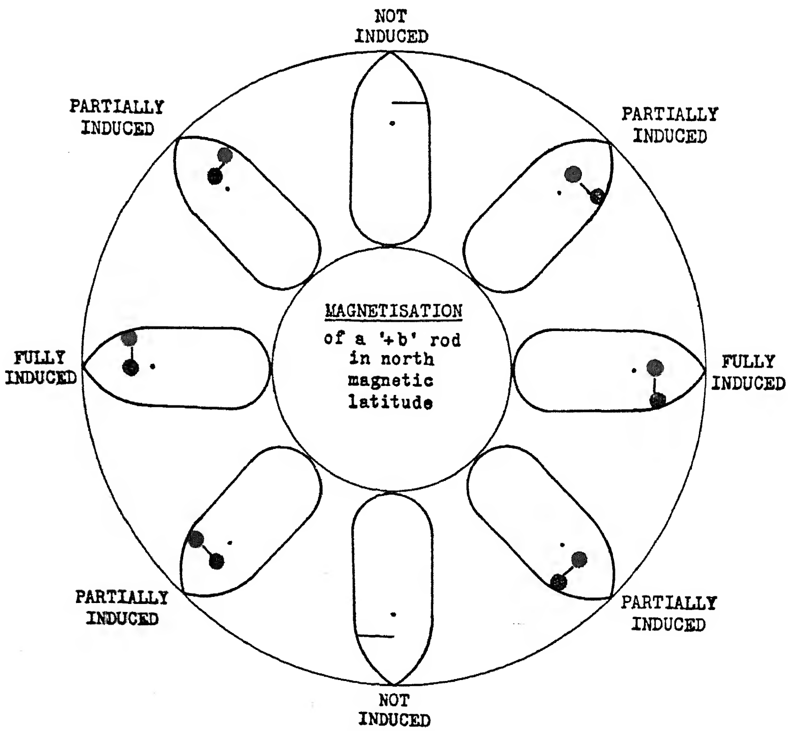


FIGURE 165.

THE ROD ' b '

Consider a ship with a ' +b ' rod.

Magnetisation. As shown in figure 165 :

On north and south the rod is lying at right-angles to the Earth's lines of force and is not magnetised.

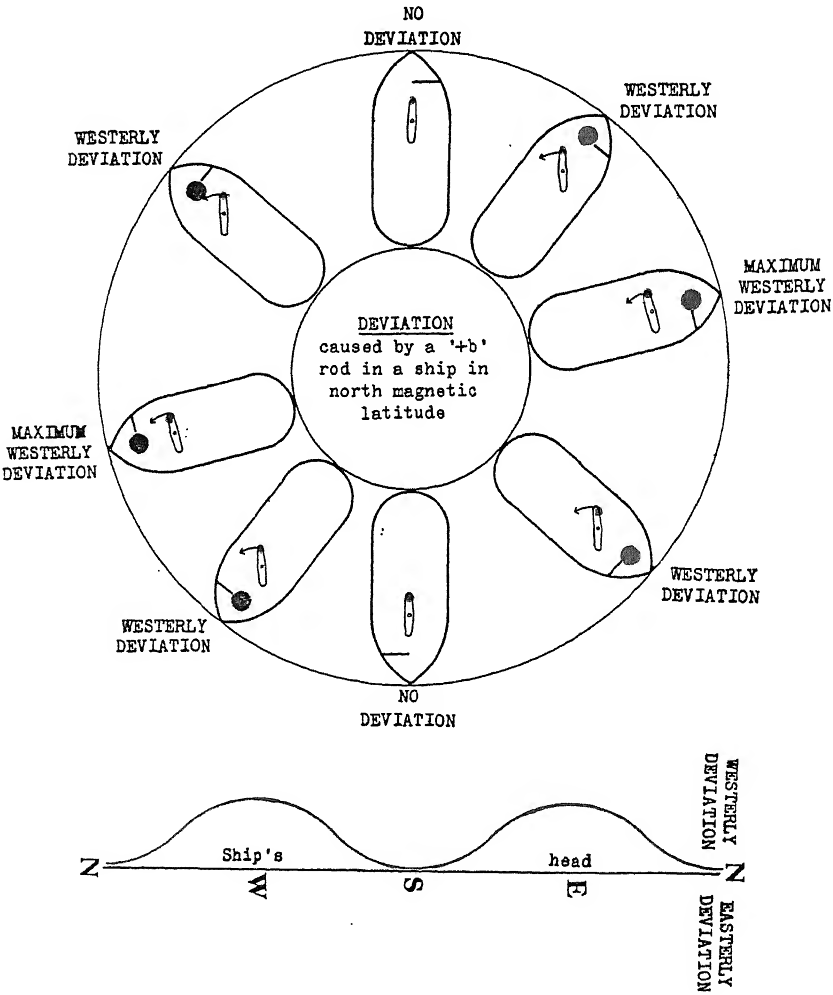


FIGURE 166.

On east and west the rod is lying along the Earth's lines of force and is fully magnetised.

On other headings the rod is partially magnetised.

Deviation Curve. As shown in figure 166 :

On north and south the rod is not magnetised.

On east and west when the pull is at right-angles to the compass needle, it is fully magnetised and causes maximum deviation.

On other points the rod is partially magnetised and causes deviation.

If a curve is drawn it will be seen that the deviation varies as cos $2\zeta'$, about an axis of fixed deviation. The variable deviation varies in opposite quadrants and is called quadrantal.

THE ROD 'd'

Consider a ship with a '+d' rod.

Magnetisation. As shown in figure 167:

On north and south the rod is lying in the Earth's lines of force and is fully magnetised.

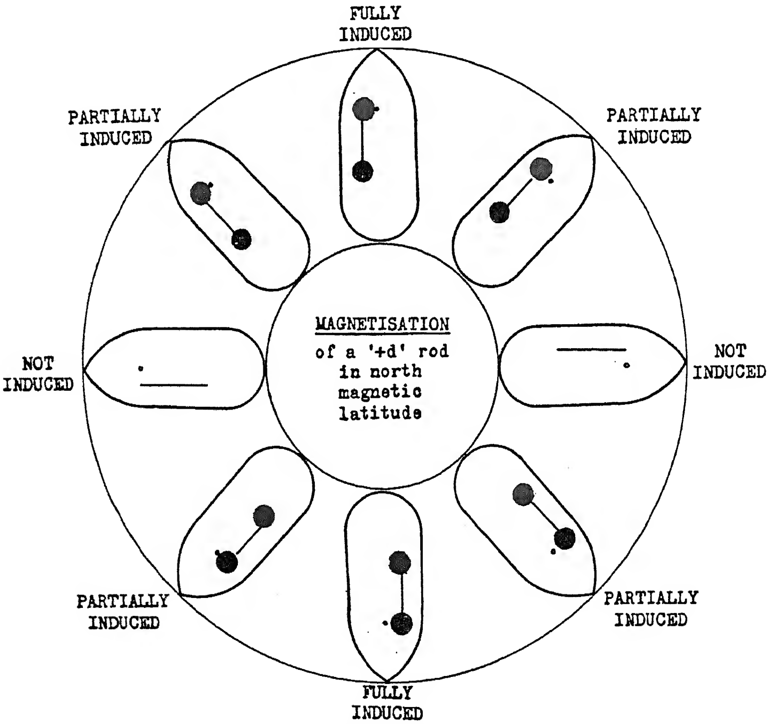


FIGURE 167.

On east and west the rod is lying at right-angles to the Earth's lines of force and is not magnetised.

On other points the rod is partially magnetised.

Deviation Curve. As shown in figure 168:

On north and south when the rod is acting at right-angles to the compass needle there is maximum induction, causing maximum deviation.

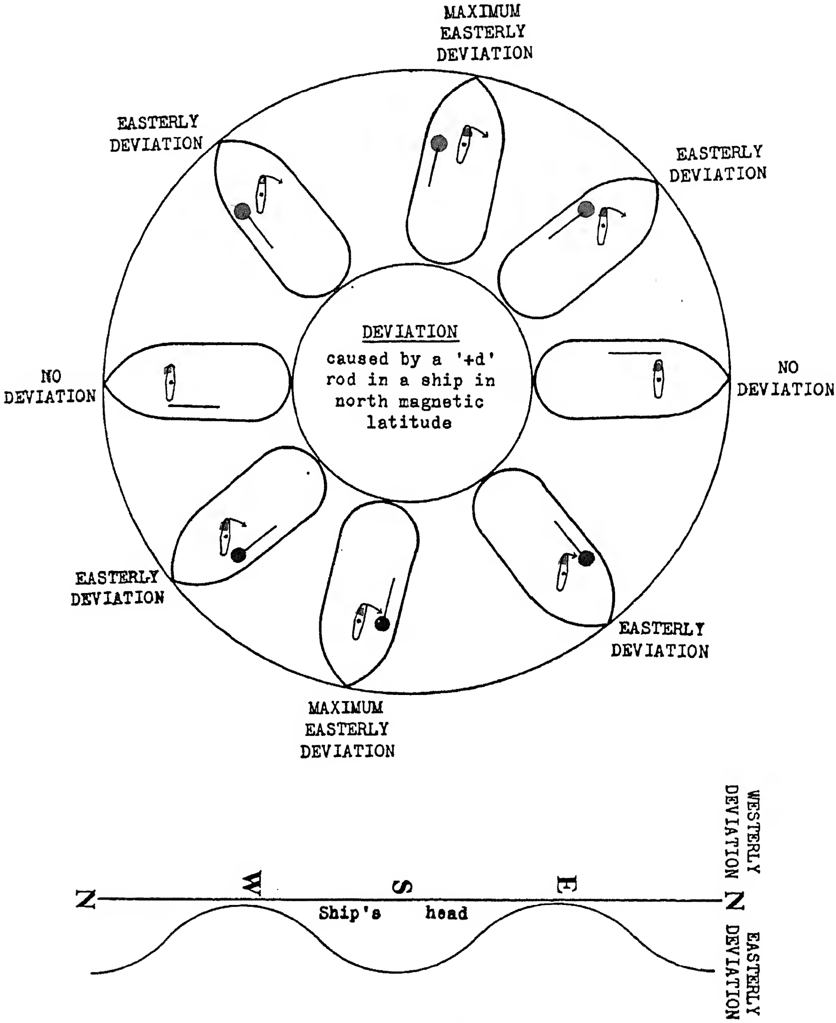


FIGURE 168.

On east and west the rod is unmagnetised.

On other points the rod is partially magnetised and causes deviation.

If a curve is drawn, it will be seen that the deviation varies as cos $2\zeta'$ about an axis of fixed deviation. The variable deviation varies in opposite quadrants and is called quadrantal.

THE COMBINATION OF 'b' AND 'd' RODS

Combined Deviation. If a curve is drawn combining the deviation caused by 'b' and 'd' rods, the result will be as shown in figure 169.

It will be seen that the combined deviation can be considered as consisting of two parts, a constant deviation and a quadrantal deviation.

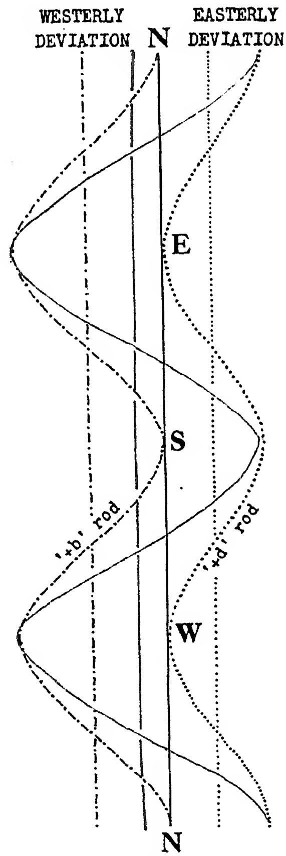


FIGURE 169.

The neutral points of the quadrantal deviation are the half-cardinal points; the points of maximum deviation are the cardinal points.

The variable part of the deviation varies as cos $2\zeta'$.

Correction of the Quadrantal Deviation Caused by 'b' and 'd' Rods. (Approximate coefficient E. See page 349.)

If the 'b' and 'd' rods are decreased at different rates until they are equal and opposite, then quadrantal deviation will be

eliminated and a constant deviation only (dependent on the value to which they have been reduced) will remain. This effect is shown in figure 170.

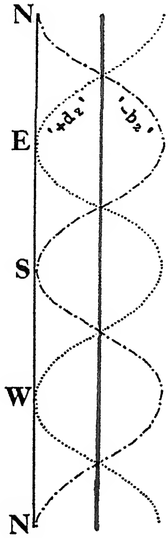


FIGURE 170.

The correction is made by slewing the spheres at an angle to the athwartship line.

Thus on north the spheres will be induced, as shown in figure 171, and will cause a pull similar to a ' —d ' rod.

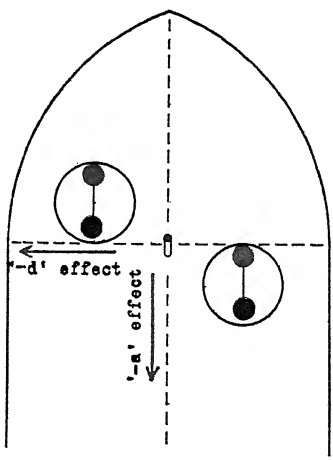


FIGURE 171.

On east the spheres will be induced, as shown in figure 172, and will cause a pull similar to a ' —b ' rod.

On south and west they have similar effects.

On other points their effects are similar to those of ' —d ' and ' —b ' rods, partially induced and acting simultaneously.

The actual angle of slew is small and, therefore, as with ' a ' and ' e ' rods of the spheres, described on page 321, the ' —b ' rod is twice the value of the ' —d ' rod. When the correct angle of slew is used, the rods of the ship and spheres combined are equivalent to ' —b ' and ' +d ' rods of the same value, and the quadrantal deviation is therefore corrected, as shown in figure 170.

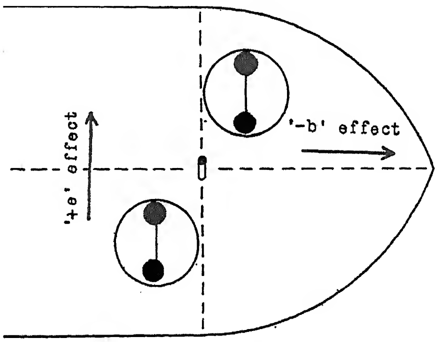


FIGURE 172.

The Correction of Fixed Deviation. (Approximate coefficient A. See page 341.)

Even after spheres have been adjusted, there is still a fixed deviation that cannot be corrected.

At a steering compass the lubber's line can be moved to allow for this deviation.

It must be emphasized that after the correction of the quadrantal deviation caused by ' b ' and ' d ', the value (and sign) of the fixed deviation changes.

The Effect of Deviation Caused by ' b ' and ' d ' Rods when the Ship Changes Her Magnetic Latitude. The magnetisation of the rods and the Earth's directive force on the compass needles both depend on the horizontal intensity of the Earth's field.

As the directive force increases, the pull exerted by the rods increases and vice versa, the fixed deviation and the quadrantal deviation caused remain constant.

THE ROD ' f '

Consider a ship with a ' —f ' rod, as for example an island-type aircraft carrier, with the compass on the starboard side as shown in figure 173.

Magnetisation. While the ship is upright the rod is always lying in the Earth's vertical field and is induced by the vertical force (Z), which does not change on any alteration of the ship's head.

Thus in north magnetic latitude the rod causes a pull to port on the compass needle, and in south magnetic latitude it causes a pull to starboard. On the magnetic equator the rod is unmagnetised.

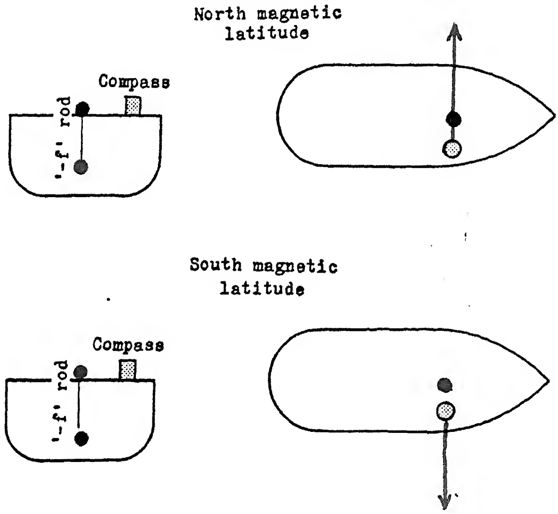


FIGURE 173.

Deviation Curve. In north magnetic latitude the pull is the same as a $-Q$.

In south magnetic latitude the pull is the same as a $+Q$.

The deviation is, therefore, semicircular and varies as cos ζ'.

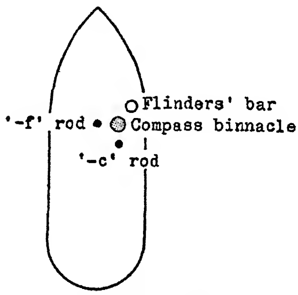


FIGURE 174.

Correction of the Semicircular Deviation Caused by an ' f ' Rod. The Flinders' bar is slewed at an angle to the fore-and-aft line, and introduces a ' +f ' counteracting the effect of the ' —f ' rod. To counteract a ' —f ', the Flinders' bar must be slewed to starboard, as shown in figure 174.

A ship with a ' +f ' rod has similar semicircular deviation

which is corrected by slewing the Flinders' bar to port, as shown in figure 175.

The Effect on Deviation Caused by an 'f' Rod when the Ship Changes Her Magnetic Latitude. The pull of the rod on the compass needle depends on the inducing force, that is, the Earth's vertical force Z.

The directive force on the compass needle varies as the Earth's horizontal force H.

As the ship moves to higher latitudes Z increases and H decreases. Hence the deviation increases. As the ship moves to lower latitudes the deviation decreases and is nil on the magnetic equator.

The deviation thus varies as Z/H or tan (dip).

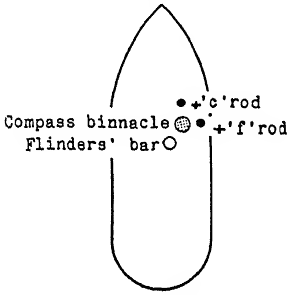


FIGURE 175.

Example. With the ship's head on S.40°E. (compass), the deviation caused by an 'f' rod at Gibraltar ($H=0{\cdot}25$; $Z=+0{\cdot}32$) is 10°W.

(1) What is the maximum deviation caused by this rod at Gibraltar ?

Maximum deviation is caused when the ship's head is north or south. Also the deviation varies as cos ζ'. Therefore, while the ship remains at Gibraltar :

$$-10=-k \cos 40^\circ$$

i.e. $$k=10 \sec 40^\circ$$

Deviation on north $=10 \sec 40^\circ \cos 0^\circ=13^\circ$E.

Deviation on south $=10 \sec 40^\circ \cos 180^\circ=13^\circ$W.

(2) What will be the deviation with the ship's head S.75°W. (compass) at the Falkland Isles ($H=0{\cdot}26$; $Z=-0{\cdot}28$) ?

The deviation varies as tan (dip) and also as cos ζ'.

Deviation on north at Gibraltar $=+13$

,, ,, ,, Falkland Isles $=+13\times\dfrac{\text{tan dip at Falkland Isles}}{\text{tan dip at Gibraltar}}$

$$= 13\times\frac{(-0{\cdot}28)\times(+0{\cdot}25)}{(+0{\cdot}26)\times(+0{\cdot}32)}$$

$$=-11$$

∴ Deviation on S.75°W. (compass) at Falkland Isles $=-11 \cos 255^\circ=11 \cos 75^\circ$

$$=+\ 2{\cdot}9 \text{ or } 2^\circ{\cdot}9\text{E.}$$

DIRECTIVE FORCE

On any magnetic course ζ, the forces P and Q and the rods 'a', 'c' and 'e' will all affect the directive force felt by the compass needles at a well-placed compass.

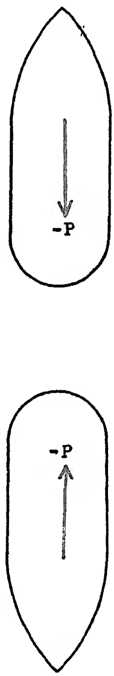


FIGURE 176.

The Forces —P and +Q. The force —P causes a loss of directive force on north and an equal gain on south, as shown in figure 176.

On east and west magnetic it causes no alteration in directive force.

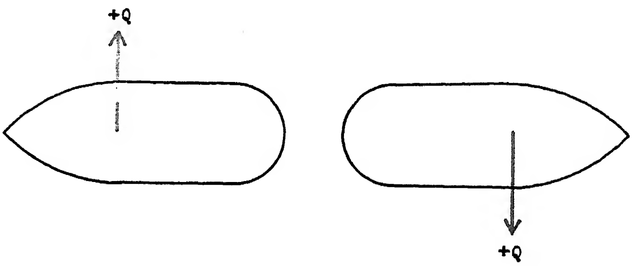


FIGURE 177.

On other magnetic courses it causes a partial gain or loss.

Similarly a +Q causes a loss of directive force on east and an equal gain on west, as shown in figure 177. On north and south

magnetic it causes no alteration in directive force, but on other magnetic courses it causes a partial gain or loss.

The Rod 'c'. This rod causes the maximum alteration in directive force on north and south magnetic, no alteration on east and west magnetic and a varying alteration on other magnetic courses.

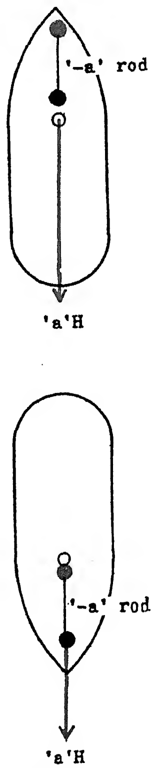


FIGURE 178.

The Rod '—a'. This rod causes a maximum loss of directive force on both north and south magnetic, as shown in figure 178, no loss on east and west magnetic, and a varying loss on other magnetic courses.

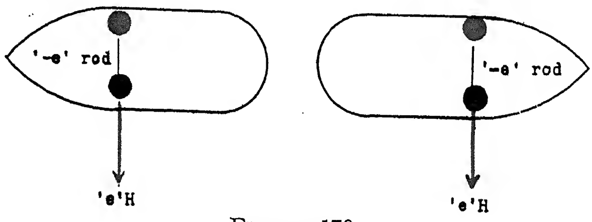


FIGURE 179.

The Rod '—e'. This rod causes a maximum loss of directive force on both east and west magnetic, as shown in figure 179, no loss

on north and south magnetic, and a varying loss on other magnetic courses.

The Ratio λ (lambda). If the mean of the directive forces to north magnetic is taken, the ratio of the mean directive force at the compass to the directive force of the Earth is called λ.

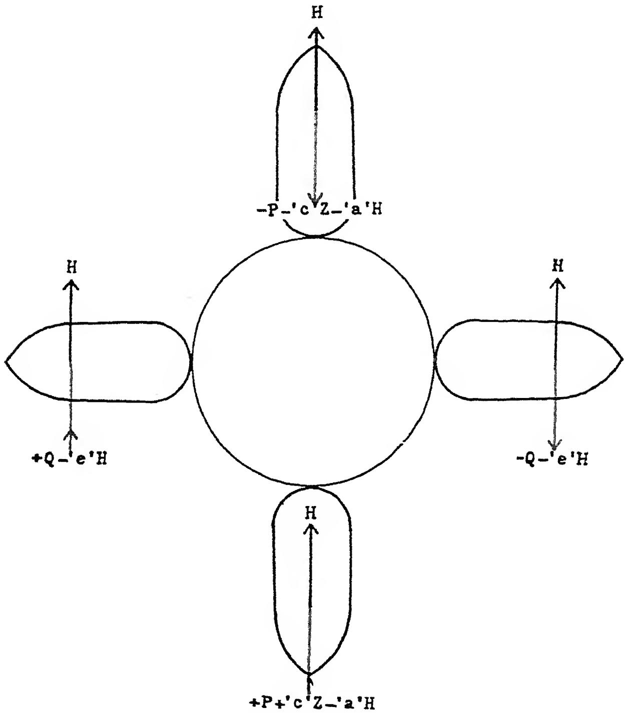


FIGURE 180.

The mean of the directive forces to north magnetic can be obtained by finding the directive force to the north on each cardinal point, as shown in figure 180.

NOTE. The compass is considered to be well-placed, and 'b', 'd' and 'f' have not been considered.

On north (magnetic) : directive force to north $=H-P-\text{'c'}Z-\text{'a'}H$
On east ,, : ,, ,, ,, $=H-Q\quad -\text{'e'}H$
On south ,, : ,, ,, ,, $=H+P+\text{'c'}Z-\text{'a'}H$
On west ,, : ,, ,, ,, $=H+Q\quad -\text{'e'}H$

$$\text{Mean directive force to north}=H-\frac{(\text{'a'}+\text{'e'})H}{2}$$

$$\lambda=\frac{\text{mean directive force to the north at the ship}}{\text{directive force to the north of the Earth}}$$

$$=1-\left(\frac{\text{'a'}+\text{'e'}}{2}\right)$$

The Ratio λ_2. When the spheres are correctly placed, the remaining '$-a$' rod equals the remaining '$-e$' rod. These are called '$-a_2$' and '$-e_2$' rods, and the mean directive force to the north (magnetic) is given by :

$$H-\frac{('a_2'+'e_2')H}{2}$$

$$=H-\frac{2'e_2'H}{2}$$

$$=H-'e_2'H$$

The value of λ is therefore :

$$\frac{H-'e_2'H}{H}$$

$$=1-'e_2'$$

This value of λ, called λ_2, can be defined as the ratio of the mean directive force at the compass resolved in the magnetic meridian, to the directive force of the Earth, when the spheres have been correctly placed.

It may be as much as 0·9 for a good compass position but as low as 0·4 at between-deck compasses, and it is always greater than λ.

NOTE. If P, Q and 'c' rod have been completely corrected and the spheres correctly placed, the deviation on every course is nil and the directive force to north becomes the directive force along the compass needle and is the same on any heading.

Changes in λ_2. The force inducing the 'a' and 'e' rods is the same as the directive force on the compass needles, and λ_2 does not, therefore, vary when the ship changes her magnetic latitude.

TO FIND λ

Since λ, in actual fact λ_2, is the ratio of the mean directive force at the compass to the directive force ashore, it is clearly necessary to take observations both on board and ashore. Also, by definition :

H=the Earth's directive force in the horizontal plane on shore.

H'=the directive force at the compass on any particular direction of the ship's head.

$H' \cos \delta$=the directive force on the compass needle in the magnetic meridian.

The mean directive force on the compass is therefore the mean of $H' \cos \delta$ for all directions of the ship's head, and since all hard and soft-iron components (except '$-a$' and '$-e$') increase the directive force in one semicircle and decrease it in the other, the

mean value of $H' \cos \delta$ can be found by observing the deviation on four equidistant magnetic points. This is done by means of a horizontal vibrating needle.

The Horizontal Vibrating Needle. This instrument consists of a wooden box, the base of which is marked in degrees. The box has a glass cover with cross lines engraved to assist in making observations. A pivot and a magnetised needle are supplied in a separate box.

To Set Up the Instrument. Ship the pivot in the centre of the base of the box and place the needle on the pivot. The needle is constructed so that it lies horizontal. When the needle has settled, ship the glass cover so that one of the cross lines, engraved on the glass, is in line with the needle.

Observations Ashore to Find a Measure of H. Set up the instrument as already described, in a place free from magnetic disturbance. Set the needle 40° away from the meridian and count the time of 10 vibrations. Repeat this procedure and obtain a mean time, T seconds.

Observations on Board to Find a Measure of H′. Unship the compass and set up the instrument in the position normally occupied by the compass needles. Repeat the procedure carried out ashore and obtain a mean time T′ seconds.

The horizontal force varies inversely as the square of the rate of oscillation of the vibrating needle, that is :

$$H \propto \frac{1}{T^2} \quad \text{and} \quad H' \propto \frac{1}{(T')^2}$$

The ratio of H′ to H when H′ is resolved in the magnetic meridian, with the ship's head on any course, is therefore :

$$\frac{T^2}{(T')^2} \cos \delta$$

If, therefore, this procedure is carried out on four equidistant points, then :

$$\text{mean} \frac{H'}{H} = \lambda$$

Example. Ashore, the mean time of ten vibrations was found to be 20 seconds. On board :

Ship's head by compass	*Deviation*	*Magnetic heading*	*Mean time of 10 vibrations*
N. 3°E.	3°W.	N.	22 seconds
N.89°E.	1°E.	E.	19 seconds
S. 2°E.	2°E.	S.	17 seconds
N.89°W.	1°W.	W.	25 seconds

Required the value of λ_2.

On north (magnetic) : $\dfrac{H'}{H}=\dfrac{20^2}{22^2}\cos 3^\circ \qquad =0{\cdot}825$

east (magnetic) $\dfrac{H'}{H}=\dfrac{20^2}{19^2}\cos 1^\circ \qquad =1{\cdot}108$

south (magnetic) : $\dfrac{H'}{H}=\dfrac{20^2}{17^2}\cos 2^\circ \qquad =1{\cdot}383$

west (magnetic) $\dfrac{H'}{H}=\dfrac{20^2}{25^2}\cos 1^c \qquad =0{\cdot}640$

$4|3{\cdot}956$

$\therefore$ Mean of $\dfrac{H'}{H}=\lambda_2=0{\cdot}989$

SUB-PERMANENT MAGNETISM

The causes of sub-permanent magnetism in a ship, and its effect on an upper-deck compass when the ship alters course, were dis-

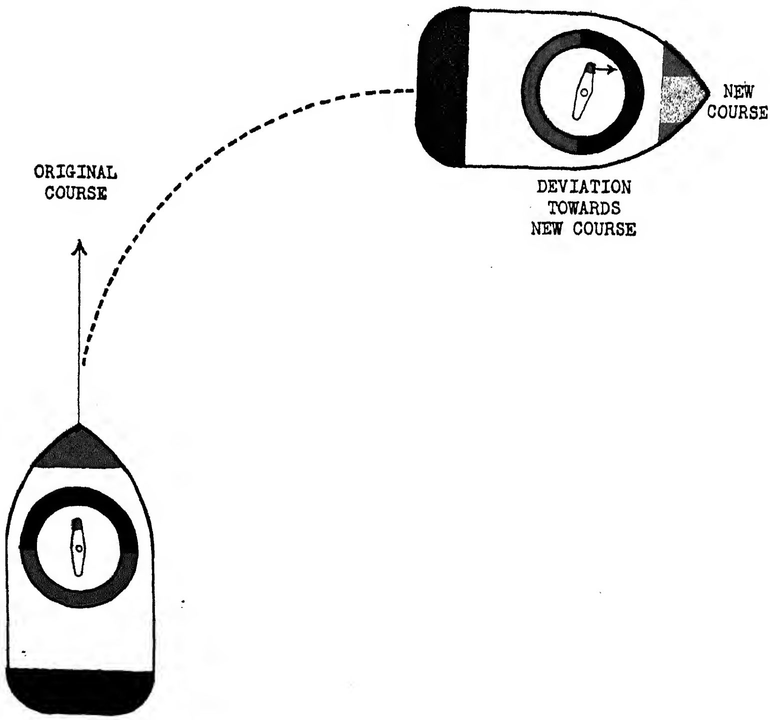


FIGURE 181.

cussed in Volume I, and it was seen that when a ship alters course, the effect of the ship's sub-permanent magnetism at an upper-deck

compass is to produce a diminishing deviation towards the original course.

At between-deck compasses the opposite may occur, as can be seen in figure 181.

NOTE. The effect of sub-permanent magnetism is greatest when a ship alters course from east or west, because the sub-permanent athwartship poles are closer to the compass than the fore-and-aft poles.

The Effect of Sub-permanent Magnetism when a Ship Changes Her Geographical Position. When a ship changes her geographical position, not only will the magnetism of the horizontal intermediate iron alter gradually, but the magnetism of the vertical intermediate iron will also change. Thus a ship crossing the magnetic equator will still have a temporary and gradually diminishing pull in the vertical plane, as can be seen in figure 182.

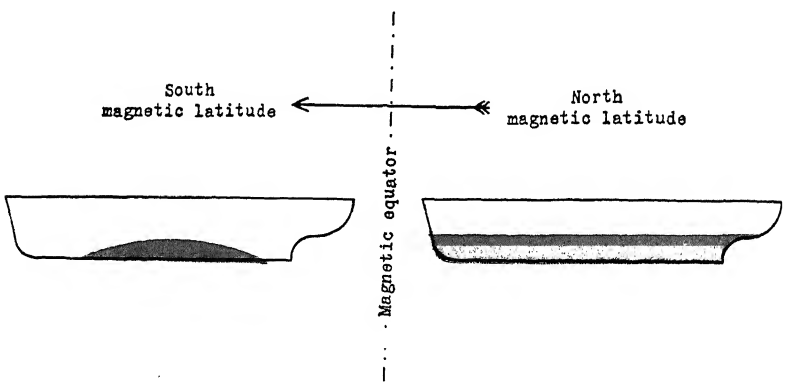


FIGURE 182.

ANALYSIS OF THE DEVIATION

Before the deviation can be analysed it must be brought within reasonable limits. If there are no previous records of the ship, or of another ship of the same class, it is necessary to insert correctors to counteract the effect of P, Q, and 'a', 'c', and 'e' rods, by approximation. Information concerning the position of these correctors can be obtained from the Admiralty Compass Department, Slough. It is usual for a representative of this department to place the correctors in position.

The preliminary corrections should be made in the following order :

(1) Correct 'c' rod by placing an approximate length of Flinders' bar.

(2) Correct 'a' and 'e' rods by placing spheres of an approximate size at an approximate distance.

(3) Correct heeling error.

(4) With the ship heading east (compass) or west (compass), correct all deviation with fore-and-aft permanent magnets.

(5) With the ship heading north (compass) or south (compass), correct all deviation with athwartship permanent magnets.

(6) With the ship heading on one of the quadrantal points, correct all deviation by moving the spheres.

The deviation table should then be sufficiently small to be analysed.

Analysis. The resultant deviation is divided into :

(1) *Fixed deviation,* caused by the 'b' and 'd' rods.

(2) *Semicircular deviation,* caused by the forces P and Q, and rods 'c' and 'f'.

(3) *Quadrantal deviation,* caused by the 'a', 'e', 'b' and 'd' rods.

The principal divisions are further divided, and, as stated in Volume I, the constant parts of the resulting components of the deviation are denoted by letters and referred to as coefficients.

Coefficient A is a fixed deviation.
B is deviation (derived from P and 'c') that varies as $\sin \zeta'$.
C is deviation (derived from Q and 'f') that varies as $\cos \zeta'$.
D is deviation (derived from 'a' and 'e') that varies as $\sin 2\zeta'$.
E is deviation (derived from 'b' and 'd') that varies as $\cos 2\zeta'$.

The value of a coefficient is expressed in degrees.

The deviation on any course can be expressed as :

$$\delta = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta'$$

To Evaluate the Coefficients. When a ship is swung to obtain a deviation table, the deviations are observed on N., N.E., E., S.E., S., S.W., W. and N.W. (compass), as described in Volume I.

Also, by substitution in the expression :

$$\delta = A + B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta' + E \cos 2\zeta'$$

—it is seen that these deviations are given by :

Compass North	$\delta_{N.}$	$= A \qquad\qquad + C \qquad\qquad\qquad + E$
N.E.	$\delta_{N.E.}$	$= A + B \sin 45° + C \cos 45° + D$
E.	$\delta_{E.}$	$= A + B \qquad\qquad\qquad\qquad - E$
S.E.	$\delta_{S.E.}$	$= A + B \sin 45° - C \cos 45° - D$
S.	$\delta_{S.}$	$= A \qquad\qquad - C \qquad\qquad\qquad + E$
S.W.	$\delta_{S.W.}$	$= A - B \sin 45° - C \cos 45° + D$
W.	$\delta_{W.}$	$= A - B \qquad\qquad\qquad\qquad - E$
N.W.	$\delta_{N.W.}$	$= A - B \sin 45° + C \cos 45° - D$

To Evaluate A. Take the mean of the deviations on four or eight equidistant points.

All other coefficients cancel.

To Evaluate B. Take half the difference between the deviations on east and west. Thus:

$$\frac{A+B-E-A+B+E}{2}=B$$

To Evaluate C. Take half the difference between the deviations on north and south. Thus:

$$\frac{A+C+E-A+C-E}{2}=C$$

To Evaluate D. Take one quarter of the difference between the sum of the deviations on N.E. and S.W. and the sum on S.E. and N.W. That is, evaluate:

$$\tfrac{1}{4}(\text{N.E.}-\text{S.E.}+\text{S.W.}-\text{N.W.})$$

Thus:

$$\begin{aligned}&\tfrac{1}{4}[A+B\sin 45^\circ+C\cos 45^\circ+D\\&-A-B\sin 45^\circ+C\cos 45^\circ+D\\&+A-B\sin 45^\circ-C\cos 45^\circ+D\\&-A+B\sin 45^\circ-C\cos 45^\circ+D]=\frac{4D}{4}=D\end{aligned}$$

To Evaluate E. Take one quarter of the difference between the sum of the deviations on N. and S. and the sum on E. and W. That is, evaluate:

$$\tfrac{1}{4}(\text{N.}-\text{E.}+\text{S.}-\text{W.})$$

Thus:

$$\begin{aligned}&\tfrac{1}{4}[A+C+E\\&-A-B+E\\&+A-C+E\\&-A+B+E]=\frac{4E}{4}=E\end{aligned}$$

COEFFICIENT A

Coefficient A is the fixed deviation caused by the combination of 'b' and 'd' rods, and is called + when causing easterly deviation.

It is a constant all-round deviation and cannot be corrected. In a steering compass it can be allowed for by moving the lubber's line.

NOTE. To move the lubber's line it is necessary to slew the binnacle.

After the quadrantal deviation caused by 'b' and 'd' rods, that is, coefficient E, has been corrected, the value of coefficient A will change.

Coefficient A does not change when the ship changes her magnetic latitude.

Coefficient A exists only at badly-placed compasses, but sometimes it may appear in the deviation table for one of the following reasons :

(1) Using the wrong magnetic bearing.
(2) Using the wrong variation.
(3) A faulty azimuth mirror.
(4) The binnacle of a steering compass not placed in the ship's fore-and-aft line.
(5) The pelorus and the standard compass may not be placed in the ship's fore-and-aft line when the ship is swung by comparison with a gyro compass.

NOTE. Sometimes when the ship is swung in only one direction and the swing carried out too fast, an apparent coefficient A may be found.

COEFFICIENT B

Coefficient B is the semicircular deviation caused by the force P and 'c' rod, and is called + if the deviation caused is east when the ship's head is east (compass).

Clearly, if coefficient B is to be completely eliminated, it is necessary to counteract the part caused by P by permanent magnets, and the part caused by 'c' by vertical soft iron. The effects of P and 'c' must therefore be separated.

If the deviation with the ship's head on east and west can be observed at the magnetic equator, where 'c' is unmagnetised, the entire deviation must be caused by P only, and can be corrected by permanent magnets. This having been done, any deviation subsequently found when the ship is heading east (compass) or west (compass) at any place where the Earth's vertical force is acting, must be caused entirely by 'c'.

It is seldom that H.M. ships have the opportunity of correcting the compass on the magnetic equator. It is possible, however, to separate P and 'c' if coefficient B is found when the ship is at two places of widely different magnetic latitude, and *no correctors* are moved in the interval between the two observations.

The Separation of P and 'c'. Figure 183 shows a ship in any magnetic latitude, heading east by compass. The compass needle is deflected from the magnetic meridian through an angle of δ by the force P+'c'Z. Because this force has its maximum effect when the ship's head is east, it acts at right-angles to the disturbed needle, that is, along CD.

The directive force at the compass, as already explained, is λH. This force can be resolved into two components at right-angles to each other, one acting along DC and the other along ED ; the component along DC being $\lambda H \sin \delta$.

Since the compass needle remains in equilibrium, it follows that

the disturbing force and the directing force must be equal and opposite; in other words, $\lambda H \sin \delta$ is equal to P+' c 'Z. Hence:

$$\lambda H \sin (\text{coefficient B}) = P + \text{'c'}Z.$$

If, therefore, coefficient B is found when the ship is at two places of widely different magnetic latitudes, two equations are obtained. In these equations, λ is known and the values of H and Z can be found from the charts shown in Volume I. Substitute these values in both equations and thus find the signs and values of P and ' c '.

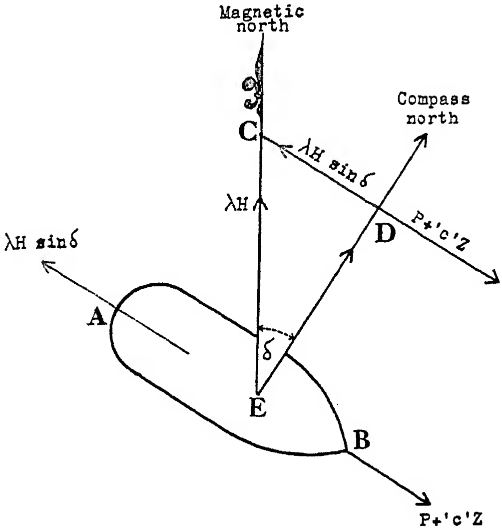


FIGURE 183.

Correction of Coefficient B. (1) That part caused by ' c ' is corrected by a Flinders' bar. Tables are given in Volume I, showing the length of Flinders' bar required to correct the value of ' c ' rod.

(2) The remaining deviation caused by force P is corrected by fore-and-aft permanent magnets.

NOTE. (a) If the two parts of coefficient B have been exactly analysed and corrected, P and ' c ' will cause no deviation, with the ship upright, wherever the ship may be.

(b) See the remarks concerning the effects of altering the length of Flinders' bar on pages 362 and 363.

Examples of Separating P and ' c '.

Example 1. At Halifax, Nova Scotia (H=0·16; Z=+0·55), a compass in a pattern 194 binnacle was corrected completely for coefficient B by $19\frac{1}{2}''$ of $3''$ Flinders' bar on the after side of the compass. Coefficient D (quadrantal deviation) was wholly corrected by spheres.

Subsequently when the ship was in position 20°S., 20°W. ($H=0{\cdot}24$; $Z=-0{\cdot}12$) coefficient B was found to be $-7°$. The value of λ_2 was 0·85. What is the correct amount of Flinders' bar required to correct fully for 'c'?

$$P+\text{'c'}Z=\lambda H \sin B$$

At Halifax $\quad P+0{\cdot}55\text{'c'}=0$

At sea $\quad P-0{\cdot}12\text{'c'}=-0{\cdot}85\times0{\cdot}24 \sin 7°$

By subtraction: $\quad 0{\cdot}67\text{'c'}=0{\cdot}85\times0{\cdot}24 \sin 7°$

$\therefore \quad$ uncorrected $\text{'c'}=+0{\cdot}037$

But the Flinders' bar already in place corrects a value of 'c'.
From table, $19\frac{1}{2}''$ of $3''$ F.B. correct a value of $\text{'c'}=+0{\cdot}12$

Uncorrected value of $\text{'c'}=+0{\cdot}037$

Ship's total value of $\text{'c'}=+0{\cdot}157$

From table, $23\frac{1}{4}''$ of $3''$ Flinders' bar will be required on the after side of the compass.

Example 2.

At Bahia ($H=0{\cdot}26$; $Z=0{\cdot}0$) coefficient B was $+5\frac{3}{4}°$.
At Trinidad ($H=0{\cdot}31$; $Z=+0{\cdot}25$) coefficient B was $+1\frac{1}{2}°$.
What is coefficient B at Capetown ($H=0{\cdot}17$; $Z=-0{\cdot}32$)?

$$P \propto \frac{1}{H}$$

At Trinidad the deviation caused by $P=\dfrac{5{\cdot}75\times0{\cdot}26}{0{\cdot}31}=+4{\cdot}8$

At Trinidad the deviation caused by $\text{'c'}=-3{\cdot}3$

At Capetown the deviation caused by 'c', which varies as $\dfrac{Z}{H}$:

$$=\frac{-3{\cdot}3\times0{\cdot}31}{0{\cdot}25}\times\frac{-0{\cdot}32}{0{\cdot}17}$$

$$=+7{\cdot}7$$

At Capetown the deviation caused by $P=\dfrac{5{\cdot}75\times0{\cdot}26}{0{\cdot}17}=+8{\cdot}8$

$\therefore \quad$ Coefficient B at Capetown$=8{\cdot}8+7{\cdot}7=+16{\cdot}5$

COEFFICIENT C

Coefficient C is the semicircular deviation caused by the force Q and 'f' rod, and is called + if the deviation caused is east when the ship's head is north by compass.

NOTE. In a well-placed compass there is no 'f' rod.

If the compass is badly placed, it will be necessary to separate Q and 'f'.

The Separation of Q and ' f '. The procedure is similar to that for separating P and ' c '. If the deviation can be observed when the ship is at the magnetic equator, where ' f ' is unmagnetised, the entire deviation must be caused by Q only. If this procedure is not possible, Q and ' f ' can be separated if coefficient C is found when the ship is at two places of widely different magnetic latitude, and *no correctors* are moved in the interval between the two observations.

From figure 184, it is clear that the equation for each of these observations will be :

$$\lambda H \sin \delta = Q + \text{'f'}Z$$

Correction of Coefficient C. (1) That part caused by ' f ' is corrected by slewing the Flinders' bar.

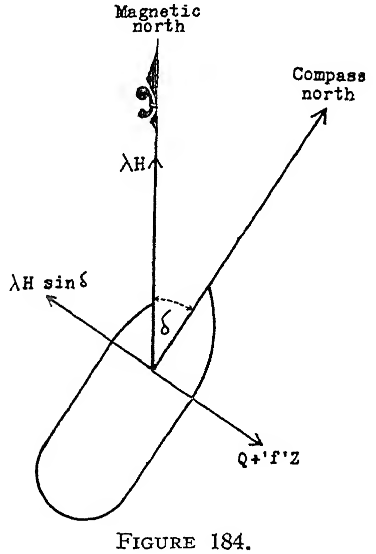


FIGURE 184.

(2) The remaining deviation caused by force Q is corrected by athwartship permanent magnets.

If the two parts of coefficient C have been exactly analysed and corrected, Q and ' f ' will cause no deviation, with the ship upright, wherever the ship may be.

To Slew the Flinders' Bar. Consider a ship with a ' —c ' rod and a ' —f ' rod, and with the compass placed to starboard of the fore-and-aft line, as shown in figure 185.

In north magnetic latitude :

' —f ' rod gives a pull to port of value ' f 'Z

' —c ' rod gives a pull aft of value ' c 'Z

The combined pull, expressed by the value $\sqrt{\text{'c'}^2 + \text{'f'}^2}$, is at an angle α to the fore-and-aft line, where tan α is equal to ' f '/' c '. Thus

a Flinders' bar is required to correct a value of $\sqrt{\text{'c'}^2+\text{'f'}^2}$, and it must be placed at an angle to the fore-and-aft line so that tan α is equal to 'f'/'c'.

The correct amount of Flinders' bar required can be found from the table in Volume I.

A '−f' requires a slew to starboard and a '+f' requires a slew to port.

NOTE. See the remarks concerning the effect of altering the length of a Flinders' bar on pages 362 and 363.

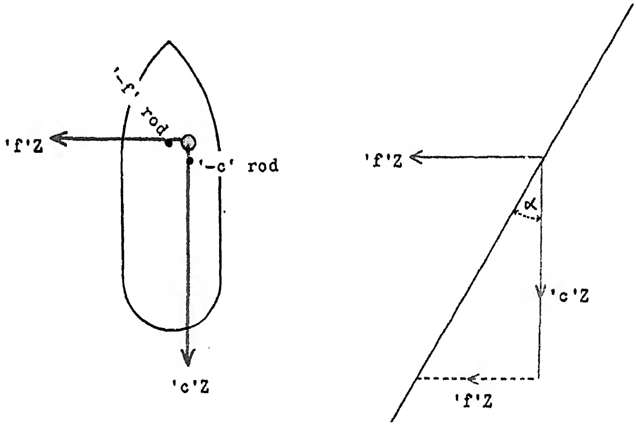


FIGURE 185.

Example of Slewing a Flinders' Bar. At Portsmouth (H=0·19; Z=0·42) a ship had 12″ of 3″ Flinders' bar on the fore side of the compass in a pattern 194 binnacle. Coefficient B was 0° and coefficient C was +1°.

At the Falkland Islands (H=0·27; Z=−0·28) coefficient B was −2° and coefficient C was +3°. No correctors had been changed. The value of λ_2 was 0·85.

What length of Flinders' bar is required and how should it be placed?

$$P+\text{'c'}Z=\lambda H \sin B$$

At Portsmouth: $P+0{\cdot}42\text{'c'}=0$

At Falklands: $P-0{\cdot}28\text{'c'}=-0{\cdot}85\times0{\cdot}27\sin 2^\circ$

By subtraction: $0{\cdot}7\text{'c'}=0{\cdot}85\times0{\cdot}27\sin 2^\circ$

∴ Uncorrected $\text{'c'}=+0{\cdot}012$

But: Corrected $\text{'c'}=-0{\cdot}05$

∴ Total $\text{'c'}=-0{\cdot}038$

$$Q+\text{'f'}Z=\lambda H \sin C$$

At Portsmouth: $Q+0{\cdot}42\text{'f'}=0{\cdot}85\times0{\cdot}19\sin 1^\circ$

At Falklands: $Q-0{\cdot}28\text{'f'}=0{\cdot}85\times0{\cdot}27\sin 3^\circ$

By subtraction: $0{\cdot}7\text{'f'}=0{\cdot}85(0{\cdot}19\sin 1^\circ-0{\cdot}27\sin 3^\circ)$

∴ $\text{'f'}=-0{\cdot}013$

The Flinders' bar will have to correct a value of $\sqrt{'c'^2+'f'^2}$.

$$\sqrt{'c'^2+'f'^2}=\sqrt{0{\cdot}038^2+0{\cdot}013^2}$$
$$=0{\cdot}04$$

From the table given in Volume I it will be seen that $10\frac{1}{2}$ inches of Flinders' bar will be required.

The angle of slew is found from the expression:

$$\tan\alpha=\frac{'f'}{'c'},$$

i.e. $$\tan\alpha=\frac{-0{\cdot}013}{-0{\cdot}038}$$

$\therefore$ $$\alpha=19°$$

Since 'f' is negative, the Flinders' bar must be slewed 19° to starboard.

NOTE. When the values of 'c' and 'f' have been found, an alternative method is to find the value of α. Then the Flinders' bar will have to correct a value of either:

$$\text{total } 'c' \sec\alpha=0{\cdot}038\sec 19°=0{\cdot}04$$

or: $$'f'\operatorname{cosec}\alpha=0{\cdot}013\operatorname{cosec}19°=0{\cdot}04$$

Example of Reslewing Flinders' Bar. A compass is fitted with $13\frac{1}{2}''$ of $3''$ Flinders' bar on the fore side of the compass in a pattern 194 binnacle, slewed 15° to port. ($\lambda_2=0{\cdot}8$.)

At Simonstown ($H=0{\cdot}19$; $Z=-0{\cdot}32$) the following deviations were observed:

Ship's head by compass	*Deviation*
N.	2°W.
E.	1°W.
S.	2°E.
W.	1°E.

At Gibraltar ($H=0{\cdot}25$; $Z=+0{\cdot}32$) the following deviations were observed:

Ship's head by compass	*Deviation*
N.	1°E.
E.	2°E.
S.	1°W.
W.	2°W.

What is the correct setting for the Flinders' bar?

At Simonstown: $B=-1$

At Gibraltar: $B=+2$

$$P+'c'Z=\lambda H\sin B$$

At Simonstown: $P-0{\cdot}32'c'=-0{\cdot}8\times0{\cdot}19\sin 1°$

At Gibraltar: $P+0{\cdot}32'c'=\ \ 0{\cdot}8\times0{\cdot}25\sin 2°$

$$0{\cdot}64'c'=\ 0{\cdot}007+0{\cdot}003$$

$$\text{Uncorrected } 'c'=+0{\cdot}015$$

At Simonstown: $C=-2$
At Gibraltar: $C=+1$

$$Q+\text{‘f’}Z = \lambda H \sin C$$

At Simonstown: $Q-0{\cdot}32\text{‘f’} = -0{\cdot}8\times 0{\cdot}19 \sin 2°$
At Gibraltar: $Q+0{\cdot}32\text{‘f’} = 0{\cdot}8\times 0{\cdot}25 \sin 1°$

$$0{\cdot}64\text{‘f’} = 0{\cdot}004+0{\cdot}005$$

$\therefore$ Uncorrected ‘f’ $=+0{\cdot}014$

$$\text{Tan } \alpha=\frac{\text{‘f’}}{\text{‘c’}} \qquad \therefore \quad \text{‘f’}=\text{‘c’} \tan 15° = \underline{0{\cdot}27\text{‘c’}}$$

From table:

Flinders' bar in place has corrected $-0{\cdot}06$, and this is equal to $\sqrt{\text{‘c’}^2+0{\cdot}27\text{‘c’}^2}$. Therefore:

Corrected ‘c’ $=-0{\cdot}058$
Corrected ‘f’ $=+0{\cdot}016$ (plus because the Flinders' bar is slewed to port).

Uncorrected ‘c’	$=+0{\cdot}015$	Uncorrected ‘f’	$=+0{\cdot}014$
Corrected ‘c’	$=-0{\cdot}058$	Corrected ‘f’	$=+0{\cdot}016$
Total ‘c’	$=-0{\cdot}043$	Total ‘f’	$=+0{\cdot}030$

Correction required by the Flinders' bar $=\sqrt{0{\cdot}043^2+0{\cdot}03^2}=0{\cdot}052$

$\therefore$ $12\frac{3}{4}''$ of Flinders' bar will be required on the fore side (because ‘c’ is minus).

$$\text{Tan } \alpha=\frac{30}{43}$$

$$\alpha=35°$$

The Flinders' bar must therefore be slewed 35° to port (because ‘f’ is plus).

NOTE. An alternative method can be used, similar to that explained in the note at the end of the previous example.

COEFFICIENT D

Coefficient D is the quadrantal deviation caused by ‘$-$a’ and ‘$-$e’ rods and is called + if the deviation caused is east when the ship's head is on north-east (compass). This is fully explained in Volume I.

It is counteracted by soft-iron spheres. If the spheres are in place when the ship is swung and coefficient D is found to be +, then the spheres are undercorrecting and will have to be moved in; if coefficient D is found to be $-$, then the spheres will have to be moved out.

The quadrantal deviation caused by coefficient D does not vary when the ship changes her magnetic latitude.

Tables giving the sizes and distances of spheres to correct quadrantal deviation, together with examples of placing spheres, are given in Volume I.

COEFFICIENT E

Coefficient E is the quadrantal deviation caused by ' b ' and ' d ' rods, and is called + if the deviation caused is east when the ship's head is on north (compass).

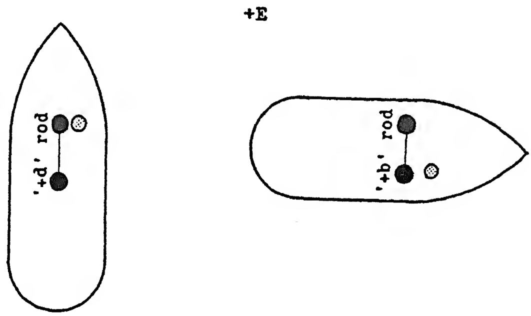


FIGURE 186.

Coefficient E is not found at a well-placed compass.

A compass placed to starboard of the fore-and-aft line has ' +b ' and ' +d ' rods, as shown in figure 186, which cause a +E, and vice versa.

Coefficient E is counteracted by slewing the spheres, as shown in figure 187.

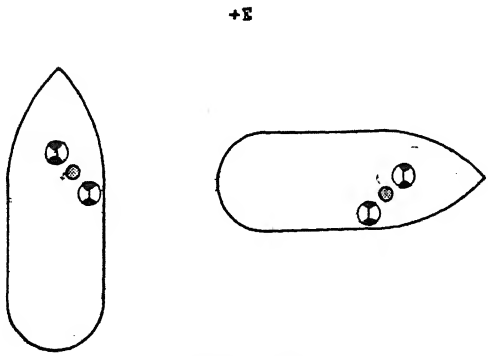


FIGURE 187.

The angle of slew is found from the formula :

$$\tan 2M = \frac{E}{D}$$

—where M represents the number of degrees the spheres are slewed from the athwartship line.

Slewing the Spheres. Consider a ship with a +D and a +E. The total quadrantal deviation is given by:

$$D \sin 2\zeta' + E \cos 2\zeta'$$

That is, since E is equal to D tan 2M:

$$\begin{aligned} & D \sin 2\zeta' + D \tan 2M \cos 2\zeta' \\ &= D \sec 2M (\sin 2\zeta' \cos 2M + \cos 2\zeta' \sin 2M) \\ &= D \sec 2M \sin 2(\zeta' + M) \\ &= \sqrt{D^2 \sec^2 2M \sin^2 2(\zeta' + M)} \\ &= \sqrt{D^2 (1 + \tan^2 2M) \sin^2 2(\zeta' + M)} \\ &= \sqrt{(D^2 + E^2) \sin^2 2(\zeta' + M)} \\ &= \sin 2(\zeta' + M) \sqrt{D^2 + E^2} \end{aligned}$$

The maximum value of the total quadrantal deviation is therefore $\sqrt{D^2+E^2}$. This occurs when $\sin 2(\zeta'+M)$ is 1. That is, when:

$$\begin{aligned} 2(\zeta'+M) &= 90° \\ \zeta'+M &= 45° \\ \zeta' &= 45° - M \end{aligned}$$

Thus the maximum quadrantal deviation is equal to $\sqrt{D^2+E^2}$ on a course ζ' given by $(45°-M)$. In order to correct this it is necessary for the spheres to be of such size and so placed that they produce an equal and opposite effect. Normally the spheres are placed athwartship and have a maximum effect when the course ζ' is equal to 45°, as shown in Chapter X of Volume I, and they must, therefore, be slewed through an angle M (where tan 2M is equal to E/D) to produce a maximum effect on a course ζ' given by $(45°-M)$.

To correct D and E, therefore, the size and position of the spheres can be found from Table I in Volume I by entering the table with a value for the quadrantal deviation of $\sqrt{D^2+E^2}$; and both spheres must be slewed through an angle M from the athwartship line. If E is positive the direction of slew is clockwise (port sphere forward, starboard sphere aft). If E is negative the direction of slew is reversed.

Example of Slewing Spheres. At a pattern 193 compass, 7″ spheres are placed athwartship at 14″, and the following deviations are observed:

Ship's head (compass)	*Deviation*
N.	$3\frac{1}{4}$°W.
N.E.	$\frac{1}{4}$°W.
E.	$2\frac{1}{4}$°E.
S.E.	$1\frac{1}{2}$°E.
S.	$\frac{3}{4}$°E.
S.W.	$1\frac{1}{4}$°E.
W.	$\frac{1}{4}$°E.
N.W.	$2\frac{1}{2}$°W.

How should the spheres be placed?

Coefficient D remaining$=\dfrac{-\frac{1}{4}-1\frac{1}{2}+1\frac{1}{4}+2\frac{1}{2}}{4}=\frac{1}{2}$

Coefficient D already corrected (from tables) $=3\frac{1}{4}$

Total coefficient D $=3\frac{3}{4}$

Coefficient E remaining$=\dfrac{-3\frac{1}{4}-2\frac{1}{4}+\frac{3}{4}-\frac{1}{4}}{4}=-1\frac{1}{4}$

Coefficient E already corrected $=0$

Total coefficient E $=-1\frac{1}{4}$

$$\text{Total quadrantal deviation}=\sqrt{D^2+E^2}$$
$$=\sqrt{3\tfrac{3}{4}^2+1\tfrac{1}{4}^2}$$
$$=+4^\circ$$

From the table it is seen that 7″ spheres must be placed at a distance of 13″·3.

$$\text{Tan } 2M=\frac{E}{D}=\frac{1\frac{1}{4}}{3\frac{3}{4}}$$
$$2M=18\tfrac{1}{2}^\circ$$
$$\therefore\ \underline{M=9\tfrac{1}{4}^\circ}$$

The spheres must be slewed anti-clockwise (starboard spheres forward) because E is minus.

Example of Reslewing Spheres. At a pattern 193 compass, 7″ spheres are placed at 14″, slewed 20° anti-clockwise (starboard sphere forward). The following deviations are observed :

Ship's head (*compass*)	*Deviation*
N.	4° E.
N.E.	$1\frac{1}{2}$°E.
E.	3° W.
S.E.	$4\frac{1}{2}$°W.
S.	$2\frac{1}{2}$°W.
S.W.	Nil.
W.	2° E.
N.W.	3° E.

How should the quadrantal deviation be corrected ?

$$\text{Coefficient D remaining}=\frac{+1\frac{1}{2}+4\frac{1}{2}+0-3}{4}=+\frac{3}{4}$$

$$\text{Coefficient E remaining}=\frac{+4-2\frac{1}{2}+3-2}{4}=+\frac{5}{8}$$

On the Old Adjustments.

$$\text{Tan } 2M=\frac{E}{D}$$
$$E=D\tan 40^\circ=0{\cdot}84\ D$$

Quadrantal deviation corrected$=\sqrt{D^2+E^2}$

$=\sqrt{D^2+0{\cdot}84\ D^2}$

$=1{\cdot}3\ D=3\frac{1}{4}°$ from table

Coefficient D corrected $\left(\frac{3{\cdot}25}{1{\cdot}3}\right)$ $=+2\frac{1}{2}$

,, remaining $=+\ \frac{3}{4}$

,, total $=+3\frac{1}{4}$

Coefficient E corrected $=-2$ (Spheres slewed anti-clockwise correct minus E)

,, remaining $=+\ \frac{5}{8}$

,, total $=-1\frac{3}{8}$

New Adjustment.

Total quadrantal deviation$=\sqrt{3\frac{1}{4}^2+1\frac{3}{8}^2}$

$=\ 3\frac{1}{2}°$

From the table it is seen that 7″ spheres must be placed at $13\frac{3}{4}$″.

$$\text{Tan } 2M=\frac{E}{D}=\frac{1\frac{3}{8}}{3\frac{1}{4}}$$

$$2M=24°$$

$$\therefore\ M=12°$$

The spheres must be slewed anti-clockwise (starboard sphere forward) because E is minus.

HEELING ERROR

When a ship is upright, the vertical pulls of the permanent, induced and sub-permanent magnetism of a ship cause no deviation, but as soon as she moves from the upright they cause horizontal pulls and consequent deviation.

At a well-placed compass additional forces caused by R, 'k', 'c', and the spheres (if placed), are introduced when a ship heels. When a ship pitches the same effects are produced, except that 'e' has no effect although 'a' acts.

At a badly-placed compass, in addition, rods 'b', 'd', 'g', and 'h' must be considered.

The system of correction used takes account of R, 'k', 'c', 'a', and 'e' only.

NOTE. The fact of placing a Flinders' bar automatically corrects the effect of 'c' when the ship moves from the upright and, therefore, this effect need not be considered when heeling error is corrected.

The Force +R. This force is the vertical component of the permanent magnetism of a ship built in the northern hemisphere, as described in Volume I.

(1) When the ship lists to port it causes a pull to starboard, similar to the effect of a $+Q$, as shown in figure 188.

With a permanent list the deviation would vary as $\cos \zeta'$. (Similar to Q.)

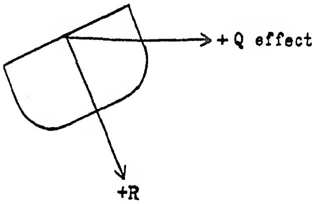


FIGURE 188.

(2) When the ship lists to starboard it causes a pull to port, similar to the effect of a $-Q$, as shown in figure 189.

With a permanent list the deviation would vary as $\cos \zeta'$.

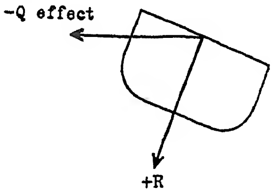


FIGURE 189.

(3) When a ship changes trim, it causes a pull similar to P, as shown in figure 190, and varying as $\sin \zeta'$.

(4) If a ship rolls or pitches, the pulls will vary from port to starboard, and from forward to aft, and cause the compass needle to wander.

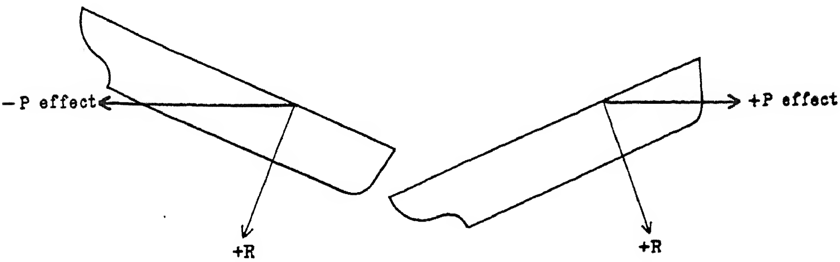


FIGURE 190.

The Rod '$+k$'. A '$+k$' causes pulls similar to those caused by $+R$, but they vary when the ship changes her magnetic latitude because the inducing force is Z. In south magnetic latitudes the

pull would oppose the pull of a +R; in north magnetic latitudes it would act with a +R, as shown in figure 191.

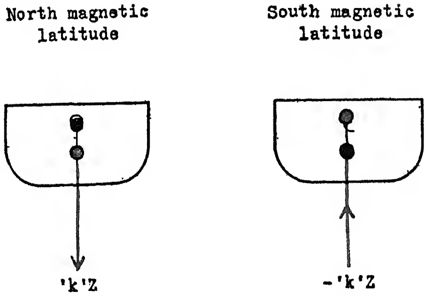


FIGURE 191.

Variation of the Horizontal Pull Caused by R and 'k'. If a ship moves an angle 'i' from the upright, the vertical pulls cause horizontal pulls of R sin 'i' and 'k' Z sin 'i', as shown in figure 192.

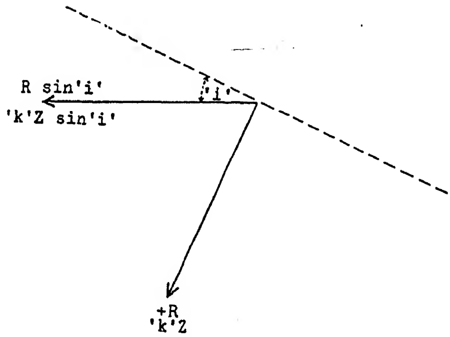


FIGURE 192.

With the spheres placed to correct coefficient D, the ship's original 'a' and 'e' rods have been reduced to 'a_2' and 'e_2'. Rods of values 'a_2' and 'e_2' will therefore cause pitching and heeling error.

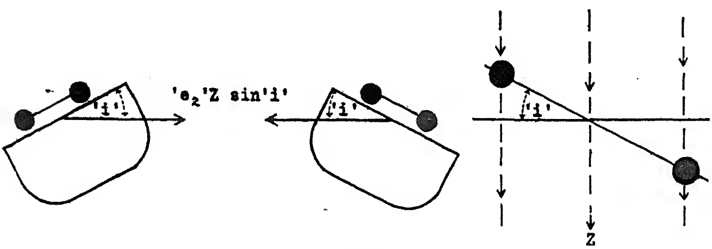


FIGURE 193.

The Rod '$-e_2$'. When a ship heels, the '$-e_2$' rod becomes induced by Z.

The amount of induction depends on the angle of the rod to the vertical lines of force, and the pull exerted is equal to '$-e_2$' Z sin 'i', as shown in figure 193.

The Rod '$-a_2$'. When a ship changes trim, a '$-a_2$' rod will cause a horizontal pull similar to that caused by '$-e_2$' rod, and equal to 'a_2'Z sin 'i', as shown in figure 194.

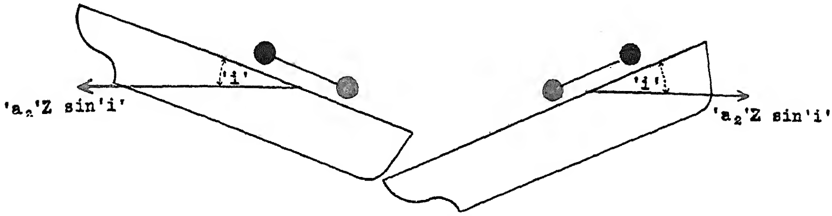


FIGURE 194.

Correction. It is not possible to separate the effect of the permanent magnetism from the effect of the induced vertical magnetism, or to place soft-iron correctors next to permanent magnets because the soft-iron correctors would have magnetism induced in them. The vertical pulls caused by R, 'k', 'e_2' and 'a_2' rods are, therefore, corrected by vertical magnets. Heeling

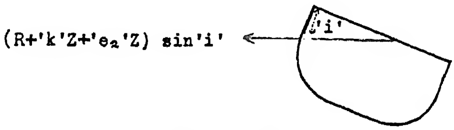


FIGURE 195.

error will therefore alter when the ship changes her magnetic latitude.

For small angles of heel, up to 20°, the horizontal pull of these rods will equal $(R+\text{'}k\text{'}Z+\text{'}e_2\text{'}Z)$ sin 'i', where '$-a_2$' equals '$-e_2$'. This occurs if coefficient D is accurately corrected, as shown in figure 195.

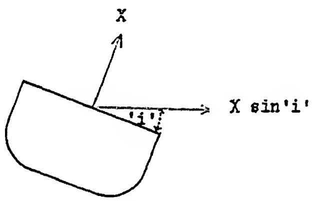


FIGURE 196.

If the pull of the vertical magnets is represented by X, then it will be necessary for them to produce a horizontal pull of X sin 'i', as shown in figure 196, so that :

$$X \sin \text{'}i\text{'} = (R+\text{'}k\text{'}Z+\text{'}e_2\text{'}Z) \sin \text{'}i\text{'}$$
$$X = R+\text{'}k\text{'}Z+\text{'}e_2\text{'}Z$$

When the ship is upright, as shown in figure 197, the vertical force at the compass after correction is :

$$Z+R+\text{'k'}Z-X$$
$$=Z+R+\text{'k'}Z-R-\text{'k'}Z-\text{'e}_2\text{'}Z$$
$$=Z-\text{'e}_2\text{'}Z$$

Therefore when the compass is corrected for heeling error, with the ship upright :

$$\frac{\text{Vertical force at the compass}}{\text{Vertical force of the Earth}}=\frac{Z-\text{'e}_2\text{'}Z}{Z}=1-\text{'e}_2\text{'}=\lambda_2$$

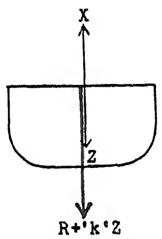


FIGURE 197.

THE SHIP'S MULTIPLIERS, μ AND μ_2

At a well-placed compass feeling only the effects of R and 'k' with the ship upright, the vertical force at the compass, $Z+R+\text{'k'}Z$, would be constant while the ship remains in the same geographical position.

At a badly-placed compass the 'g' and 'h' rods also cause vertical pulls, and the vertical force would alter as the direction of the ship's head is altered, as shown in figure 198.

With the ship's head north (magnetic), the pull would be :

$$Z+R+\text{'k'}Z+\text{'g'}H$$

With the ship's head east (magnetic), the pull would be :

$$Z+R+\text{'k'}Z+\text{'h'}H$$

With the ship's head south (magnetic), the pull would be :

$$Z+R+\text{'k'}Z-\text{'g'}H$$

With the ship's head west (magnetic), the pull would be :

$$Z+R+\text{'k'}Z-\text{'h'}H$$

Clearly, the mean pull would still be : $Z+R+\text{'k'}Z$

The ratio of $(Z+R+\text{'k'}Z)$ to Z, that is, $\frac{Z+R+\text{'k'}Z}{Z}$, is called μ.

It can be defined as :

$$\frac{\text{The mean vertical force acting on the compass}}{\text{The Earth's vertical force at the place}}$$

After heeling error has been corrected, the mean vertical force at the compass is $(Z - \text{'}e_2\text{'}Z)$, and μ, which is now μ_2, is given by:

$$\frac{Z - \text{'}e_2\text{'}Z}{Z} = 1 - \text{'}e_2\text{'} = \lambda_2$$

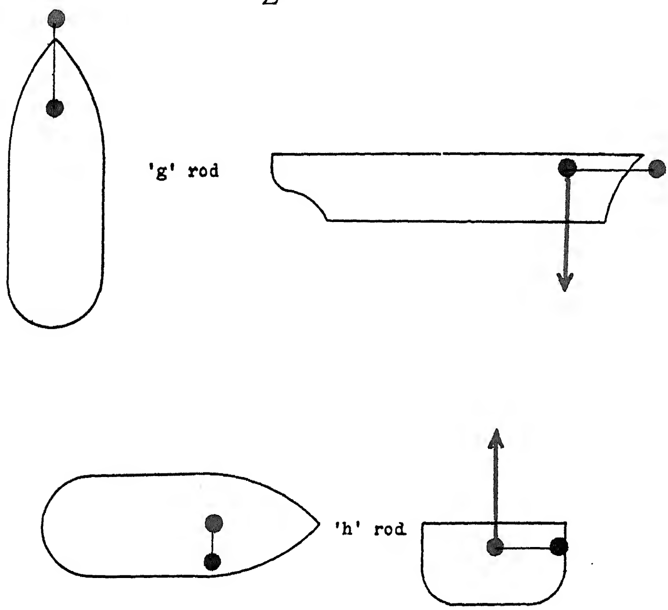


FIGURE 198.

The quantity μ_2 is called the *Ship's Multiplier*, and it may be defined as the ratio of the mean vertical force acting at the compass after heeling error has been corrected to the Earth's vertical force at the place.

NOTE. The heeling error effect of ' g ' and ' h ' rods cannot be corrected. It must be accepted. The effect of ' h ' is, however, negligible.

In order to avoid over-correction of the other factors involved in heeling error, the correction is made:

(1) *when the heeling error instrument is used*, with the ship's head on east or west when ' g ' is not magnetised.

(2) *when the correction is made at sea*, by finding a mean position of the bucket containing the vertical magnets for the positions obtained on two magnetic courses, approximately north and south, which are 180° apart.

CORRECTING HEELING ERROR WITH THE HEELING-ERROR INSTRUMENT

The heeling error is corrected by vertical permanent magnets, the number and position of these magnets being found by using the heeling-error instrument, shown in figure 199. This consists of a circular brass case AA with flat sides. A hook and chain, C, are provided to suspend the instrument when observations are made on board.

One of the sides is glazed and hinged, forming the door of the instrument. It closes with a spring catch at the upper edge.

The flat oblong foot B acts as a stand when observations are made ashore.

Inside the case are the brass bearers, which can be raised or lowered by a lifter E, worked by a milled head at the back of the case. Above the bearers are the agate planes, on which the knife edges of the needle rest when observations are made.

A level F is also fitted, and so adjusted that when the bubble is in the centre, the line DD engraved on the glass will be horizontal.

The needle is mounted on its transverse axis, knife-edged on its under side. It upper side is graduated, a dot marking every

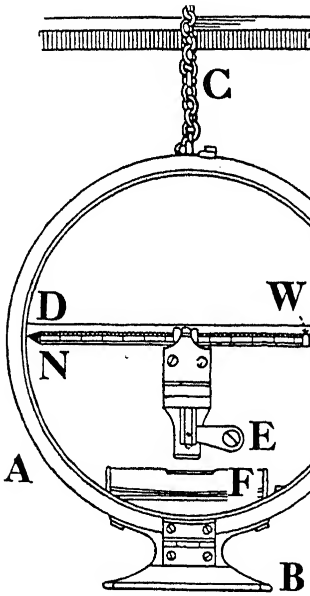


FIGURE 199.

five scale-divisions. The north-seeking or red end, N, is marked by a circle near the extremity. One or more aluminium balancing weights, W, adjustable if required, are placed on one arm of the needle. In the figure the weight is shown on the blue end, S.

It should be understood that the needle is exactly balanced on its axis, without the weight, before being magnetised, and, therefore, the weight is intended to balance only the Earth's vertical force on shore, and the vertical force of the Earth and ship on board.

The instrument is used both on shore and on board.

Ashore. Take the instrument ashore to a place free from local disturbances and place it not less than three feet from the ground, with the needle lying in the magnetic meridian. Carefully

level the instrument. Note that the lifter is up, and place the needle with the marked end to the north, and one of the aluminium weights on the opposite end. (If the observations are made in south magnetic latitude, the weights are put on the north end.) Gently lower the needle on to the agate planes and note if it is exactly horizontal when steady; if not, lift it again and shift the weight in or out as necessary, repeating the operation until it rests exactly level. The horizontal mark DD on the glass door shows when it is level. If one weight is not sufficient, another should be put on, but the two

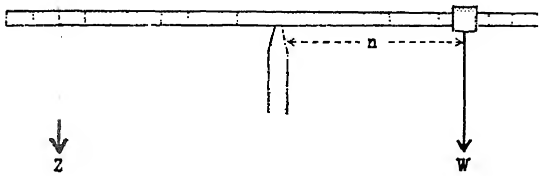


FIGURE 200.

weights must be touching. When the needle has been made to rest horizontally, note the reading on the inner edge of the weight or weights and call this 'n'. The product of the weights W and the scale reading 'n' is a measure of Z ashore. This can be seen in figure 200.

The instrument is now brought on board.

On board. Place the ship's head east or west (magnetic), so that the 'g' rod will not be magnetised. Remove the compass and place a wooden batten across the binnacle at right-angles to the meridian so that the instrument and needle lie in the meridian.

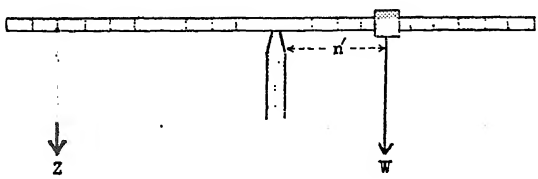


FIGURE 201.

Suspend the instrument under the rod by means of the chain, so that the needle will be in the same horizontal plane as that occupied by the compass needles, and the instrument will be vertically over the centre of the binnacle. If properly suspended by the chain, the instrument should level itself. It must lie so that the needle is in the magnetic meridian.

The vertical force on board (Z') must equal $\mu_2 Z$. Therefore the moment to produce Z' must be $W\mu_2$ 'n'.

Adjust the weight or weights to a distance μ_2'n', that is, to a distance 'n′' from the centre of the needle, as shown in figure 201.

Place the needle on the lifter, marked end to the north, and lower it gently on the agate planes. If no correction is necessary, the needle will remain horizontal. If the marked end of the needle is inclined downwards, insert magnets in the vertical bracket, red ends up, or, if repelled upwards, insert magnets in the vertical bracket with red ends down, increasing the number of magnets until the needle is horizontal; also, when necessary, raising the bucket until the needle is horizontal, but taking care not to have the magnets nearer to the compass card than twice their length.

The vertical magnets must not be allowed to touch one another, and the buckets for holding them are constructed so that this cannot occur.

It is also important that the magnets should be under the centre of the compass. Therefore, if only one magnet is necessary, it should be put in the centre compartment of the bucket. If more than one magnet is required, they should be placed symmetrically in the bucket. The bucket should be examined periodically to ensure that it has not become defective and allowed any magnets to drop out.

If the heeling error has not been corrected, the compass will be seen to swing from side to side when the ship is rolling considerably, because, as already described, the red end of the compass needle is drawn to port and starboard alternately, and the compass becomes unreliable.

When this occurs, and if no heeling-error instrument is supplied, it is possible to carry out the procedure described in Volume I for correcting heeling error at sea.

Whenever it is necessary to alter the number or position of the vertical magnets in order to correct the heeling error, it must be remembered that they will alter the induced magnetism in the Flinders' bar (if this is fitted) and so affect coefficient B. For this reason it will be most necessary to swing the ship and obtain a new table of deviations as soon as possible after the alteration is made.

Examples of Finding the Value of 'n' for a Heeling-Error Instrument.

(1) At Portsmouth ($Z=0{\cdot}42$) the heeling-error instrument is taken ashore and gives 'n' equal to 22.

If λ_2 is 0·8, what value must be used for 'n′' onboard?

'n′' $=0{\cdot}8\times22=17{\cdot}6$. (On the south end of the instrument.)

(2) What value must be used for 'n′' in the same ship at Simonstown ($Z=-0{\cdot}32$)?

'n' at Simonstown $=\dfrac{22}{0{\cdot}42}\times-0{\cdot}32=-16{\cdot}7$

'n′' at Simonstown$=0{\cdot}8\times-16{\cdot}7=-13{\cdot}3$. That is, 13·3 on the opposite side to that used at Portsmouth.

Examples of Heeling Error.

(1) A ship has a list of 9° to starboard, and the following deviations were observed at a well-placed compass:

N.	6°E.
N.E.	1°E.
E.	2°W.

What will be the deviation with the same list when the ship is heading N.60°W. (compass)?

The deviation caused by the list is the same as that caused by force Q (a different coefficient C).

$$\delta = B \sin \zeta' + C \cos \zeta' + D \sin 2\zeta'$$

$\delta_{N.} = +6$ $\therefore$ Coefficient $C = +6$

$\delta_{E.} = -2$ $\therefore$ Coefficient $B = -2$

$\delta_{N.E.} = -2 \sin 45° + 6 \cos 45° + D$ $\therefore$ Coefficient $D = 1 + 1{\cdot}4 - 4{\cdot}2$

$= +1$ $= -1{\cdot}8$

On N.60°W. (compass): $\delta = +2 \sin 60° + 6 \cos 60° + 1{\cdot}8 \sin 60°$

$= +1{\cdot}7 + 3 + 1{\cdot}6$

$= +6{\cdot}3$

$= \underline{6°{\cdot}3E.}$

(2) After complete correction of the deviation with the ship upright, she changes trim by the bow, 2° down. On N.40°E. (compass) the deviation is observed to be 4°E.

What will be the deviation on S.70°W. (compass) if she changes trim by the bow, 4° up?

The deviation caused by the change of trim is the same as that caused by force P (a different coefficient B).

$$B \text{ caused by trim} = +4 \operatorname{cosec} 40° = 6°{\cdot}2$$

$\therefore$ a trim of 1° causes a change in B of $\frac{6{\cdot}2}{2} = 3°{\cdot}1$ (since for small angles sin 'i' is proportional to 'i')

$\therefore$ a change of trim of 4° up changes B, $4 \times 3{\cdot}1 = 12°{\cdot}4$ in the reverse direction to a trim down

$= -12°{\cdot}4$

$\therefore$ deviation on S.70°W. $= -12{\cdot}4 \sin 250°$

$= -12{\cdot}4 \times -\sin 70°$

$= +11{\cdot}7$ or $11°{\cdot}7$E.

INDUCTION

Vertical Magnets and the Flinders' Bar. The chief inductive effect is that of the vertical magnets on the Flinders' bar, as shown in figure 202.

If the Flinders' bar is in the fore-and-aft line, as shown in figure 203:

(1) when vertical magnets are altered, a different force P will be produced.

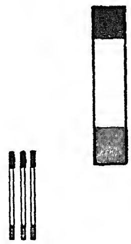


FIGURE 202.

(2) when the length of the Flinders' bar is altered, a different force P will be produced.

Similarly, if the Flinders' bar is slewed, a different force Q will also be produced, as shown in figure 204.

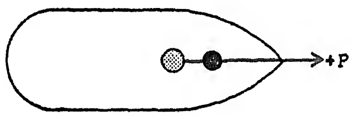


FIGURE 203.

Hence:

(1) when P and 'c', or Q and 'f', are separated, the vertical magnets must be in the same positions for obtaining values of coefficients B and C.

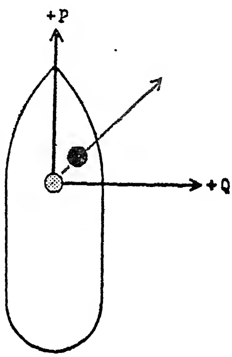


FIGURE 204.

(2) when heeling error is corrected, a different semicircular deviation will be caused and it will be necessary to check the ship's deviation table.

(3) when the length of Flinders' bar is corrected, forces P and Q will alter.

Soft-Iron Correctors and Permanent Magnets. The following arrangements are made to reduce induction :

(1) The Flinders' bar is placed as far away from permanent magnets as possible.
(2) Athwartship magnets are placed on the opposite side of the binnacle to the Flinders' bar.
(3) Permanent magnets are placed as low down as possible.
(4) Large spheres far out are used in preference to small spheres close in.
(5) Spheres and fore-and-aft permanent magnets are placed symmetrically, thus causing equal and opposite induction.

NOTE. One sphere has half the effect of two spheres. Sometimes, in drifters for instance, it is possible to fit only one sphere.

Soft-Iron Correctors and the Compass Needle. The following arrangements are made to reduce induction :

(1) The smallness of the magnetic moment of the compass is made consistent with other requirements.
(2) Soft-iron correctors are placed as far away from the compass as possible.

NOTES ON THE FLINDERS' BAR

When the Flinders' bar is placed in the fore-and-aft line, it produces a small ' +a ' rod, as shown in figure 205. Coefficient D will therefore increase.

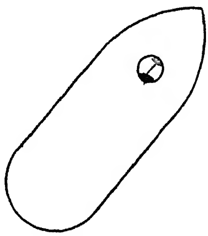


FIGURE 205.

When the Flinders' bar is placed at an angle to the fore-and-aft line it produces, in addition, ' b ' and ' d ' rods, as shown in figure 206. Coefficient E will therefore alter.

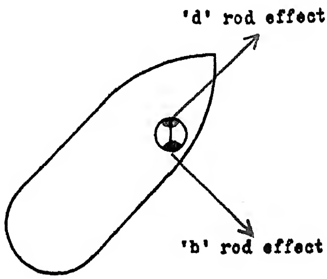


FIGURE 206.

Therefore when the Flinders' bar is altered :

(1) semicircular deviation alters.
(2) quadrantal deviation alters.

It is therefore necessary to correct the compass completely if possible. If this cannot be done, a new deviation table should be made out as soon as possible.

THE THOMPSON (OR KELVIN) DEFLECTOR

This instrument is designed to enable an approximate correction of compasses to be carried out in circumstances where the normal methods of bearings, transits, etc., are not possible. It is frequently used by adjusters of merchant-ship compasses but is not supplied to H.M. ships, because most of these have gyro compasses and the errors of deviation should never be unknown to greater limits than the accuracy of the deflector method of correction can reduce them.

After correction by the deflector, no deviation table of residual errors can be made.

On any course m :

$$H_m = H - P\cos m + Q\sin m - \text{`c'}Z\cos m - \text{`a'}H\cos^2 m - \text{`e'}H\sin^2 m$$

On north magnetic :

$$H_m = H - P - \text{`c'}Z - \text{`a'}H$$

On south magnetic :

$$H_m = H + P + \text{`c'}Z - \text{`a'}H$$

Mean $H_m = H - \text{`a'}H$

Thus if the directive force on north is equal to the directive force on south, coefficient B is corrected.

Similarly if the directive force on east is equal to directive force on west, coefficient C is corrected.

If coefficients B and C have been corrected :

On north and south magnetic : $H_m = H - \text{`a'}H$
On east and west magnetic : $H_m = H - \text{`e'}H$

$$\text{Mean } H_m = H - \left(\frac{\text{`a'} + \text{`e'}H}{2}\right)$$

If coefficient D is corrected, the directive force on north and south is equal to the directive force on east and west.

$$\text{Mean } H_m = H - \text{`}e_2\text{'}H$$

This is the principle of the Thompson deflector.

Method of Using the Instrument. The ship must be conned to the necessary headings by using a compass other than the one being corrected.

(1) *To Find the Deflection-Scale Reading with the Ship's Head North by Compass.* Place the deflector over E. by S. or W. by N., using the pointer.

Alter the force of the deflector by working the adjusting screw until the card is turned 90°, keeping the pointer over the chosen heading.

Note the scale reading.

(2) Similarly find the scale reading with the ship's head east by compass.

(3) Similarly find the scale reading with the ship's head south by compass.

If the scale reading on north is 28 and on south is 20, insert permanent magnets to increase the reading to 24, the mean of the two readings.

(4) Similarly find the scale reading with the ship's head west by compass.

If the scale reading on east is 24 and on west is 34, insert permanent magnets to decrease the reading to 29, the mean of the two readings.

(5) Repeat the above procedure and move the spheres to produce a reading of $26\frac{1}{2}$—the mean of the two mean readings—on all four cardinal points.

If the final readings on all four cardinal points are in close agreement and it can be assumed that there are no coefficients A and E, then the compass may be considered to be approximately corrected.

Note. When the ordinary commercial instrument is used, it will be necessary to carry out a second swing if the original readings on opposite points differ by more than 10 divisions.

SWINGING SHIP

The various methods of swinging ship and the procedure for carrying out a normal swing, together with examples of normal swings for obtaining a deviation table, are given in Volume I.

Procedure for Swinging a Hitherto Uncorrected Compass in a Ship of a New Class.

(1) Steady the ship for at least four minutes on the quadrantal points, N.E., S.E., S.W. and N.W. (compass). Obtain the deviations and find the coefficient D remaining; hence the total coefficient D to be corrected. Adjust the position of the spheres.

(2) Steady the ship on four equidistant magnetic points and find λ_2, as described on page 338.

(3) Steady the ship on east or west (compass) and correct the heeling error, using the heeling-error instrument, as described on page 357.

(4) Steady the ship for at least four minutes on a cardinal point. Raise, lower, or alter the number of corrector magnets and remove all the deviation.

NOTE. The magnets to alter are those at right-angles to the compass needle.

(5) Steady the ship for at least four minutes on the next cardinal point. Alter the other set of corrector magnets and remove all deviation.

(6) Repeat the procedure in paragraphs (4) and (5) and remove half of the remaining deviation.

(7) Analyse coefficient E and calculate the total quadrantal deviation and angle of slew. Reset the spheres.

(8) Steady the ship for at least four minutes on N., N.E., E., S.E., S., S.W., W. and N.W. (compass), and obtain the deviations. Then repeat this procedure, swinging the ship in the opposite direction. Mean the results of the two swings and obtain the deviation table.

The deviation table thus obtained will be reasonably accurate, if the ship does not change her magnetic latitude. A large change of magnetic latitude will be necessary before it is possible to separate P and 'c' and Q and 'f'.

Example of Checking a Deviation Table at a Compass where Coefficient A is Zero, or a Known Amount, by Swinging Ship by an Object, the True Bearing of which is not known. At a well-placed compass the bearings of a lighthouse, distant 16 miles, were found to be:

Ship's head (compass)	*Compass bearing*
N.	S.$23\frac{1}{2}$°E.
N.E.	S.$26\frac{1}{2}$°E.
E.	S.$28\frac{1}{2}$°E.
S.E.	S.$28\frac{1}{2}$°E.
S.	S.$28\frac{1}{2}$°E.
S.W.	S.$26\frac{1}{2}$°E.
W.	S.24°E.
N.W.	S.24°E.
	8|210 =S.$26\frac{1}{4}$°E.

The deviation table, will therefore be:

Ship's head (compass)	*Deviation if coefficient $A=0$*	*Deviation if coefficient A is known to be $+\frac{1}{2}$*
N.	$2\frac{3}{4}$°W.	$2\frac{1}{4}$°W.
N.E.	$\frac{1}{4}$°E.	$\frac{3}{4}$°E.
E.	$2\frac{1}{4}$°E.	$2\frac{3}{4}$°E.
S.E.	$2\frac{1}{4}$°E.	$2\frac{3}{4}$°E.
S.	$2\frac{1}{4}$°E.	$2\frac{3}{4}$°E.
S.W.	$\frac{1}{4}$°E.	$\frac{3}{4}$°E.
W.	$2\frac{1}{4}$°W.	$1\frac{3}{4}$°W.
N.W.	$2\frac{1}{4}$°W.	$1\frac{3}{4}$°W.

Example of Swinging a Badly-Placed Compass. A ship left harbour with heeling error corrected and the following correctors in place at a pattern 193 compass:

$8\frac{1}{2}''$ spheres at 14″ placed athwartship.
$8\frac{1}{4}''$ Flinders' bar on the fore side of the compass.
Fore-and-aft magnets with blue ends placed forward.

On the First Swing.

Ship's head (compass)	*Deviation*		
N.E.	3°W.	Coefficient D remaining	$=-\frac{1}{2}$
S.E.	2°E	,, corrected	$=+5\frac{1}{4}$
S.W.	Nil	,, to be corrected	$=+4\frac{3}{4}$
N.W.	3°W.	∴ Place the spheres at $14\frac{1}{2}''$.	

On the Second Swing.

N.	2°E.
E.	4°W.
S.	Nil
W.	Nil

Half this deviation was taken out with permanent magnets, leaving:

N.	1°E.	
E.	2°W.	Coefficient $E=+1\frac{1}{2}$
S.	1°E.	∴ Total quadrantal deviation
W.	2°W.	$=\sqrt{4\frac{3}{4}^2+1\frac{1}{2}^2}$

$$=5{\cdot}1$$

$$\text{Tan } 2M=\frac{E}{D}=\frac{1\frac{1}{2}}{4\frac{3}{4}}$$

$$2M=19°$$

∴ $M=9°$. Spheres slewed anti-clockwise and placed at $14\frac{1}{2}''$.

On the Final Swing.

N.	1°E.
N.E.	½°E.
E.	1°E.
S.E.	Nil
S.	½°E.
S.W.	½°W.
W.	Nil
N.W.	½°E.

$A=+\frac{3}{8}$ $B=+\frac{1}{2}$ $C=+\frac{1}{4}$ $D=-\frac{1}{8}$ $E=+\frac{1}{8}$

VARIATION

The changes in variation are described in Volume I.

H.M. ships, particularly on the East Indies and West Indies stations, should obtain at every opportunity observations for finding the variation.

Observations on Shore. The methods of choosing a suitable position ashore and making observations to find the variation by means of a theodolite, landing compass, or sextant, are described in the *Admiralty Manual of Hydrographical Surveying.*

Observations on Board. Observations should be made with the standard compass on eight or sixteen equidistant points, the ship being steadied for at least four minutes on each point, and bearings obtained of the Sun, or another heavenly body, or of a distant object.

If the azimuth of the heavenly body is worked out, the difference between this true bearing and the compass bearing will give the total compass error.

This mean total compass error, corrected for coefficient A if necessary, will give the variation.

Two sets of observations should be obtained, one set with the ship swinging to starboard and the other set with the ship swinging to port. The results should be meaned.

Observations should be recorded on form S 374A—*Record of Observations for Deviation*—which should be amended as necessary, and a copy forwarded to the Hydrographer in accordance with K.R. and A.I.

Example. A ship was known to have a coefficient A of —1. The following observations of the Sun were made :

Ship's head (compass)	*Compass bearing*	*True bearing*
N.	S.70°W.	260°
N.E.	S.70°W.	260½°
E.	S.70°W.	261°
S.E.	S.71°W.	261½°
S.	S.71½°W.	261¾°
S.W.	S.73°W.	262°
W.	S.73½W.	262¼°
N.W.	S.74°W.	262½°

Required the total compass error.

Ship's head (compass)	*Total compass error*
N.	10°E.
N.E.	$10\frac{1}{2}$°E.
E.	11°E.
S.E.	$10\frac{1}{2}$°E.
S.	$10\frac{1}{4}$°E.
S.W.	9°E.
W.	$8\frac{3}{4}$°E.
N.W.	$8\frac{1}{2}$°E.

Mean total compass error $= 8\overline{\smash{)}78\frac{1}{2}}$

$= 9°{\cdot}7$E.

Coefficient A $=$ 1° W.

$\therefore$ Variation $=$ $10°{\cdot}7$E.

Notes on Inspecting the Magnetic Compass Equipment in Ships Under Construction in Private Yards Before Trials.

(1) Check the distances of magnetic materials and electrical fittings from the various compasses and see that they conform with the tables given in Volume I.

(2) No bells, wires or other electrical fittings, other than the compass light, should be attached to binnacles.

(3) Electric lighting leads to binnacles must be led through the tube provided at the base and not through a hole in the side of the binnacle.

(4) Binnacles must be lined up in the fore-and-aft line, and deck plates must be fitted so that some adjustment is possible in each direction.

(5) Voice pipes must be attached to binnacles so that the doors for inserting correcting magnets can be hinged right back.

(6) Lighting arrangements to be in order.

(7) Check that the spheres, Flinders' bars, and corrector magnets have been placed in position in accordance with the information supplied by the Compass Department, Slough.

(8) Check that the correct compasses have been supplied and that the cards and pivots, etc., are undamaged.

CHAPTER XXII

THE BROWN AND ANSCHUTZ GYRO COMPASSES

The Sperry gyro compass, in general use in H.M. ships, is fully described in the *Admiralty Manual of the Sperry Gyro Compass.* In addition, a brief description of the compass is given in Volume I.

Brown and Anschutz compasses are found in some ships of the Merchant Navy and in some foreign warships.

The Brown Gyro Compass. This compass is considerably smaller and lighter than the Sperry type in use in H.M. ships. The master compass weighs approximately 27 pounds.

The gyro is a small light wheel (4 lbs.) run in air at high speed (14,000 r.p.m.). The case in which the wheel runs is free to tilt in a vertical ring on knife edges. The vertical ring is free to turn in azimuth about a vertical axis and is supported at the end of this axis by a jet of oil maintained by a small electric pump. The card is secured on top of the vertical ring and is therefore part of the sensitive element.

The control is of the liquid type, oil being used. Two pairs of bottles, each connected north–south, are fitted on the wheel case. One pair supplies the north-seeking control and the second pair the damping, or north-settling property. When the wheel tilts, the oil in the first pair is forced against gravity by an air blast, the pressure for which is obtained by the revolution of the wheel in its case. In the damping bottles the oil is blown in the opposite direction, but the flow is restricted by an adjustable needle valve so as to make it out of phase with the tilt of the wheel.

The repeaters are worked by a transmitter operated through a relay by an air vane that is maintained in alignment with the vertical ring. The air jet to control the air vane is supplied from the wheel case by the same method used for the control bottles.

The whole compass is gimballed in the usual way, being made pendulous by the oil pump which maintains the oil jet for supporting the sensitive element.

The Anschutz Gyro Compass (F.M. type). This compass is approximately the same size and weight as the Admiralty Sperry compass.

The sensitive element consists of two gyro wheels that are carried inside a sphere which is filled with gas and then sealed. These wheels are linked together and spring-controlled round their vertical axes. They are arranged so that their axes of spin, which

are in a horizontal plane, are at an angle of 120° to each other when the rotors are not spinning. When the gyro is running, this angle varies slightly through the action of the gyros in stabilising the sphere against the motion of the ship. The line bisecting the angle between the two axes defines the meridian line on the sphere. The sphere is bottom heavy about its central horizontal plane and so provides a solid gravity control to make the compass north-seeking.

Damping is produced, as in the Brown compass, by a restricted oil flow between bottles secured north and south on the upper half of the sphere.

The sensitive sphere has slight negative buoyancy in acidulated water, through which the necessary electric currents are supplied to it, contained in an outer (or follow-up) sphere. Inside the lower half of the sensitive sphere is a horizontal coil which is supplied with alternating current and causes a repulsion effect between the two spheres and so centralises and supports the inner in the outer.

The outer sphere is kept in alignment with the inner sphere electrically, on the Wheatstone-bridge principle. It thus corresponds to the phantom in the Sperry compass but is non-hunting. The card is carried on this follow-up sphere, which also operates the transmitters.

The whole is enclosed in an outer container which is connected with the liquid between the spheres and is cooled by salt water. The rate of flow of cooling water is controlled by a thermostat, so that the sensitive element is maintained at a constant temperature.

The container is gimballed fore-and-aft only. The transmission is on the rotating field principle.

This compass, which was given Service trials, has a performance very similar to the Admiralty Sperry. Its particular merit is the consistency of its settling position in undisturbed conditions.

The Anschutz Gyro Compass (A.B.C. type). This compass was designed to provide a unit stabilised about all three axes. The compass element (stabilisation in azimuth) is the same as the F.M. type except for the arrangement of the gyros in the sensitive sphere. The two gyros are not linked but are spring-controlled round their vertical axes and so arranged that their axes of spin, which are in a horizontal plane, point at each other when the rotors are not spinning. When running, the gyros set themselves at an angle to each other that varies with the latitude. This arrangement automatically gives the compass a constant period in all latitudes. The line bisecting the angle between the two axes defines the meridian on the sphere.

CHAPTER XXIII

HYDROGRAPHIC SURVEYING

Although the majority of regular surveys at the present time must be carried out by specially equipped surveying vessels, much valuable work can still be undertaken by navigating officers when opportunity offers. In the past many excellent charts have been produced from surveys made by navigating officers, and the information gathered by them has added considerably to hydrographic knowledge in general.

Whenever it is proposed to undertake a marine survey, the navigator is well advised to communicate with the Hydrographic Department at the Admiralty if there is sufficient time. Correspondence should be addressed direct to the Hydrographer, who is in a position to give valuable advice about the execution of the proposed survey and may be able to supply useful trigonometrical (triangulation) data from previous surveys made in the same area.

The Department is usually able to lend a certain number of surveying instruments—observing and sounding sextants, sextant stands, measuring tapes, theodolites, scales and straight edges, for example—to surveying expeditions, and application should be made for any instrument that will facilitate the work by augmenting the ordinary navigational equipment on board. Small supplies of paper of the various qualities required in surveying work can also be supplied at short notice and will be forwarded by air mail, if necessary.

The guiding principles that must be followed in any hydrographic survey are given in the *Admiralty Manual of Hydrographic Surveying* and other books.

In this chapter it is necessary to do no more than consider how the navigator can modify normal practice to suit his own limited resources and still make a useful survey. He will, as a rule, have no qualified assistants, and it may be assumed that the time at his disposal will be short enough to render impossible the refinements in observation and calculation that are customary in properly organised work.

The 'navigational' survey will nearly always cover a comparatively small area and will seldom be used as a basis for subsequent extension. For this reason it will be sufficiently accurate to use five-figure logarithm tables in all calculations.

If the triangulation provides a framework free from any *plottable* error on the particular scale in use, the results will be all that can normally be expected and will satisfy requirements.

Surveys that navigators can undertake fall into three categories.

(1) *Original Surveys.* These should be confined to anchorages and little-known harbours, where the chart shows only the barest detail or no detail at all, or is on too small a scale to be of practical value. When this occurs, the chart cannot be used in any way as a basis for the projected survey, and some regular form of triangulation is necessary. Surveys of this type are particularly useful, and in their execution the object should be a careful and systematic examination of a small area in preference to a less thorough survey covering much ground. The latter, though of some value, does not wholly fulfil navigational requirements and might be the inadvertent means of leading ships into dangers they would otherwise have avoided.

(2) *Re-surveys.* These may be regarded as surveys of areas about which the charts give information that is either insufficient or inaccurate. They should be made with the object of increasing or correcting that information. Such re-surveys are often of great value and, as they can usually be based largely on points already shown on the published chart, they are considerably easier to carry out than wholly original surveys.

(3) *Running and Sketch Surveys.* These can be considered to include all work that can be conveniently forwarded as a Hydrographic Note accompanied by a tracing. No regular triangulation is involved and the results must depend to a large extent on the accuracy with which distances can be ascertained by log, compass bearings and other methods of measurement.

Before a small survey is started, careful attention should be given to the means by which the results can be incorporated on the charts covering the area. Many surveys, accurate in themselves, have been spoilt by the lack of any connexion with the detail already shown on the charts. In consequence it has been impossible to 'locate' the survey satisfactorily in relation to previous work. In surveys of out-of-the-way anchorages and harbours which are perhaps shown in the barest outline on the charts, it may be impossible to find any recognisable points with which to connect the new work. The location of the survey must then rely on astronomical observations which at best do not solve the problem satisfactorily. In all other surveys, however, every endeavour should be made to include, if possible in the triangulation, one or more points of detail, such as hill-summits, buildings, well-defined land-marks and islets, by which the work can be fitted in its correct position relative to previous surveys. This is especially important when the principal object is to test the accuracy of

soundings over an area which has already been surveyed. Attention to this point not only helps the cartographer but often provides a valuable check on the accuracy of the survey as a whole.

The survey of a small area on a moderate scale can be undertaken without much difficulty with the navigational instruments normally available on board. It will be necessary to improvise only surveying marks, etc. If anything more elaborate is contemplated, it is advisable to obtain a theodolite for use in at least some part of the triangulation, a pocket compass, a metal scale, beam compasses, a straight edge and a supply of linen-backed paper for the plotting sheet. Linen-backed paper and tracing cloth can often be obtained from local land-survey offices abroad. Further additions to the equipment, such as steel tapes, can be made, but full advantage of these additions can be taken only if time is not restricted.

Scale of the Survey. In the absence of definite instructions about the scale to be used for the survey, one of the standard scales enumerated on Admiralty Chart Misc. 19 should be chosen. For the small surveys which are being considered here, the most useful scales are probably 1/25,000 (about 2·9 inches to a mile) which is sufficiently large to show anchorages satisfactorily, and 1/50,000 (about 1·45 inches to a mile) which is suitable for open coasts. For very small harbours scales of 1/12,500 or even 1/6,250 may be desirable, but much skill and very careful observations are needed to ensure accurate results on these large scales, and the inexperienced surveyor will be well advised not to attempt anything too ambitious.

In choosing a scale it is also necessary to consider the instruments available for plotting. Large sheets can be plotted with accuracy only if metal scales, straight edges and beam compasses are used. If the plot has to be made entirely with station pointers, the scale should be such that the marks of the survey are all contained within a sheet of moderate size, about 15 inches square, say.

In the absence of a metal scale, there is no exact method of measuring distances on the paper and the natural scale can only be approximate. This, however, is not important since the *relative* positions of every point in the survey can be correctly plotted, and the scale can be determined at any time if the actual length on the Earth of one or more sides of the triangulation is stated.

Base Measurement. The method of using a steel tape for base measurement is described in Chapter II of the *Manual of Hydrographic Surveying*. It may be assumed, however, that a steel tape will not, as a rule, be available, and that the navigator will usually have to rely on measuring his base by masthead angle or rangefinder. For a small survey either of these methods should give sufficiently good results. The rangefinder method is usually the more convenient, and, if a two-metre or larger instrument is fitted,

its use is also preferable on the score of accuracy. An example of base measurement by masthead angle, which is equally applicable to the rangefinder, is given in Chapter II of the *Manual of Hydrographic Surveying*.

In this example the ship is anchored in such a position that she forms a well-conditioned triangle with two shore marks from which horizontal angles are observed simultaneously with the ranges, and the measurement of the distances of both marks affords a valuable check on the accuracy of the work. The principle illustrated by the example can almost always be applied in the survey of a small harbour, strait or anchorage, and it may often be possible to measure the longest side of the triangulation in this way since it will probably be only 3 to 4 miles. If this can be done, all the remaining marks can be plotted, starting from the ' long side ', without any mathematical calculation other than that required for finding the length of this side from the rangefinder measurements and converting it to the corresponding length in inches proportional to the scale of the survey. This saves considerable time and labour.

When a theodolite is available, a good base measurement can be made by the subtense method described in Chapter II of the *Manual of Hydrographic Surveying*. The subtended distance must be measured with great care if accurate results are to be obtained, and an engineer's fifty-foot steel tape is recommended. Linen tapes are so inaccurate that they must not be used for any form of base measurement.

Triangulation. The regular methods of triangulation and the elaborate adjustment of the observations described in Chapter III of the *Manual of Hydrographic Surveying*, cannot be carried out thoroughly unless ample time and a full equipment of surveying instruments are available. As a rule, the navigator must measure all angles with a sextant, and in the small surveys with which he is generally concerned, he should reduce his system of triangulation to the simplest possible form. For this type of survey the following points should be noted :

(1) The number of triangulation stations should be the least that will provide a framework to cover the area of the survey. For a small anchorage or harbour, half a dozen stations should generally suffice. From these it should be possible to shoot in any additional marks required for fixing the soundings and topography.

(2) The stations should be grouped as far as possible in quadrilaterals or polygons with central stations. By so doing, each will be connected by at least *three* shots from adjacent points, and a check is provided against errors of observation and plotting.

(3) Natural marks may be used to a large extent, and efforts should be made to include as part of the triangulation a selection of well-defined points already shown on the chart. This will greatly

facilitate the subsequent incorporation of the work with other material appearing on the chart plate.

(4) The angles of a triangulation should be measured in the horizontal plane. The errors to which sextant measurements between terrestrial objects are susceptible, can be largely eliminated by proper methods of observation. It may not be possible to site all the stations at the same or nearly the same level, but it should be remembered that, if the subtended angle is 90°, no error will be introduced into sextant angles by the difference in the elevation of two objects. This fact will often enable the sextant observer to obtain an accurate series of angles between stations at different elevations by arranging the connexion between the high and low objects to be made through a subtended angle of about 90°.

(5) No hard and fast rule can be laid down about the accuracy required in the measurement of the angles of the triangulation when a sextant is used. It should, however, be possible to 'close' the triangles with an error not exceeding 2′.

(6) Complete adjustment of the angular observations is rather laborious and for small sextant triangulations serves little purpose. It will be sufficient before plotting to close the separate triangles so that the values of the angles add to 180°.

(7) If plotting is effected by the 'long side' method, it is unnecessary to calculate the rectangular co-ordinates of the stations. If required, these can be found at a later date.

(8) Once a framework of triangulation has been provided, any additional marks required for sounding, etc., can often be shot from boat stations, a method which saves a lot of time.

(9) It is unlikely that a small triangulation, made in the way described in this chapter, will be subsequently used for extending the work. For this reason it is unnecessary to adjust the quadrilaterals or polygons.

Surveying Marks. (See Chapter V of the *Manual of Hydrographic Surveying.*) For shore marks, whitewash marks or boathook staves with small flags will, with the help of natural objects, fulfil all the requirements of a small survey. Floating marks are sometimes useful for fixing part of the survey area, and the ship or a boat on a taut moor can often be used with advantage if the scale of the survey is not too large. Alternatively, a small beacon made with a pole and two lime-juice casks can be improvised and anchored in a suitable place.

Plotting and Graduation. The most satisfactory method of plotting the triangulation is by co-ordinates, as described in detail in Chapter VIII of the *Manual of Hydrographic Surveying*. This, however, involves a considerable amount of mathematical calculation which is hardly justified unless a complete outfit of plotting instruments is available. When station pointers only are used, it is advisable to plot from a long side, according to the method

explained in the same chapter. This method is simple and involves a minimum of calculation, especially if, as previously stated, a direct measurement of the long side can be made by rangefinder or masthead angle. If possible, a piece of linen-backed paper should be obtained and used as a plotting sheet. Otherwise the plot of a small survey can be satisfactorily drawn on cartridge paper pasted on a flat board or the back of an ordinary chart. The method of graduating a sheet, plotted from a long side, is described in the same chapter of the *Manual of Hydrographic Surveying*. Unless, however, a proper equipment of scales, straight edges and beam compasses is used, no attempt should be made to graduate the sheet, because the ordinary navigational instruments are insufficient to enable this graduation to be done with the degree of accuracy that is essential. The graduation can always be added at a later date if necessary. At the time of plotting :

(*a*) the latitude and longitude of at least one trigonometrical station are recorded or given by reference to the chart.

(*b*) a true meridian is drawn through one of the stations.

(*c*) the lengths of the sides of the triangulation are given.

For work in the field, Whatman drawing boards or field boards are used. The latter can easily be made in the ship. Normally they are mounted with cartridge paper, but any good quality paper —the back of a chart, for example—will serve the purpose.

Rectangular Co-ordinates. When time permits, it is useful to calculate the rectangular co-ordinates of the main stations of the triangulation since they provide a convenient method of defining relative positions over a small area and they can be used for plotting the survey. A description of the method and principles involved will be found in Chapters III and VIII of the *Manual of Hydrographic Surveying*. Only a brief summary is given here.

One station, at which a true bearing has been observed, is selected as the *point of origin*. The true meridian through this station is the y-axis of the co-ordinates, and a line through the station at right-angles to this meridian is the x-axis. The positions of the other stations are defined by their distances in feet (or metres) from these axes, y co-ordinates being regarded as positive when the stations lie north of the x-axis and negative when they lie south, and x co-ordinates as positive when the stations lie east of the y-axis and negative when they lie west. The co-ordinates of the point of origin are $x=0$, $y=0$. If the geographical position of the point of origin is known, that of any other station can be calculated from its rectangular co-ordinates without difficulty.

In figure 207 O, A, B and C are four main stations, and a true bearing of the side OA has been observed from O which is therefore selected as the point of origin. OY is a true meridian through O and therefore the y-axis ; OX is a line at right-angles to OY and therefore the x-axis. Aa and Bb are drawn parallel to OX to meet

the y-axis at a and b respectively. Ac is drawn parallel to OY to meet Cc drawn parallel to OX in c. The rectangular co-ordinates of the stations are then defined by the following lengths:

O:	$x=0$;	$y=0$
A:	$x=Aa$;	$y=Oa$
B:	$x=Bb$;	$y=Ob$
C:	$x=Aa+Cc$;	$y=Oa+Ac$

The lengths of the sides OA, OB, . . . and the angles of the triangles OAB and ABC are known from the triangulation, and the

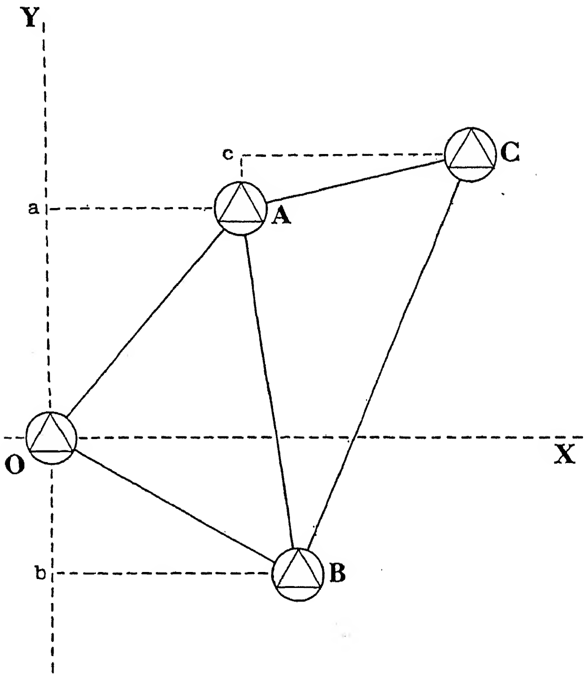


FIGURE 207.

true bearing OA—the angle AOY, that is—has been observed. It is thus clear that the co-ordinates of A are:

$$x_A=OA \sin AOY; \qquad \text{and } y_A=OA \cos AOY$$

The true bearing of the side OB (measured from north) is the sum of the angles AOY and AOB, and the co-ordinates of B are:

$$x_B=OB \sin (AOY+AOB); \qquad \text{and } y_B=OB \cos (AOY+AOB)$$

Bearings taken from points not on the original meridian are not true bearings, and are called co-ordinate bearings since they are measured from co-ordinate meridians, such as Ac, which are lines drawn parallel to OY. The co-ordinate bearing of AC is the

angle CAc, and this can be deduced from the true bearing of OA and the angles OAB and BAC. Thus:

$$\angle CAc° = (180° + \angle AOY) - (\angle OAB + \angle BAC)$$

Hence the co-ordinates of C are:

$$x_C = x_A + AC \sin [(180° + AOY) - (OAB + BAC)]$$
$$y_C = y_A + AC \cos [(180° + AOY) - (OAB + BAC)]$$

It is seen that the co-ordinates of C can be found equally well through B or those of B through A, and the formulæ can be stated in general terms thus: if d is the length of a side joining two stations, α its bearing measured clockwise from the co-ordinate meridian, and $\triangle x$ and $\triangle y$ the required differences between the co-ordinates of the two stations:

$$\triangle x = d \sin \alpha$$
$$\triangle y = d \cos \alpha$$

If the co-ordinates of the first station are (x_1, y_1), those of the second station are:

$$x_2 = x_1 + d \sin \alpha$$
$$y_2 = y_1 + d \cos \alpha$$

In the application of these formulæ, all addition must be algebraic. For example:

x is $\frac{\text{positive}}{\text{negative}}$ when the station is $\frac{\text{east}}{\text{west}}$ of the y-axis.

y is $\frac{\text{positive}}{\text{negative}}$,, ,, ,, $\frac{\text{north}}{\text{south}}$,, x-axis.

x is $\frac{\text{positive}}{\text{negative}}$ when α lies between $\frac{0° \text{ and } 180°}{180° \text{ and } 360°}$

y is $\frac{\text{positive}}{\text{negative}}$,, ,, ,, $\frac{270° \text{ and } 090°}{090° \text{ and } 270°}$

By the reverse process, if the co-ordinates of the two stations are given, $\triangle x$ and $\triangle y$ are at once deduced from their differences, and the co-ordinate bearing and length of the side are found from the formulæ:

$$\tan \alpha = \frac{\triangle x}{\triangle y}$$

$$d = \triangle x \operatorname{cosec} \alpha = \triangle y \sec \alpha$$

Plotting by Co-ordinates. When the stations are plotted by their co-ordinates, a rectangle is chosen, the sides of which are parallel to the x and y axes, and the dimensions of which are such that it will contain all the stations within its sides, suitable co-ordinates being assigned to the four corners to effect this. The first stage in the plot is to construct this rectangle on the scale of the survey.

If, for example, the difference between the co-ordinates of its east and west sides is 100,000 ft. and the scale 1/25,000, the north and south sides are represented by lines 4 feet or 48 inches in length on the plotting sheet. The diagonals of the rectangle are drawn and the sides bisected so that their mid-points can be joined. The result is shown in figure 208.

The co-ordinates of the corners being known, those of the mid-points and centre can be deduced and the bearings of the diagonals calculated. The four corners, mid-points and centre provide a series of points from which the stations can be plotted by choosing three or more shots that will make a good cut at each of them. The station *A*, for example, can be plotted by shots from the north-

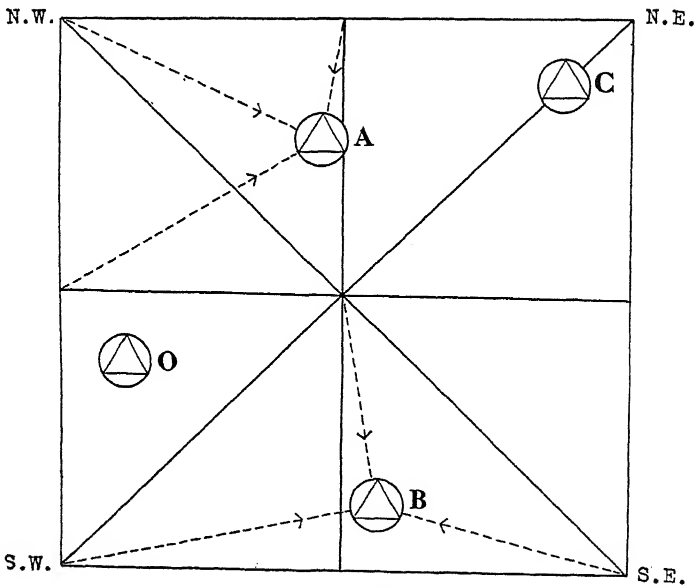


FIGURE 208.

west, mid-north and mid-west, its co-ordinate bearing being calculated from each of these points and the shots protracted by laying off (with chords) the angles they make with the vertical, horizontal or diagonal of the rectangle. The three (or more) shots should intersect in a point. Similarly *B* can be plotted from the south-west, south-east and central points, and the other stations also from appropriate points.

The rectangle provides a rigid framework on which to base the plot, and good cuts at the stations are always obtainable. This is a great improvement on the 'long side' method of plotting, and has the advantage that, once the co-ordinates of the stations have been fixed and accepted, any failure to obtain perfect results

in the plot reveals itself in errors that can result only from inaccuracies in the calculation of the bearings or their protraction; it cannot be caused by errors of observations in the field.

NOTE. Rectangular co-ordinates are used for gridding topographical maps used by artillery.

Sounding. (Chapter IX of the *Manual of Hydrographic Surveying*.) In a regular hydrographic survey a thorough examination of the water area by systematic and closely-spaced lines of sounding is the most essential feature. It cannot be expected that this idea will always be achieved in the type of survey considered here; nevertheless, efforts in this direction should be made so far as time and circumstances permit. Sounding over a small area sufficiently intense to give a reasonable assurance that no shoals remain undetected is generally preferable to more open sounding over a larger area. The latter is useful for showing the general depths to be expected but is no guarantee against the existence of dangers in the space between the lines of sounding.

In general the navigator will have to use a boat and hand-lead for most of the sounding. If, in addition, the ship can use her echo-sounding apparatus, the volume of work can be greatly increased.

For boat sounding the following fittings (which can be improvised on board) are essential:

(*a*) *Lead Lines* marked at every foot and fitted with wire hearts. Soundings below 11 fathoms (reduced) must be accurate to the nearest foot and lead lines must be marked accordingly. The fitting of wire hearts (Kelvin wire) takes a little time, but this is more than repaid by the advantages of having lead lines of unvarying length.

(*b*) *Chains* of some description should be rigged in the boat. Accurate soundings cannot be obtained unless the leadsman can stand up in comfort and give his whole mind and strength to the job in hand without having to balance himself precariously at the same time. Heaving a lead in a boat is *never* a simple operation, and it is essential to make the leadsman's work as easy as possible by sounding at a moderate speed so that there is ample time to get good up-and-down soundings. In power boats that cannot be throttled down to a low speed, a drogue should be towed.

(*c*) *A Table* on which the field board and instruments can be placed. This can usually be rigged without difficulty across the stern sheets of the boat after the canopy, which is liable to be in the way of the sextant observers, has been removed.

The usual methods of running lines of sounding approximately at right-angles to the depth contours are described in Chapter IX of the *Manual of Hydrographic Surveying*. There must be two observers with sextants in the boat, and at first a good deal of

difficulty is likely to be experienced, especially on the larger-scale surveys, in running even approximately straight lines. The process of 'starring', used for rounding a point of land, may often be applied with advantage to objects such as a buoy or a boat at anchor. Compared with the usual method of running parallel lines, it is much easier and it covers the ground in a reasonably systematic way.

All soundings must be reduced to a common datum, and it is therefore essential to erect a tide pole (which can be made on board) at some convenient place in the survey area and to record its readings at half-hourly intervals during every period of sounding. Predicted heights are not sufficiently reliable for the surveyor's purpose except for running and sketch surveys. The subject of chart datum is discussed in Part III of the *Admiralty Tide Tables.* In most parts of the World there is now sufficient information (see Parts I and II of the *Admiralty Tide Tables*) to enable the surveyor to decide on a suitable datum plane for his soundings from a comparison between a short series of observations on a tide pole at and near low water (preferably at springs) and the predicted heights of the corresponding low waters at neighbouring ports. If there is any doubt, it is obviously better to choose a datum that is too low, rather than one that is too high.

Coastline and Topography. (Chapters XI and XII of the *Manual of Hydrographic Surveying.*) For surveying a coastline, the navigator will be dependent as a rule on a sextant and a ten-foot pole which can be made on board. The work is usually plotted by means of short traverses between adjacent sounding marks, supplemented by station-pointer fixes when obtainable. The method of using the sextant and ten-foot pole for traverse work is described in Chapter VI of the *Manual of Hydrographic Surveying.* The whole coastline of the survey should preferably be walked over and fixed in this way, but where the local features prevent this from being done, it may be possible to 'coast' along the shore and obtain a series of station-pointer fixes from a boat. Failing this, the coastline can often be fixed with sufficient accuracy—at least on small scales—by shooting as many recognisable and closely-spaced points on it as possible from a series of boat stations off-shore. This method is not altogether satisfactory since important features—small river entrances, for example—may escape observation. Nevertheless, where heavy surf prevents landing, it is the best that can be done without the help of aerial photographs.

The method of determining heights from sextant angles is described in Chapter VII of the *Manual of Hydrographic Surveying.* If a pocket aneroid is available, this can also be used when circumstances permit.

In the small surveys that are being considered here, it is unlikely that much time can be devoted to mapping topographical features other than the coastline. It is, however, important to

show on the chart all marks that may help the mariner to fix his position and to indicate the general appearance of the land by means of form lines. Prominent marks, such as hill summits and buildings, should be fixed by angles from the triangulation stations and sounding marks. The remaining topographical details can be drawn as a rule with the aid of sketches made from a series of boat stations at anchor.

Sketches of the coast may be supplemented by photographs that are useful for reproduction in the *Sailing Directions*. It is often possible to obtain good land maps of the locality and, if a few points on the map can be recognised and fixed in the survey, the other topographical detail can be re-drawn on the required scale.

Astronomical Observations. Some means must be provided to enable the cartographer to locate any particular survey in its proper relation to the charts of the area in question. If possible, a direct connexion should be made through triangulation between the trigonometrical stations of the survey and some of those used in former surveys of the locality, a procedure which necessitates the recovery of the exact positions of the old stations. When this can be done, the geographical co-ordinates of any point in the new survey can be calculated.

If there are no recognisable and well-defined features on the chart that can be incorporated in the new work, the location of the survey must be settled by means of astronomical observations. The following methods of making these are described in Chapter XIV of the *Manual of Hydrographic Surveying*.

(1) Sights with an astrolabe, an instrument which is easy to use and gives good results.

(2) Latitude by circum-meridian altitudes of north and south stars and longitude by equal altitudes of the Sun or of a star east and west of the meridian, observations being made with a sextant and an artificial horizon. These sights require practice if good results are to be obtained, and a sextant stand should be used, if possible.

(3) Sea-horizon sights from the ship at anchor. This method is less accurate than the others, and it is essential that the ship should be anchored in a position where she can be accurately fixed in relation to the trigonometrical stations of the survey, and that a sea-horizon is available over the greater part of the circle.

Observations for True Bearing. The survey as a whole must be oriented by finding the true bearing of one side of the triangulation. A description of the methods employed in these observations, when either a theodolite or sextant is used, is given in Chapter XIV of the *Manual of Hydrographic Surveying*. True bearings obtained from gyro or magnetic compasses ashore or afloat, are not sufficiently reliable or accurate for normal surveys. They can be accepted only

for running and sketch surveys where there is probably insufficient time for theodolite or sextant observations.

Fair Chart. (See Chapter XVI of the *Manual of Hydrographic Surveying*.) Fair charts of all except running and sketch surveys should be drawn on linen-backed paper. Fair charts of running and sketch surveys may be drawn on tracing paper. Tracing *cloth* is liable to distortion and, if it must be used, the necessary data for re-plotting the principal points should be recorded on the tracing. When suitable plotting instruments are available for graduating the sheet accurately, the borders should be meridians and parallels; otherwise, plain borders drawn parallel and at right-angles to a meridian through one of the trigonometrical points may be used. The standard patterns of border, graduation and scales that should appear on the fair chart are shown on Admiralty Chart Miscellaneous 19. All work should be enclosed *within* the borders or graduation.

Sailing Directions. (See Chapter XVII of the *Manual of Hydrographic Surveying*.) All matter appearing in the *Sailing Directions* relevant to the locality of the survey should be carefully checked, and any new information that may be of use to the mariner should be added. This information should be recorded under the appropriate headings and in the sequence adopted in the *Sailing Directions*.

To Render Triangulation Data. In a regular survey it is necessary to record the triangulation data in great detail. For the small survey, considered in this chapter, it is sufficient to forward the following information:

(1) A tracing showing the coastline, the positions of all trigonometrical stations and the geometrical figures forming the triangulation network.

(2) A brief description of the triangulation telling whether a theodolite or sextant was used to measure the angles, whether the survey was carried out in a regular way and whether it was extended from, or based on, any previous trigonometrical points.

(3) The rectangular co-ordinates of the stations, if calculated.

(4) A list of the triangles with the observed and working values of their angles.

(5) A list of the sides with their lengths in feet.

(6) The geographical co-ordinates of the stations. Only those of the most important stations need be calculated.

(7) A brief description of the method and results of the base measurements.

(8) A description of the marks left at any permanently-marked trigonometrical stations.

(9) The results of any astronomical observations for latitude, longitude and true bearing, with a description of the way in which the observation spot was connected to the triangulation.

SEQUENCE OF OPERATIONS

For the convenience of the navigator who proposes to undertake a small survey, the approximate sequence of operations is briefly summarised thus :

(1) *Preparations before the survey area is reached.*

(*a*) If time permits, apply to the Hydrographic Department at the Admiralty for advice and information and for the loan of any instruments and stores likely to be required.

(*b*) If the chart is on a scale large enough to furnish any help, make tentative selections of a suitable place for base measurement and decide whether this is to be done on shore or from the ship at anchor. Decide whether the base is to be measured by rangefinder or masthead angle. Develop a tentative scheme of triangulation from the information available.

(*c*) Check the chronometer rate.

(*d*) See that sextants and station pointers are in adjustment.

(*e*) Spread and weight the paper for the plotting sheet, and mount field boards.

(*f*) Prepare gear for sounding marks, etc.

(*g*) Make a ten-foot pole and table.

(*h*) Fit a plotting table and chains in the motor boat and make lead lines.

(2) *Tide Pole.*

A tide pole should be set up as soon as the survey area is reached. Readings should be made at low water to establish a suitable datum for the subsequent soundings.

(3) *Sketch Survey.*

When the chart is on a small scale or is known to be inaccurate, make a sketch survey to assist in planning the triangulation. This can usually be done with sufficient accuracy in a few hours by sketches from the ship and two or three boat stations with the aid of a compass and portable rangefinder. Pick out suitable natural objects for inclusion in the triangulation. Decide the scale of the survey.

(4) *Base Measurement.*

Mark the ends of the base. If measuring it on shore, clear the line. Proceed and measure the base.

(5) *Triangulation.*

Mark all the necessary stations first and then observe the angles.

(6) *Calculation of the Triangulation*

(7) *Plotting.*

Plot the main stations and sounding marks. Make tracings and prick off on the field boards.

(8) *Coastline and Sounding.*

It is immaterial which work is done first, but it is usually convenient to fix that part of the coastline off which sounding is to be carried out.

(9) *Topography.*

This can be done simultaneously with the sounding.

(10) *Material for the Sailing Directions.*

This should be collected during the whole progress of the survey.

(11) *Astronomical Observations.*

(12) *Graduation of the Plotting Sheet.*

(13) *Drawing the Fair Chart.*

(14) *Preparation of Triangulation Data to send to the Hydrographer.*

NOTE. *Shadwell Testimonial.* In memory of the late Admiral Sir Charles F. A. Shadwell, a prize consisting of a sextant or other instrument for use in navigation or marine surveying is presented for the most creditable plan of an anchorage or for any other marine survey, accompanied by sailing directions. Details are given in the Appendix to the Navy List.

CHAPTER XXIV

WAVE MOTION AND THE BEHAVIOUR OF SHIPS IN HEAVY WEATHER

The definitions of terms used in connexion with waves, a brief explanation of how waves are formed and their dimensions in different localities, are given in Volume I. In this chapter, the subject of waves is dealt with in greater detail.

Trochoid Waves. When a volume of water is disturbed in such a way that wave motion results, the actual movement of the particles of water is obscured by the forward movement of the wave crests. These, to a casual observer, appear to be carrying the water in the direction of travel, but if a small floating object is carefully watched, it is seen that there is no definite forward

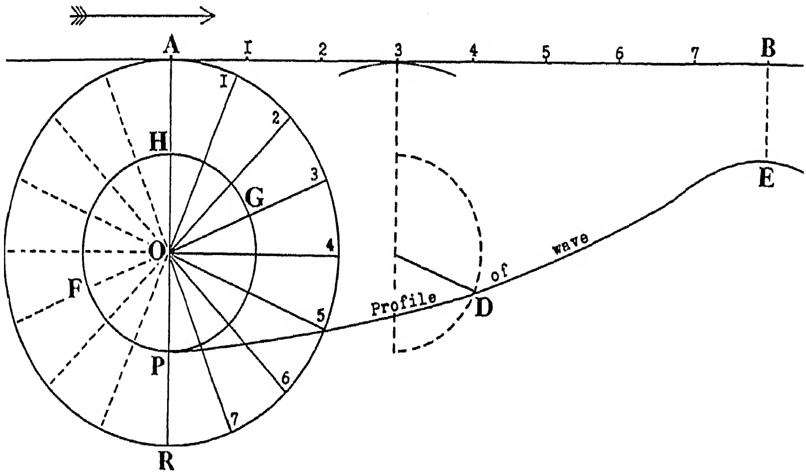


FIGURE 209.

translation. As the wave crest passes, the object simply rises and falls and has a small movement to and fro.

The profile of an ocean wave is a curve that approximates closely to a trochoid, a trochoid being the curve traced in the vertical plane by a fixed point on the spoke of a wheel as the wheel rolls along.

In figure 209, the marking point P is situated on the radius OR of the large circle AR, OP (the distance from the centre) being called the tracing arm. The arrow shows the direction in which the circle rolls and the wave is supposed to be travelling. AB is the base, that is the straight line along which the circle is to roll, the length

AB being equal to AR, which is half the circumference of the circle. As the circle rolls, 3 (say) of the circle reaches 3 of the base. The semicircle FPG is then in the position shown by the dotted semicircle, and the marking point P coincides with the point D, having described part of a trochoid PD. When the circle has completed half a revolution, the marking point P coincides with E, having described the trochoid curve PDE, which is half a wave length. The diameter POH represents the height of the wave. The nearer the marking point is to the axle of the wheel, the flatter is the trochoid.

To secure this trochoidal form and at the same time create an advancing wave crest, each particle of water must describe a circular orbit in a plane that is perpendicular to the wave profile, the time taken to complete one revolution being equal to that taken by the crest to advance one wave length.

At the wave crest the motion of the particles is wholly horizontal, advancing in the same direction as the wave ; at mean level on the front slope it is wholly upwards ; in the trough it is again horizontal but in the opposite direction to the travel of the wave, and at mean level on the back slope it is wholly downwards.

The disturbance set up by wave motion must necessarily extend for some distance below the surface ; but its magnitude decreases very rapidly in accordance with a definite law, the trochoids becoming flatter and flatter as the depth increases. At a depth equal to two wave lengths, it is less than one five-hundredth part of what it is at the surface, so that the water at that depth may be considered undisturbed. The disturbance resulting from the largest ocean waves is inappreciable at even moderate depths.

Waves formed in deep sea are modified as they reach shoal water. When the depth is reduced to less than half the wave length, the orbits of the particles become flattened and grow more elliptical as the water shoals. The period of the wave remains unchanged, but the length and speed are reduced, and the height is increased. Finally, when the depth is not sufficient for the complete formation of the wave profile, the bottom of the wave is retarded by friction of the sea bottom, the top is thrown forward and the wave breaks into surf.

Where shoaling is very steep, the change in the appearance of the waves is very rapid. Probably the most marked example of this is where, during a westerly gale, the long waves from the deep water of the Southern Ocean are suddenly shortened by the edge of the Agulhas Bank. Here the sea is much worse on the edge than either on the bank or in the deep water outside it. This notoriously steep sea is also caused by the action of the prevailing south-west wind against the Agulhas current.

The foregoing explanation of the structure of trochoidal waves with a smooth even profile closely approximates to the actual observations of swell observed in the open sea ; but the fact that

a ship rolling among waves is subjected to a certain amount of drift in the direction of the waves' advance, suggests that ocean waves do not conform entirely to this pattern. This subject, however, requires further investigation before a definite explanation can be given.

Relation Between Length, Period and Velocity of Waves. Certain relations have been established between the length, period and velocity of trochoidal waves, the principal of which are:

Length in feet = Velocity in feet per second × Period in seconds
Period in seconds = Velocity in feet per second ÷ $5\frac{1}{8}$
Period in seconds = $\sqrt{\text{Length in feet} \div 5\frac{1}{8}}$

By the use of these formulæ, if any one of the above elements is measured, the other two can be calculated, but for the convenience of observers the following table is given:

TABLE FOR FINDING THE VELOCITY OF WAVE TRANSMISSION

Wave Length in Deep Sea	Wave Period	Velocity of Transmission of *Individual Waves* in Deep Sea		Velocity of Transmission of the *Groups of Waves* in Deep Sea	
Feet	Seconds	Feet per Second	Knots	Feet per Second	Knots
25	2·2	11·3	6·7	5·7	3·4
50	3·1	16·0	9·5	8·0	4·8
75	3·8	19·6	11·6	9·8	5·8
100	4·4	22·6	13·4	11·3	6·7
150	5·4	27·7	16·4	13·9	8·2
200	6·3	32·0	19·0	16·0	9·5
300	7·7	39·2	23·2	19·6	11·6
400	8·9	45·2	26·8	22·6	13·4
500	9·9	50·6	30·0	25·3	15·0
600	10·9	55·4	32·8	27·7	16·4
700	11·8	59·8	35·4	29·9	17·7
800	12·6	63·8	37·8	31·9	18·9
900	13·3	67·7	40·1	33·9	20·1
1,000	14·1	71·4	42·3	35·7	21·2

Waves at sea are seen to occur generally in series or groups, the region between successive groups consisting of comparatively calm water. If the motion of the first wave of the group is followed, it will be found that this motion dies out, and that the wave next behind takes the lead. If on the other hand the last wave of the group is watched, another wave will be seen to appear behind it. The new waves constantly rise in rear as rapidly and as constantly as those in front die out, so that the general appearance of a group of waves remains unchanged. The group *as a whole* has a definite velocity of propagation, which has been found to be half of that of the individual waves comprising the group, as shown in the table given above.

Connexion Between Ocean Waves and Wind. It is known that waves result from the action of wind on the sea, and that there must be some connexion between the speed and direction of the wind, and the dimensions and periods of the resulting waves. It is not proposed to trace the action of wind in the actual formation of waves, but to deal simply with the effects produced by the wind.

At present comparatively little is known of the precise numerical relations between the dimensions of waves and the force of the wind that produces them. Apart from the scarcity and incompleteness of measured observations, there is the fact that the waves observed from a ship at any given time are not wholly caused by the wind then actually prevailing, but also depend on the previous direction, force and duration of the wind at that particular spot or at a distance.

The following facts of the effect of wind on waves are based on actual observations made by several investigators.

(1) The length of waves is increased when the length of the sheet of water to windward is increased. This explains why the waves in enclosed or narrow seas fail to attain such large dimensions as those attained in the ocean.

(2) The wave-raising power of the wind is much greater when operating on water already in waves than on smooth or nearly smooth water.

The height of a wave increases rapidly with an increase of wind, and soon attains its maximum height for any given wind velocity. It also diminishes more rapidly than any given element of wave motion, when the wind drops. Thus during a squall the height of the waves is seen to increase quite appreciably, and to drop quickly as the squall passes away. The length of waves increases much more slowly, but much more persistently, and with a constant wind may take four days or more to reach its maximum.

Dr. Vaughan Cornish, who has spent many years in the study of waves, gives the following table for calculating the length and height of waves finally produced in the open sea, far from sheltering land, by the action of winds of different Beaufort forces 6 to 12 :

Wind			Waves			
Seaman's Description of Wind	Force by Beaufort Scale	Velocity in Knots	Period in Seconds	Length in Feet	Height in Feet	Length ÷ Height
Strong breeze	6	24	7·2	262	17·5	15·0
Strong wind	7	30	8·9	404	21·7	18·6
Fresh gale	8	37	10·6	575	25·9	22·2
Strong gale	9	44	12·6	813	30·8	26·4
Whole gale	10	52	15·2	1,180	37·1	31·8
Storm	11	60	18·3	1,720	44·8	38·4
Hurricane	12	Above 65	22·0	2,489	—	—

The figures are for average waves. When their speed is equal to that of the wind, there is not the great variation in height that occurs when the wind has a velocity less than that of the swell left by a preceding storm.

Swell. When wave motion is once set up in the ocean, it continues for a considerable time after the originating cause has ceased or passed away, persisting until the energy imparted to the wave is absorbed by the effect of gravity, friction, etc. Groups of waves travelling away beyond the limits of the wind that raised them, thus retain their direction unchanged so long as they travel in deep water. The height will rapidly diminish, but the length and velocity will remain the same, and they assume the appearance of long low regular undulations of the water known as swell. They may ultimately appear as *rollers* or *breakers* on shores far distant from their place of origin.

The swell often observed at sea, even during calm weather, frequently has a length far in excess of the waves observed during a storm. When the storm waves travel away from their source of origin, there is no reason why they should increase in length, and it can only be supposed that these waves of extreme length are actually present during the storm, but are masked by the dominant and steeper storm waves.

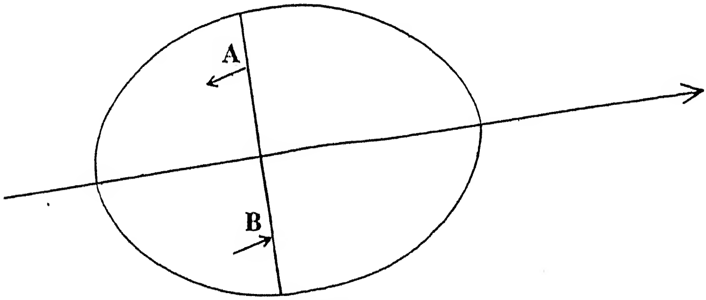


FIGURE 210.

Wave Development in Cyclonic Gales. In northern latitudes the greatest development of waves usually takes place in the right-hand rear quadrant of a cyclone or depression.

In the position A shown in figure 210 the direction of the wind is opposite to that of the travel of the depression; consequently the depression is continually receding from the waves that its winds create, and no prolonged development of waves in this quadrant takes place even though the winds are strong, if the depression is moving east.

On the other hand, at B, where the wave-direction approximates to that of the advance of the storm, the waves themselves remain under the influence of the wind for a much longer period, and will

obviously attain much greater dimensions with a given wind force than at A. Moreover the strongest winds in a cyclone are usually developed in rear of the trough, and thus add still further to the waves of this area.

It is usually found, therefore, in reports of abnormal seas, that they have occurred in the right-hand rear quadrant of a cyclone in the northern hemisphere, or in the left-hand rear quadrant in the southern hemisphere.

From this reasoning it follows that another important factor in wave development is the rate of progression of the storm. It is clear that waves which are slower than the travelling storm are left behind all the time, whereas waves that are faster run ahead of it, beyond the influence of the winds that raised them and thus become *swell*.

When the wave velocity is equal to that of the travelling storm, the waves are continuously subject to the action of the wind, and these conditions are, therefore, favourable for exceptional development of waves.

Swell Originating in a Tropical Revolving Storm. For the reasons just explained, the action of the violent sustained winds in the right-hand rear quadrant of a cyclone in the northern hemisphere (or the left-hand rear quadrant in the southern hemisphere) blowing mainly in the direction of the line of advance of the storm, develops large waves that pass on beyond the limits of the storm as swell. This swell is carried to great distances, and as it travels at a much greater velocity than the cyclone, an observer may be forewarned of the existence of the cyclone by as much as two or three days. The swell that comes approximately from the direction of the storm centre thus frequently gives not only the *first warning* of a tropical revolving storm but also its bearing, before the indications of wind, barometer or cloud are sufficiently definite to act upon.

Storm Swells of Temperate Latitudes. Much can be learned from observations of the swells set up by ordinary gales in the temperate latitudes. Although a heavy swell does not necessarily indicate a coming storm, the seaman, by noting the changes in its direction and intensity as the ship continues her passage, can obtain a very fair idea of what weather changes are taking place.

Coastal Swell. Many of the islands and shores of the Atlantic Ocean are subject to periodic or permanent swells, some of which assume the nature of *rollers*. The best known of these are perhaps the rollers of Ascension and St. Helena.

The Resaca of Rio de Janeiro. The great storm waves, locally called *Resacas*, which occasionally visit the bay of Rio de Janeiro and the adjacent coasts, afford another interesting example of a swell that travels for a long distance, and its conversion from long smooth undulations of the water into leaping and destructive waves.

Solitary Waves. Huge solitary waves (commonly, but erroneously, called 'tidal waves') are occasionally met with at sea, often in otherwise perfectly calm water, and have caused both loss of life and damage to many ships. There is a strong probability that such waves are due to submarine seismic disturbances, and are similar to the 'earthquake' waves that do an enormous amount of damage when breaking on the coast.

The Dimensions of Waves. These vary in different localities, and with different velocities and directions of the wind. The largest wave recorded is one 2,600 feet in length and 23 seconds in period. The longest waves are encountered in the South Pacific, where their lengths vary from 600 to 1,000 feet, and their periods from 11 to 14 seconds. Waves of from 500 to 600 feet long are occasionally encountered in the Atlantic, but more commonly the lengths are from 160 to 320 feet and the periods from 6 to 8 seconds. The relation between the length of a wave and the velocity and direction of the wind is not yet fully understood.

THE EFFECT OF WAVE MOTION ON A SHIP'S STABILITY

The various effects and their periods can be defined as follows:

The period of encounter is the time interval in seconds between the passage of two successive crests past the ship or an observer.

Rolling is the athwartship oscillation.

Pitching is the fore-and-aft oscillation.

The period of roll or pitch is the time interval which elapses while the ship completes a double oscillation, that is, while she rolls from starboard to port and back again or vice versa (twice through the vertical position).

Heaving is the vertical motion imparted by waves.

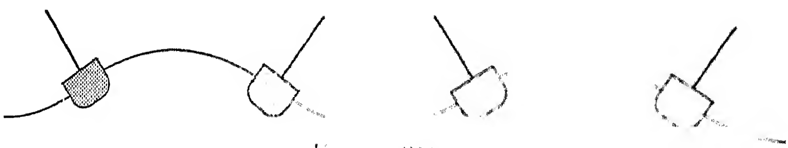


FIGURE 211.

Rolling. The angle of roll is the inclination of the ship from the vertical, measured in the plane at right angles to the fore-and-aft line. The rate of roll is the rate at which this angle is changing at any given instant.

The way in which a ship rolls in a seaway has a great bearing on her safety and behaviour. Every ship has her own period of rolling in still water, and it is important that this period should be known in order to predict her probable behaviour at sea.

When a ship's still-water period is small compared with the period of encounter, she will lie with her deck parallel to the wave slope as shown in figure 211. She will tend to keep her masts

normal to the effective wave slopes and will ship little water, but her motion will be rapid and jerky, and in stormy weather she may suffer violent and heavy rolling, with excessive straining of her structure.

When a ship's still-water period is large compared with the period of encounter, she is likely to roll slowly and move independently of the waves, inclining only through moderate angles from the upright as shown in figure 212. The waves will thus break over or against her.

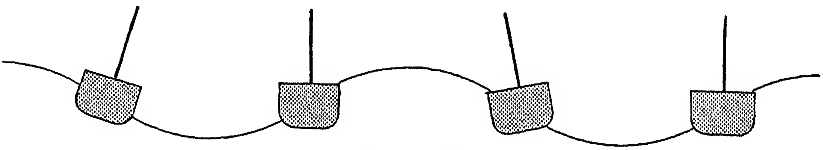


FIGURE 212.

A critical situation occurs when the ship's still-water period and the period of encounter are nearly the same, because the ship will then roll heavily, as shown in figure 213, and tend to capsize. This she would do, were it not that :

(1) her resistance to rolling increases with the roll.
(2) her still-water period increases with the roll.

When a ship is rolling excessively on account of this synchronism, the effective period of encounter relative to the ship's period will be altered by changes of course and speed ; but whenever there is

FIGURE 213.

synchronism, a ship of long still-water period is better situated than one of short period because the waves keeping time are then longer and less steep.

Were all sea waves of the same length, period, and height, it would be quite possible to design ships, not liable to changes of weight distribution, with a still-water period that would give great steadiness at sea. This would be more difficult for a merchant ship because of the effect on her behaviour of the nature and stowage of her varying cargoes.

Sea waves, however, vary greatly in their dimensions at different times and places as already stated, and records of these variations are urgently required. The methods of recording dimensions are described later in this chapter. The analysis of this recorded

information is expected to give reliable information to the ship designer.

Slight rolling is an advantage because it prevents the shipping of heavy seas and reduces the force of the waves.

NOTE. A Japanese professor (K. Suyehiro) has recently demonstrated from model experiments in a tank, that rolling induces drift that reaches a maximum when synchronism is present. He states that with a wave slope of $2\frac{1}{3}°$, synchronous rolling, of itself, will produce a drifting force corresponding to that of a moderate breeze, and that with storm waves having a maximum slope of 9° or so, the drifting force would be enormous. Whether these experimental conclusions can be applied to the circumstances affecting a ship in a gale or heavy sea, has yet to be proved.

Pitching. The combination of a ship's pitching period and period of encounter is similar to the effects described for rolling.

MEASURING WAVES

Onboard a moving ship it is very difficult to measure accurate dimensions. In a confused sea it is almost impossible to attempt any measurements of the sea or swell. Only in seas caused by a well-defined wave motion of some uniformity is it possible to take observations.

Height. The usual method of finding the height of a wave is to observe, from a known height of eye, when the crest is in line with the horizon and the ship is in the trough. The height of eye horizontal through the trough is then the height of the wave.

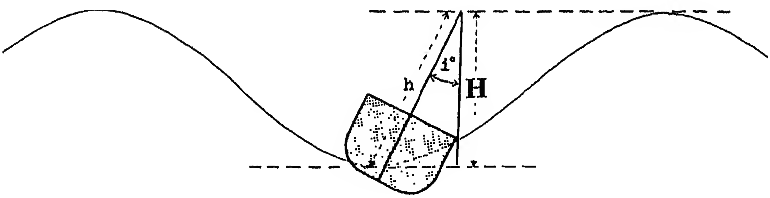


FIGURE 214.

When the ship is rolling, the apparent height will be greater than the true height, as shown in figure 214.

h=the height of eye
i=the angle of roll
H=the true height of the wave=$h \cos i$

H can be found by observing the angle of roll at the time of observation, or by continuing observations and taking the minimum height of eye required, which will be when the ship is upright.

When the ship pitches, the waterlevel of the ship is changed considerably, as shown in figure 215, and this should be allowed for. Under these circumstances there is less chance of error if the observer is equidistant from the bow and stern.

If the length of the ship is such that she cannot lie completely in the trough, an estimate of the draught should also be made.

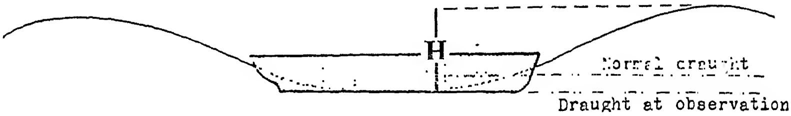


FIGURE 215.

Period. To measure the period, observe the rise of a piece of spent foam, and when it reaches the crest of a wave start a stop watch; then observe the descent of the foam to the trough, and stop the watch when the foam reaches the top of the next crest.

Length.

(1) If the ship is the same length, as shown in figure 216, or is longer than the apparent wave length, find the length of the apparent wave by comparing with the length along the fore-and-aft line.

Then the length of the wave is equal to the apparent wave length multiplied by cos i, where i is the angle of inclination of the ship to the waves.

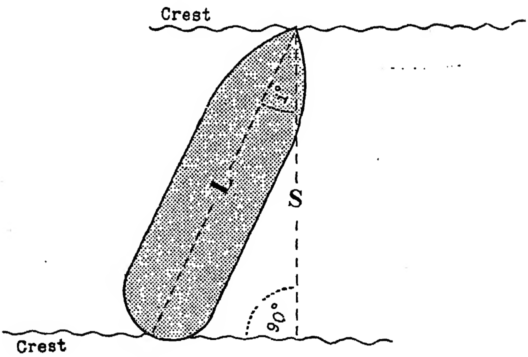


FIGURE 216.

(2) If the ship is not as long as the apparent wave length, an easy method of measurement is to tow astern a buoy or other mark, paying out sufficient line so that when a wave crest passes the stern, the buoy is on the crest of the next wave. The length of the wave is then found as explained in the preceding paragraph.

Velocity. If the length and period of a wave are known, the velocity can be found from the formula:

$$V = \frac{0{\cdot}6L}{p}$$

—where: V=wave velocity in knots
L=length in feet
p=period in seconds

Recording Information (K.R. & A.I. Article 1145). H.M. ships are supplied with Form S 561, *Rolls by the Vertical Battens*, in which is kept a record of rolling, pitching, and the dimensions of waves.

Full instructions for observing, recording, and forwarding information are given in the book.

STRUCTURAL STRAINS

Sagging. If the angle of incidence to the waves is such that the ship has the same length as the apparent length of the wave, as shown in figure 217, then the bow and stern are lifted on a crest and the centre droops. This is called *sagging*.

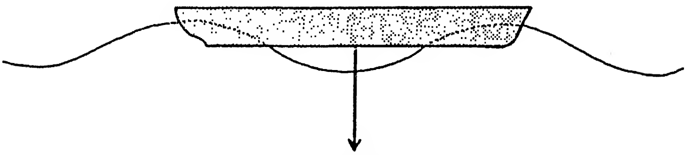


FIGURE 217.

Hogging. If the bow and stern are in a trough and the centre is lifted on a crest, the bow and stern will droop, as shown in figure 218. This is called *hogging*.

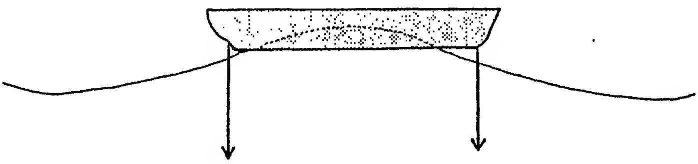


FIGURE 218.

Panting. A ship's plating is subject to varying water pressure caused by the force of the waves and the change in draught, and is worked in and out. This is called *panting*.

Pounding or Bumping. Excessive panting is sometimes caused by the stem's or bow's falling into the water when the ship pitches. This is called *pounding* or *bumping*.

When a ship rolls or pitches, the weights in her tend to continue their motion after she has come to rest and begun to move in the opposite direction.

All these strains tend to buckle the plates and frames, snap the masts, and corrugate or break the hull.

Pooping. If a ship falls into a trough and does not rise with the wave, or if she is falling as the wave is rising, the wave may break over her stern or *poop* her, causing considerable damage. This is called *pooping*.

Broaching To. A ship tends to turn into the wave front, an action that is corrected by the rudder. When a ship is in a following sea, the fast-moving crest reduces the stream of water past the rudder, steering becomes difficult and, if the rudder has little effect, the ship will *broach to*.

Racing Propellers. If the propellers are partly out of the water, they will race and cause severe strain on the engines.

Heaving. The momentum resulting from the vertical motion of the waves causes the apparent weight of the ship to be less when she is on the crest of a wave, and more when in a trough.

The righting moment and draught are therefore altered and on the crest of a wave are both less. The stability is thus reduced, and the ship is more liable to capsize.

HANDLING SHIPS IN HEAVY WEATHER

General Considerations.

(1) In a beam sea an alteration of speed does not affect rolling.

(2) In a sea abaft the beam an increase of ship's speed :

(*a*) increases the period of encounter and vice versa.
(*b*) decreases the force of the waves and vice versa.

(3) In a sea before the beam an increase of ship's speed :

(*a*) decreases the period of encounter and vice versa.
(*b*) increases the force of the waves and vice versa.

(4) An alteration of course towards the sea decreases the period of encounter and vice versa.
(5) A slight roll or pitch is usually an advantage.
(6) Racing screws and bumping usually occur at low speed with the sea before the beam. These actions can be reduced by increasing speed or altering course to bring the sea broader on the beam.
(7) The force of the waves is greatest at the crests and least at the troughs.

(8) In a following sea the rudder feels :

(*a*) the fore-and-aft orbital velocity when the crest passes it.
(*b*) the slip stream minus the fore-and-aft orbital velocity when the trough passes it.

(9) When the ship is steaming at high speed with the sea abaft the beam, the stern wave may combine with the sea wave and cause pooping.

It is best on all occasions, if possible, to have the period of encounter more than the still-water rolling period of the ship.

HANDLING WARSHIPS IN HEAVY WEATHER

The handling of warships in heavy weather is dependent on so many factors that it is not possible to lay down any definite rules.

A knowledge of the behaviour of a particular ship in various circumstances of sea and swell must be obtained from past records and *is of the utmost importance*. For this reason as much information as possible concerning her behaviour in various circumstances should be noted in the *Navigational Data Book*.

Battleships. Only in very heavy gales will battleships encounter waves possessing a period of encounter longer than the period of the ship.

With the sea on the bow or abeam they roll and pitch slightly. Hence they will ride easily. When the sea is on the quarter and the period of encounter is nearly equal to the rolling period, synchronism may occur. Synchronism is most likely to occur when the ship proceeds at high speed with the sea on the quarter.

Cruisers and Ships of Medium Period. In moderate gales these ships have a rolling period near the period of encounter of the waves, and therefore roll heavily when the sea is near the beam.

With the sea on the quarter it is necessary to increase speed to break synchronism, but when this is done the stern wave and sea wave may combine and cause pooping.

With the sea on the bow it is unlikely that synchronism for rolling will occur, but there will be heavy pitching and breaking seas will be shipped.

Destroyers and Ships of Small Period. In the open sea synchronism does not occur for rolling except at high speeds with the sea before the beam, or at low speeds with the sea ahead. With the sea on the quarter it is unlikely that synchronism will occur.

In narrow waters the periods of rolling and encounter are most likely to coincide when the sea is ahead or before the beam.

Notes on Course and Speed in Heavy Weather.

Course. The course steered should be :

(1) such that hogging and sagging do not occur.
(2) such that synchronism does not occur.
(3) as broad as possible to the waves so as to reduce their force.

Speed. The ship's speed should be :

(1) sufficient to give steerage way.
(2) such that synchronism does not occur.
(3) such that the force of the waves is minimised. (It must be remembered that this does not, necessarily, mean slow speed.)

NOTES ON THE BEHAVIOUR OF SHIPS OF THE MERCHANT NAVY IN HEAVY WEATHER

It cannot be stressed too strongly that the handling of all ships in heavy weather depends upon circumstances and a knowledge of the behaviour of the particular ship.

The following remarks are based on general experience, and they must not be assumed to apply to any particular ship.

During Gales. The best position for a deeply laden cargo vessel is head to sea. The engines should be kept running ahead as slowly as possible, and they should be stopped on the approach of a heavy breaking wave. The safety of the ship depends on giving her as little headway as possible.

A critical situation occurs if, for any reason, it becomes necessary for a ship that has been running before a gale to turn head to sea, because the cargo is liable to shift. It is thus always advisable to consider whether or not the ship can be kept running before the gale.

During Very Heavy Gales. It may be necessary to adopt one of the following procedures :

(1) *Heave to with the ship head to sea.* Watch for a smooth patch, slow down and finally stop the engines. Pour some heavy oil over the bows and float a drogue or sea anchor. If necessary, use the engines to bring the ship head to wind.

(2) *Heave to with the wind and sea on the quarter.* This procedure is most suitable for ships with a high forecastle. Stop the engines and allow the ship to take up her own position, which will be with the sea abaft the beam. Pour oil over the side to windward.

It will be necessary to batten down securely as it is probable that heavy seas will be shipped.

(3) *Steam with the stern to wind and sea and the engines dead slow ahead.*

(4) *In shallow water let go cable.* Unshackle both anchors and veer both cables up to eight shackles. This procedure will keep the ship head to wind and she will drift slowly to leeward.

CHANGE OF DRAUGHT ON PASSING FROM SEA TO RIVER WATER

The difference between the specific gravity of the sea and of river water is of considerable importance in navigation, particularly when a ship has to proceed to a dock that opens into a river, because the volume of displacement of the ship varies inversely as the specific gravity of the water in which she floats. The weight of a cubic foot of river water may be taken as 63 lbs. and of sea water as 64 lbs.

If W is the displacement tonnage, the volume of water displaced, in cubic feet, is :

$$\frac{W \times 2240}{63} \text{ in river water}$$

$$\frac{W \times 2240}{64} \text{ in sea water}$$

Therefore, if A is the waterplane area in square feet, the increase of draught is :

$$\frac{2240W}{A}\left[\frac{1}{63}-\frac{1}{64}\right] \times 12 \text{ inches,}$$

$$=\frac{20W}{A} \text{ inches.}$$

Let T be the number of tons required to sink the ship 1 inch when floating in sea water (tons per inch immersion). Then :

$$T \times 2240 = \frac{64A}{12}$$

i.e.

$$A = 420T$$

Therefore the increase of draught is $\frac{20W}{3 \times 420T}$ or $\frac{W}{63T}$ inches.

If, for example, a ship of 44,000 tons displacement and 120 tons per inch immersion is proceeding from sea to the river Clyde, her increase of draught on arrival at Glasgow will be :

$$\frac{44,000}{63 \times 120}$$

$$=5\tfrac{3}{4} \text{ inches}$$

CHAPTER XXV

TAUT-WIRE MEASURING GEAR

General. Taut-wire measuring gear provides a method of measuring the distance run over the sea-bed by means of a wire that is anchored to the sea-bottom and paid out astern of the ship.

It was first used in cable ships to find a fault in the cable when the distance of the fault from the end of the cable had been found electrically. Since large distances were involved, two machines were installed alongside each other, the second being started about 2 miles before the first had run its wire off.

The gear is of great value on all occasions when it is required to measure, accurately, the distance run over the sea-bed :

(1) when a ship is proceeding to a position for laying mines out of sight of land, and it is not possible to obtain astronomical observations.

NOTE. The sinker would be streamed on leaving harbour and the departure taken from a reliable fix.

(2) for fixing the position of an object out of sight of land, as explained later in the chapter.

(3) for assisting to check the *Speed-Revolution Table* for the state of the ship's bottom.

(4) for measuring the base in certain hydrographical surveys.

COMPONENTS (shown in figure 219)

Rollers. One set of four rollers (two vertical and two horizontal) is fitted for guiding the wire on to the counter wheel; another set of three rollers (two vertical and one horizontal) for guiding the wire over the stern.

After-Lead Wheel. This is a roller wheel that is free to revolve, and it guides the wire through the after rollers.

Tensionmeter. The tensionmeter or dynamometer consists of a spring-loaded roller wheel by means of which the tension in the wire can be measured. A scale shows the amount of tension in lbs.

Cyclometer Wheel. The cyclometer or counter wheel registers the amount of wire run off. It consists of a wheel two yards in circumference actuating a counter gear which registers to 1/1000th of a mile.

Forward-Lead Wheel. This is similar to the after-lead wheel and is provided for guiding the wire from the counter wheel on to the dynamometer wheel.

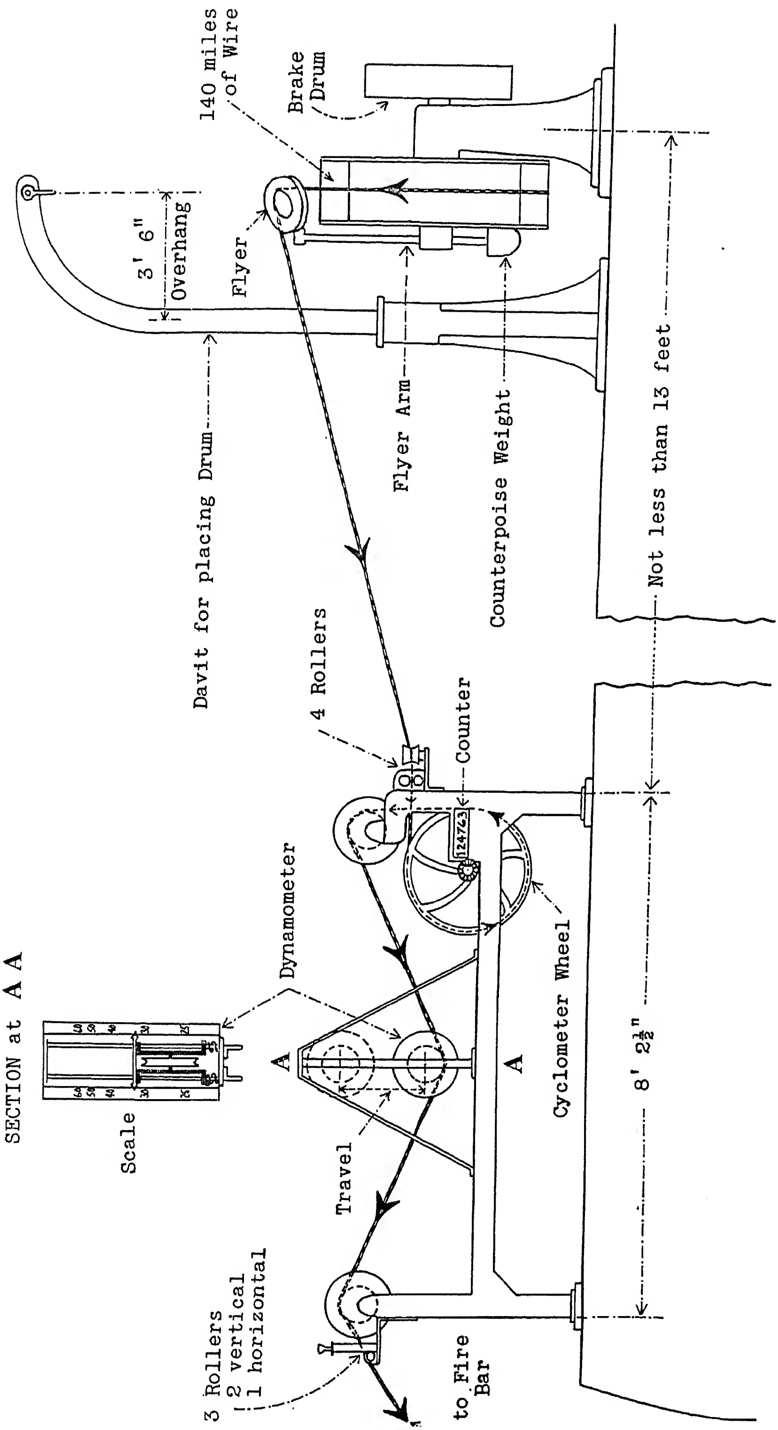

140 miles
of Wire
Brake
Drum
3' 6"
Overhang
Flyer
Davit for placing Drum
Flyer Arm
Counterpoise Weight
Not less than 13 feet
4 Rollers
Counter
124763
SECTION at A A
Dynamometer
Scale
A
A
Travel
Cyclometer Wheel
8' 2½"
3 Rollers
2 vertical
1 horizontal
to Fire
Bar

FIGURE 219.

Drum. The drum carries 140 miles of piano wire. It is fixed on a standard and does not revolve.

Drum Standard. The drum standard carries the drum. It is also fitted with a spindle, carrying at its fore end a hand brake that passes through the centre of the drum, the unwinding mechanism being secured to the after end of this spindle. The hand brake is operated by two butterfly nuts and is used to adjust the tension on the wire.

Unwinding Mechanism. This consists of a revolving arm, called the *flyer arm*, secured to the after end of the spindle carried on the drum standard. The flyer arm is fitted with a counterpoise weight at one end, and a special wheel called the *flyer* at the other end.

Davit. A small davit is fitted in a suitable position for lifting the drum on the standard by means of a Weston purchase.

To Prepare for Use. A full drum of wire is placed in position on the drum standard, and the flyer arm is connected to the brake spindle, as shown in figure 219.

The wire is led over the flyer, between the rollers, over and three-quarters round the large counter wheel, up over the first lead wheel, under the dynamometer wheel, over the after-lead wheel, and between the stern rollers.

The end of the wire is secured to a small length of rope that is, in turn, secured to the sinker.

Special grapnels, with multi-pronged hinged flukes, are supplied as sinkers. If these are not available, a suitable sinker can be made onboard by bolting together, in the form of a cross, two half fire bars or other scrap metal.

Any slack should be hove back until the rope is at the stern rollers. Then the brake should be set up, and the counter gear re-set to zero.

The wire from the flyer to the counter mechanism traces out a cone, and this space should be roped off to avoid accidents.

The gear is now ready for running.

NOTE. When reeving the wire, take care that:

(1) the wire from the large counter wheel to the small lead wheel is brought up on the port side at the part already led on to the large counter wheel, otherwise the ends of the splices in the wire will foul.

(2) the wire when being passed under the dynamometer wheel is led above the split pins connecting the two sides of the dynamometer.

(3) when the wire is rove a slight friction is kept on the brake, otherwise it may kink.

(4) the flyer is always left in such a position that the oil hole in the lubricating ring is at the top when the gear is idle, and that the ring is itself temporarily filled with waste to prevent the accumulation of grit and cinders.

To Run the Gear. Remove the waste from the oil ring and lubricate freely, except for the brake straps which require only a drop of oil.

Lower the sinker by hand to the water, and take off the brake a sufficient amount to allow the wire to be paid out. Careful control of the brake is most necessary because if the brake is taken right off, the flyer arm will overrun the wire and throw it off. Should this occur, the only thing to do is to cut the wire and start again with another sinker.

The sinker does not reach the bottom for some time, and on reaching the bottom it is dragged for some distance before holding. Consequently it is advisable to start the gear at least two miles before arriving at the point of departure.

On arrival at the point of departure, the ship is fixed and a reading taken simultaneously.

The counter gear will now register the distance run to 1/1000th of a mile.

The tension should be increased slightly after two miles by means of the brake, and after five miles raised to about 32 lb.

As the tension in the wire increases, the dynamometer wheel travels upwards in the guides, the amount of the tension in lbs. being marked on the guides.

It has been found that about 32 lbs. is the right tension for depths up to 1000 fathoms.

Procedure if the Wire Should Part During a Run. Note the reading and the exact time. Bend on a fresh sinker and restart.

Run for about 2 miles to allow the new sinker to reach the bottom and hold. Take a reading and observe the time again. The run between the two times must be estimated by the most accurate method available.

To Cut the Wire. When it is desired to cut the wire at the end of a run, choose a point where the wire is running horizontally.

Hold a piece of iron underneath the wire and tap quickly with a cold chisel, at the same time putting on the brake.

Precautions.

(1) Always keep the brake covers on: a heavy shower of rain on the brakes and straps will cause the flyer arm to be released suddenly and the wire will be thrown off.

(2) Should it be required to run the whole length of wire off the drum, the end that is fixed to the drum by a wooden peg should be examined previously to ensure that it is not jammed, otherwise the flyer may be bent.

(3) Before a run, while the sinker is hanging over the stern, it is advisable to lock one of the lead wheels, by means of a stick placed through the spokes, to prevent the machine's starting itself through vibration.

(4) Although the outer layers of the wire are well greased, it will be found advisable to run off the first 5 miles of a new drum before use, because it is liable to part.

(5) A difficulty that is sometimes experienced is that the wire parts soon after the sinker reaches the bottom. Unless an incorrect procedure has been followed, this is nearly always caused by rusty wire. Wire that has once been used should never be replaced on a drum because it is almost certain to rust.

(6) No other ship should pass within half a mile of the stern of a ship running taut-wire measuring gear.

(7) It is a good plan to keep the fire-main running on deck while the gear is in use in order to carry away lumps of grease and spots of oil.

Care and Maintenance. In harbour the flyer arm should always be kept unshipped.

Spare drums must be kept well covered and chocked up clear of the deck in order to prevent rusting.

All moving parts should be kept clean and well lubricated.

All gear must be kept covered and protected from the weather as much as possible.

Remarks.

(1) Small alterations of course will not affect the running of the wire. Large alterations of course, however, will affect the running, and for this reason it will be more accurate to plot the turn from turning data. Note the reading of the counter gear before starting to turn and again after the ship has run a short distance, say 1 mile, on the new course.

(2) Remember that the gear only measures the distance run over the sea-bed. The ship's position at the end of a run will thus be influenced by any cross current or tidal stream.

A fix at the end of a run and a knowledge of the currents and tidal streams experienced are, therefore, most important when the gear has been used and the ship's track and positions have to be verified.

(3) Taut wire gives a position line that is the arc of a circle, drawn from the point of departure, with radius the distance run by the ship.

Example 1. To Fix a Turning Point.

The ship, shown in figure 220, runs taut wire from a fix at A, steering 135° for 40 miles to B'. Arriving at B, she alters course to 225°. On making land at C, she obtains a fix. The distance run by taut wire from B to C is 30 miles, the first mile after the turn at B having been estimated by engine revolutions.

It is required to know the accurate position of the point B.

From figure 220 it is clear that the intersection of arcs drawn with radius 40 miles from A, and 30 miles from C, gives the position of the point B.

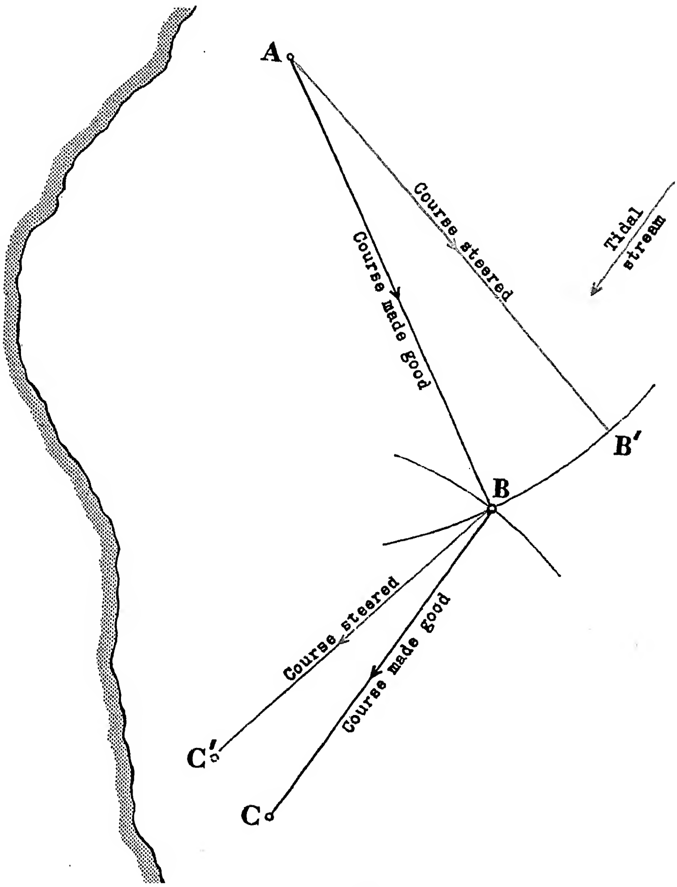


FIGURE 220.

Example 2. To Fix the Position of a Buoy Out of Sight of Land.

It is assumed that the position of the buoy is known approximately, and that if the ship steers for its estimated position she will pass close enough to sight it.

The ship, shown in figure 221, runs taut wire from a fix at A and steers for the buoy.

On arrival she reads the taut-wire gear when the buoy is abeam.

She then runs on two miles past the buoy and returns on a course making a large angle with her outward course.

She then reads the taut-wire gear when the buoy is again abeam, on the return journey, and again on making the land and obtaining a fix at *C*.

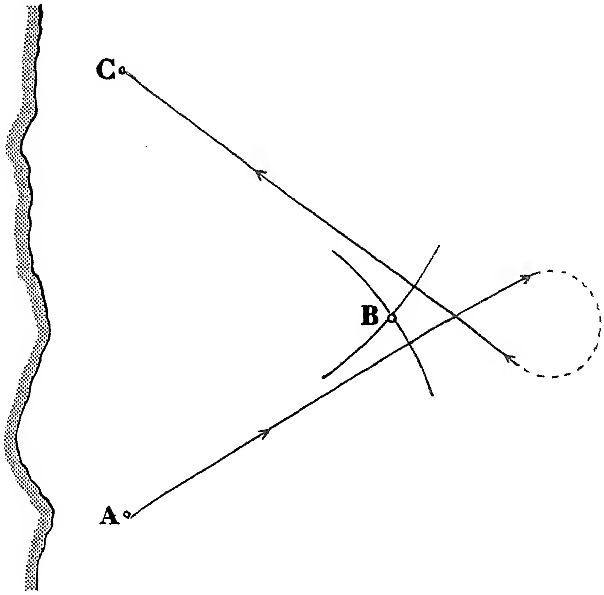


FIGURE 221.

The position of the buoy *B* is then obtained by the intersection of the arcs from *A* and *C*.

CHAPTER XXVI

SOUND RANGING

Sound ranging is a method of locating an under-water explosion such as that of a mine, torpedo, or depth charge, by means of the pressure wave that proceeds from it.

The sound wave travels outwards in all directions from the point of origin approximately at uniform speed, until it meets the surface or dies away. Where the wave meets the surface, or the bottom, it is reflected. Shoals that come between the explosion and the receiver reduce the strength of the waves that pass them, particularly if they are steep. When, however, a shoal does not rise too close to the surface, and the explosion and receiver are reasonably far removed from it, the wave will pass over the shoal and reach the receiver. The path of the sound wave from the point of origin to the receiver is not necessarily a great circle and depends on the sound velocity gradients that are present. Sometimes the waves reaching the receiver arrive after one or more reflections from the bottom and surface. In deep water, reflections may possibly occur at intermediate depths where sudden changes in the water conditions cause reflecting layers. The maximum reliable range in the English Channel is about 60 miles with a large charge. In similar circumstances a range of about 40 miles is obtained with a 9 oz. charge of high explosive. In deep water ranges of 200 miles have been obtained with large charges. Since the velocity of the sound waves is independent of the size of the charge, the amount of explosive used can be varied according to the range and sensitivity of the receiving apparatus. The charge should be exploded at a depth of not less than 20 feet, but 40 to 100 feet below the surface is usually a good depth for sound ranging. The best depth depends on local factors, which are difficult to determine, such as temperature gradients, reflecting layers and depth. The results are not affected by rough weather or fog.

The Velocity of Sound in Salt Water. An accurate knowledge of the velocity of sound is essential, because it is the basis of all methods of sound ranging.

The velocity of sound in sea water depends on the temperature, salinity and pressure. Salinity is expressed as the weight of solid matter contained in 1,000 parts by weight of sea water. In the seas round the British Isles it can be taken as 35 and, at a temperature of 53° F., the velocity is approximately 4,900 feet per

second. The salinity varies in different parts of the world from about 31, in such places as the Bering Sea and Gulf of Pechili, to about 40 in the Gulf of Suez and off the coasts of Palestine. This increase corresponds to a variation in the velocity of sound of about 34 feet per second. Pressure itself varies with the temperature, salinity and also, to a certain extent, with the latitude on account of the variations of gravity.

The work of calculating the velocity of sound from formulæ is extremely laborious. In practice, all that is required can be found in the *Tables of the Velocity of Sound in Pure Water and Sea Water for use in Echo-Sounding and Sound Ranging* (H.D.) 282, published by the Hydrographic Department. Tables with the necessary corrections are given for 23 large areas, as for instance, the Gulf Stream and the Mediterranean.

There are two types of sound ranging.

(1) *Submarine Sound Ranging* whereby a submarine sound-ranging station locates an explosion without the aid of W/T.

(2) *Radio Sound Ranging* whereby a submarine sound-ranging station locates a ship, or where one ship locates another, using W/T in addition to an explosion.

Submarine Sound Ranging. A submarine sound-ranging station on shore is equipped with three or four hydrophones laid out some miles apart on the sea-bed, in places, the positions of which are found with great accuracy. All the hydrophones are connected by electric cables to the station, where the exact time of arrival of the sound wave at each hydrophone is observed accurately to 1/1,000th of a second by a photographically recording galvanometer. From this record is taken the time interval between the arrival of the sound wave at any two hydrophones. This time interval, when multiplied by the velocity of sound in water, gives the difference in the distance of the explosion from the two hydrophones. The position of the explosion is fixed by finding the intersection of the hyperbolas obtained from the records of pairs of hydrophones, a hyperbola being the locus of a point which moves so that the difference between its distances from two fixed points, called the foci, is constant. The fourth hydrophone record provides an independent check, because the three hyperbolas obtained from three hydrophones are not independent, and will cut in a point if the arithmetic and plotting have been correctly done, even though errors have been made in the velocity used or in the surveyed position of any of the hydrophones.

In practice a certain amount of the necessary calculations can be made beforehand and kept available on graphs or in tables at each station, and a quick method of drawing the relevant positions of the hyperbolas is used. For rapid navigational work this reduces the time necessary to obtain a fix from several hours to about

ten minutes, and the errors introduced do not amount to more than five cables. Greater accuracy is attained when full calculations are made. When more than one station is available, the results can be compared, and so provide a still further check on the accuracy of the work, both in submarine sound ranging and also in radio sound ranging.

Radio Sound Ranging. In this method a W/T signal is sent out at the same time as the explosion takes place. Since the W/T signal travels at the rate of 186,000 miles per second, the time of its reception is practically the same as its time of origin. The interval between the arrival of the W/T signal and of the sound in a hydrophone, when multiplied by the velocity of sound in water, is thus a measure of the range of the explosion from the hydrophone.

(*a*) *Sound-ranging station locating a ship using W/T.* Arrangements have to be made in the ship for the W/T signal to be sent simultaneously with the firing of the charge, and in the shore station for the time of receipt of the W/T signal to be accurately recorded. From the time intervals between the arrival of the W/T signal and the sound in each hydrophone the distance of the ship from each hydrophone is calculated. Then by drawing a circle round each hydrophone with a radius equal to the range obtained, a fix is obtained at the intersection of these circles. Thus a fix can be obtained by ranges from two hydrophones, but in practice at least one other hydrophone is used to give an independent check on the other two. Again, as in submarine sound ranging, some of the calculations are already available for reference at the station, and the method of plotting the fix is thus also simplified. There is no departure from the principle, however, that the position line is the arc of a circle drawn about the hydrophone as centre with radius equal to the range obtained. The fix is where two or more of these circles cut. In practice, when the most rapid means of calculation and plotting are used, a vessel that has asked for a fix can be given her position, correct to half a mile, in ten minutes from the time her request is received. For survey work where accuracy is of great importance and speed of no account, positions can be found with a very high degree of accuracy. Apart from ordinary use in surveying, it has been suggested that the radio method is particularly suitable for checking the positions of light-vessels and buoys out of sight of land and fixing ships uncertain of their position when making a landfall or in foggy weather.

One of the main sources of error is the timing of the W/T signal. When great accuracy is required the charge firing-key is closed simultaneously with the W/T key, and the record at the station is automatic, but ordinarily the ship's operator presses the key when he feels the shock of the explosion. Thus there is a lag of about 0·3 of a second that has to be allowed for.

(*b*) *One ship locating another.* The sending ship fires a charge and makes a simultaneous W/T signal, as previously described.

The receiving ship, which has to be fitted with a hydrophone and D/F set, takes the time interval between the receipt of the W/T signal and the arrival of the sound wave in her hydrophones, using a stop watch. At the same time a D/F bearing is taken of the W/T signal. Since the velocity of sound through the water is known, the range can be calculated, and the bearing is found by W/T—D/F. Actually great-circle bearings and ranges are obtained by this method, but, owing to the unavoidable small inaccuracies and the short limiting range of operation, it is sufficiently accurate to treat them as mercatorial bearings and distances.

CHAPTER XXVII

TESTING NAVIGATION LIGHTS

Responsibility for Tests.

(1) In H.M. Dockyards the tests are carried out by the staff of the Captain of the Dockyard.

(2) In private yards the Captain Superintendent of Contract-Built Ships is responsible for the tests, and he may order them to be carried out by the navigating officer of the particular ship.

Instructions for Testing Navigation Lights. When navigation lights are tested to ensure their compliance with the *Regulations for the Preventing Collisions at Sea*, the following instructions (taken from the Board of Trade publication *Instructions as to the Survey of Lights and Sound Signals*, 1927) are to be carried out.

OIL LIGHTS

Side Lights. (*Screening abaft the beam.*) The wick or wicks of a side light must be placed at an angle of 112½ degrees with the fore-and-aft line of the ship ; in other words they must be parallel to the direction two points abaft the beam. The burner must be so placed that a line drawn in this direction from the after edge of the wick in a single burner, and of the forward wick in a duplex burner, shall cut the edge of the housing of the lens.

Side Lights. (*Screening forward.*) The screens of side lights, the length of which should never be less than thirty-six inches from the flame to the chock or its equivalent, must always be placed parallel to the line of the keel. The chocking must be so arranged to show a ‘ thwartship value ’ of at least one inch of wick in a forward direction ; that is to say, a person looking past the edge of the chock in a line parallel to the keel must be able to see at least one inch of wick.

Steaming Lights. In a masthead light the wick or wicks must be at right-angles to the line of the keel, and the setting must be such that lines drawn from the centre of the after edge of the wick in a single burner and of the forward wick in a duplex burner, in directions two points abaft the beam on each side, shall cut the edges of the housing of the lens.

Stern Lights. If a fixed stern light is fitted, the wick, which must be a single one, should be set as for the masthead light and so screened that a line drawn from the centre of the edge of the

wick nearest the back of the lantern in a direction two points abaft the beam cuts the edge of the housing of the glass front of the lantern.

NOTE. The screening arrangements for the stern lights should be so constructed that the screen can be removed to give an all-round light in harbour.

ELECTRIC LIGHTS

Side Lights. (*Screening abaft the beam.*) Lamp sockets should be placed in the lantern cases so that a line drawn in a direction two points abaft the beam, touching the forward edge of a circle five-eighths of an inch in diameter, concentric with the socket, will cut the edge of the housing. The centre of the lamp sockets should be placed five-sixteenths of an inch abaft the centre from which the curvature of the lens is struck.

Side Lights. (*Screening forward.*) The screens must be placed in a similar way to the oil side lights described above. The chocking must be arranged so as to show a ‘ thwartship value ’ of at least one inch of filament in a forward direction.

Steaming Lights. (*Screening.*) Lamp sockets should be placed in the lanterns so that lines drawn in directions two points abaft the beam on each side and touching the forward side of a circle five-eighths of an inch in diameter concentric with the lamp socket, will cut the edges of the housing of the lens. The centre of the lamp socket should be placed five-sixteenths of an inch abaft the centre from which the curvature of the lens is struck.

MISCELLANEOUS

(1) **Steaming Lights.**

(*a*) Take care to see that proper fittings are provided for the masthead light on or in front of the foremast—or on an independent stay—and that the lantern will be so fixed that the light is projected in the direction required by the regulations, as described above, and is not obstructed by any fittings, such as derricks housed vertically.

(*b*) The height of the masthead light should be measured from the weather deck (on the fore side of the bridge) through which the foremast passes.

(*c*) Masthead lights should be in line with and in the same vertical plane as the keel. Where this is not practicable the lights should be positioned as closely as possible to the centre line.

(*d*) No glare from either masthead light is to be noticeable on the bridge.

(*e*) The fore steaming light is not to be visible from a point six feet above the eyes of the ship.

(2) **Side Lights : Construction and Position of Screens.**

(*a*) When the screens are of wood they should be well seasoned and not less than one and a quarter inches thick ; the chock should be rounded off, and when the set screw of the cleat is screwed hard up, the back of the lantern should fit closely against the back of the screen, and the side of the lantern should be parallel to the side of the screen.

(*b*) Side lights should be fitted on the bridge-ends whenever possible.

(*c*) The screens are never to be secured to the rigging except in small sailing vessels. When the screens are attached to moveable davits or to outriggers extending outwards over the side of the vessel, they should be fitted with stop pins or distance rods so arranged that when the stop pins are in their places the screens are parallel to the line of the keel.

(3) **Not-Under-Command Lights.** The two red lights, prescribed by Article 4 of the collision regulations to be used by vessels when not under command, must be visible all round the horizon for at least two miles.

(4) **Anchor Light.** All vessels under 150 feet long must be provided with one anchor light, and all vessels 150 feet long or over with two anchor lights. The lantern or lanterns must be constructed so as to show a white unbroken light, visible all round the horizon, for at least one mile.

When two lanterns are carried, it is desirable that they should be of the same description and the light of the same power.

(5) **Jackstaff Light.** The pin light on the jackstaff should be visible from the quartermaster's position at the wheel.

Before Testing.

(1) Mark off the positions of the light boxes from a sketch prepared from the latest *Instructions as to the Survey of Lights and Sound Signals*, issued by the Board of Trade.

(2) Make certain that the ship will not yaw during the trials. It may be necessary to tauten the securing hawsers.

(3) Check the following :

- (*a*) the fore-and-aft centre line of the ship and the squared line through the centre of the filaments of each lamp.
- (*b*) the distance of each lamp from the centre line of the ship.

(4) Erect in a rigid position over each lamp, so that it is convenient for sighting, a standard wood mould with sighting battens on angles of 90 degrees, $112\frac{1}{2}$ degrees and $123\frac{3}{4}$ degrees (two points and three points abaft the beam). Take care to keep the base of the mould on a fore-and-aft line and the mould pitched correctly for the centre of the filament.

(5) Take sights along each sighting batten to points ashore as far as possible from the ship:

(*a*) two and three points abaft the beam from the bow lights.

(*b*) one and two points abaft the beam from the stern light position.

Mark the points with paint, whitewash, or a flag, etc.

(6) Work out the distance at which the side lights should cut-out, ahead of the ship, and having measured this distance with a tape measure, mark the point in the way described in paragraph 5 above.

Note. When the side lights have a ' thwartship value ' of one inch, the distance at which they should cut-out will be equivalent to eighteen times their distance apart, measured along the centre line ahead of the ship from a point midway between the lights.

(7) Hoist the oil steaming lights into position to check the lead of the jackstays.

To Test the Side Lights.

(1) Switch on the lights.

(2) Note that the lights both cut-out at the mark ahead of the ship.

(3) Make any necessary adjustment and subsequently fix the adjustable screens on the fore end of the light boxes.

(4) Check the starboard (or port) light by noting that at the 90 degree mark there is a full light. Proceed to the $112\frac{1}{2}$ degree mark (two points abaft the beam) and check that at this point the light begins to cut-out. Proceed further to the mark three points abaft the beam where the light should be completely cut-out. Make any necessary adjustment and fix the adjustable screen on the after side of the lightbox.

(5) Repeat the procedure for the port (or starboard) light.

To Test the Steaming Lights.

(1) Carry out the procedure for testing bow lights, already described, on each side of the ship.

(2) From a point six feet above the eyes of the ship, check that the fore steaming light is not visible.

(3) From the bridge, check that there is no glare from each steaming light.

To Test the Stern Light. Proceed from the stern, where there should be full light, to the mark two points abaft the beam where the light should begin to cut-out. Proceed further to the mark one point abaft the beam where the light should be completely cut-out. Make any necessary adjustment and fix the adjustable screen on the fore side of the lightbox.

To Test the Jackstaff Light. From the steering position, check that the pin light on the jackstaff is visible.

It is preferable for the tests to be done during darkness, but if this is not possible they can be done in daylight with the aid of binoculars.

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